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[M]r W. P. Shepherd-Barron, Past-President, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the [C]hairman, the thanks of the Division were accorded to the Author.

Hydraulics Paper No. 8

EXPERIMENTS WITH HYDRAULIC MODELS OF PORT LYTTELTON

by

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SYNOPSIS

The Hydraulics Research Station has conducted model experiments on behalf of the [L]yttelton Harbour Board into the behaviour of alternative designs for a new harbour. [T]he proposed harbour is to be built alongside the existing inner harbour and will consist [e]ssentially of a single straight breakwater, perpendicular to the coastline, behind which [sh]ips shelter at their berths. This arrangement was always expected to provide less [pr]otection from storm waves than the existing inner harbour which, except for an opening [.]0 ft across, is totally enclosed. However, it was thought that, in view of the increase [in] the size of ships that has taken place since the existing harbour was designed, the [pr]otection might prove to be adequate.

Two models were built. One was suitable for studying the penetration of storm waves [in]to the harbour, whilst the other was suitable for studying the penetration of the very [lo]ng waves that are likely to cause ships to range at their berths. In the absence of reliable [in]formation on the characteristics of real waves in that area, the suitability of the proposed [ha]rbours was judged by comparing their behaviour with that of the existing harbour.

The Paper contains examples of the two different methods adopted for presenting data

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on the protection afforded by harbours. One consists of a diagram, known as a response curve, that shows how the wave height at one fixed point varies as the wave period varies. The other, known as an iso-wave-height plan, is a plan of the harbour showing the height of waves in every part of it under one set of wave conditions. Examination of many diagrams of both types led to the conclusion that the best of the open-type harbours would be satisfactory. Mention is made of the difficulty of making enough measurements in any wave disturbance model to ensure that the worst condition has been examined. Resonance as a factor affecting the surges in this and other harbours is put in its correct perspective.

An investigation into the dredging that would be needed to keep the harbours open is described in the Paper. Forecasts are based on experiments with the smaller of the two models, in which a mobile bed of china clay was caused to move under the influence of waves and tidal currents. Proving experiments, made with the harbour in its present form, gave silting rates in good enough agreement with the known rates of silting in Port Lyttelton to justify basing forecasts on the results of the model experiments. It is shown that the building of the new harbour would almost double the rate at which dredging now needs to be done.

The present harbour and its environment

Lyttelton harbour is situated half-way up one of the many tidal inlets that are characteristic features of the Banks Peninsula in the South Island of New Zealand. The inlet known as Port Lyttelton is about 8 miles long and about 1 mile wide at the point where it meets the ocean. The aerial photographs in Figs 1 and 2 give impressions of the surrounding terrain.

Port Lyttelton, a plan of which is shown in Fig. 3, shares with its neighbouring inlets the peculiarity of having an exceedingly soft muddy bed lying at a gentle

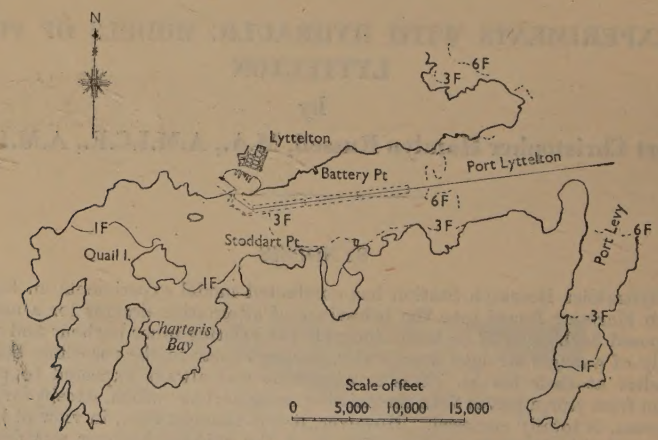


Fig. 3.—Plan of Port Lyttelton

almost straight slope from the exposed beach at the head of the inlet to the 8-fathom contour, where it meets the sea. Towards their seaward ends the inlets are bordered by steep plunging cliffs that meet the soft bed several fathoms below the water level. Cross-sections across Port Lyttelton show a bed that is practically horizontal from cliff to cliff. Both the flatness of the bed and the absence of beaches are factors favouring the penetration of storm waves up the inlet with little loss of height. In

Fig. 1.—Aerial view looking northwards across the seaward end of Port Lyttelton

Fig. 2.—Aerial view of the inner harbour, looking northwards across Port Lyttelton

Fig. 6.—View of larger model with the measurement of generated waves in progress

e absence of a shelving cross-section waves are not refracted towards the sides as rmally occurs in estuaries and, in the absence of beaches, the waves do not expend eir energy in breaking. It is likely that certain waves actually increase in height they travel up the inlet, owing to shoaling of the bed.

The existing inner harbour provides satisfactory protection against wave action. the whole history of the port there has been only one occasion when a vessel has d to leave the wharf for fear of damage due to ranging. With this exception, it s very rarely been necessary even to suspend cargo handling during periods of nge. In view of this very high standard of protection it was thought reasonable the proposed extensions to the harbour to provide a somewhat lower standard; , for example, would be provided by a single breakwater running from a point the northern shore to another, one-third of the way across the inlet.

The present navigable depths in Port Lyttelton are maintained by dredging. om the straight approach channel, the turning basin, and the inner harbour mbined, 600,000 tons are removed annually. Although it was hoped that the nstruction of the harbour extension would not greatly add to the need for dredging me increase in dredging was acceptable, and the cost of dredging was not expected be of paramount importance in deciding which of the alternative extensions should built. The present rate of expenditure on dredging is said to capitalize at a sum approximately £500,000, and this sum is not a large fraction of the cost of building e new extension.

The harbour extensions

The four alternative designs for the harbour extension that were tested by models e shown in plan in Fig. 4.

Scheme A was a design favoured by the Lyttelton Harbour Board as being onomical to build, and yet having the necessary length of wharves and the neces-ry area of reclaimed land for the first stage of development. It was planned at further extensions would be added in later years either by building out jetties rmal to the scheme A wharves or by constructing another similar harbour to the stwards, like the scheme B extension. Scheme A was the first to be tested, and is shown to be a tolerable harbour both from the silting and wave-disturbance pects. The experiments showed that the open type of harbour, offering protection ly from the east, could be satisfactory.

The scheme B extension was next investigated. It was shown to be a bad harbour, ving little protection to ships lying at the wharves.

The experiments then returned to modifications of the scheme A design which, signated KI and KIII, differed from one another only in the length of the main eakwater. KIII, the harbour with the longer breakwater, was found to be very od, offering protection as good as that afforded by the present harbour; whilst the otection afforded by KI was slightly less satisfactory and comparable with the otection afforded by scheme A.

The protection afforded by the various harbours against waves can only be scribed accurately in terms of a mass of data that it is impossible to condense. or making comparisons between the harbours, however, condensed data are neces-ry, which must necessarily involve broad statements of somewhat imperfect curacy. Bearing in mind the limitations of all the following broad statements, e heights of waves to be expected at the wharves of the various harbours could be scribed as follows. Storm waves reaching the quays in scheme A would be half as gh again as those in the existing harbour. The height of storm waves reaching the

quays in the scheme B extension would be three times as great as in the existing harbour. Storm waves in the KI harbour would be slightly higher and in the KIII harbour slightly lower than equivalent waves in the existing harbour.

Protection against local waves

The statements above refer only to waves coming up the inlet from the ocean. They were thought to be the only important ones, and were the only waves fed into the models.

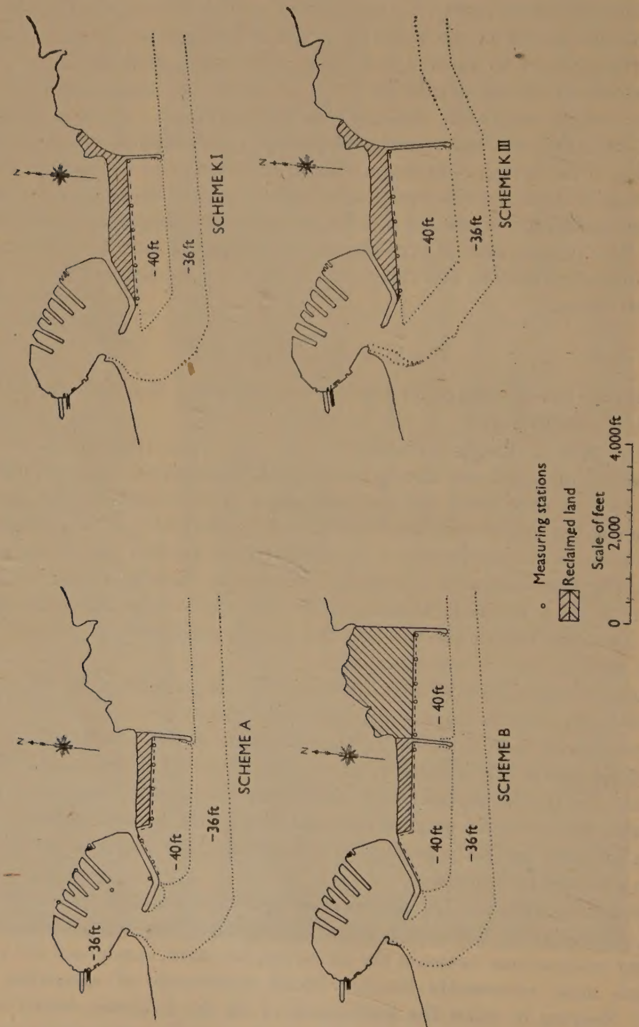


Fig. 4.—Plan of the four harbours investigated

It is implied in the design of all the proposed harbours that no protection is required against any of the waves that come from the comparatively short fetches

the south and west. The predominant wind over Port Lyttelton is reported to from the south-west quarter, and this wind is said to raise waves 4 ft high at the trance to the present harbour on those occasions when it blows strongly over the ıgest fetch, which is from the direction of Charteris Bay.

From the comparative shortness of the fetch one would judge that the waves ıuld be so short compared with the principal dimensions of the ships subjected to əm, that although 4 ft high they would be innocuous. A United States Army ach Erosion Board publication [1] enables the period of waves generated over allow fetches to be calculated, and incidentally corroborates the figure of 4 ft for e wave height in storms. According to these data a wave 4 ft high would be built over a fetch 18,000 ft long in water 15 ft deep—these are reasonable figures the fetch from Charteris Bay—by a wind of 47 knots. Now putting a wind ength of 47 knots into the same set of data one can calculate the wave period at e end of the fetch to be 3·5 sec. In 44 ft of water, which is the mean depth at the posed new berths, a 3·5-sec wave would be 63 ft long. Thus 63 ft is the length the longest waves to which the ships, using any of the new harbours, might be ly exposed.

The models

Two models were built. Fig. 5 shows a plan of the large basin into which they were ed. The smaller of the two reproduced almost the whole of the Lyttelton inlet

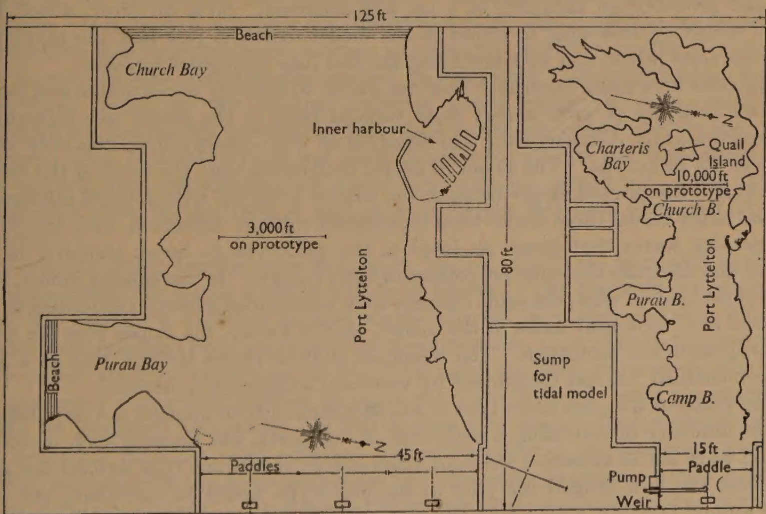


Fig. 5.—Plan of wave basin showing arrangement of models

a horizontal scale of 1/600 and a vertical scale of 1/100; whilst the larger model roduced the immediate vicinity of the inner harbour to a horizontal scale of 1/180 a vertical scale of 1/90.

The horizontal scale of 1/180 was arrived at as a consequence of a desire to include

[1] The references are given on p. 27.

to as large a scale as possible all the bays and headlands that reflect waves towards the harbours, some space for paddle-generated waves to settle down in, and some space for the waves to be slowly dissipated in. If the experiments had been made with an undistorted model the analysis of the experimental results would have been easier, but unfortunately a vertical scale of 1/180 would have resulted in such shallow depths that waves would have been excessively damped by friction as they travelled up the model. It was calculated by a method due to Hunt[2] that if the vertical scale were 1/90 the damping of the waves would be just tolerable. For example, waves of 1-sec period (19 sec in the prototype), travelling from the generator to the harbour entrance a distance of 40 ft (7,200 ft in the prototype) in water 3 in. deep ($22\frac{1}{2}$ ft), would lose 14% of their height. In the prototype the loss of wave height would be negligible.

The horizontal scale of the smaller model (1/600) was selected so as to enable the whole inlet to be reproduced in a length of 80 ft. The same argument, concerning the damping of the model waves in small depths, led to the choice of 1/100 as the vertical scale.

Construction of the models and divergence from perfect scale modelling

The charts from which the models were to be built were contoured in 2-ft intervals, and the areas between adjacent contours were built on the model as horizontal terraces. The steps between terraces were later filled in, so as to make a more or less continuous surface. An accuracy of $\frac{1}{16}$ in. in the levels of the terraces was aimed at, and probably an accuracy of $\frac{1}{8}$ in. was achieved. Such an error, about 1 ft when scaled up according to either of the model scales, is just half the maximum possible error introduced by simplifying the bed into horizontal terraces.

The cliffs were made vertical. This was partly to simplify construction and partly because reflexion of wave energy from vertical faces in the model was thought to be insignificantly different from the reflexion of wave energy from the very steep cliffs of the prototype. The slopes of flatter inclination, e.g., the faces of the breakwaters and the pitched slopes beneath the timber wharves, were treated differently. In the larger model these slopes were reproduced without distortion, with the object of enabling waves that break on them in the prototype to break similarly in the model and dissipate the same proportion of their energy. In the smaller model, with the same object in view, the same features were reproduced distorted according to the vertical and horizontal scales of the model; that is to say, the slopes were six times as steep as in the prototype. The steepness of these slopes is dealt with in a later section entitled "Errors introduced by vertically exaggerated models."

The piling of the piers and wharves was idealized in both models; not only because of the difficulty of providing a multitude of fine wires, but because such procedure would have led to excessive losses of wave energy in viscous flow around the piles. Whereas the flow around the piles in the prototype would be turbulent, the same flow around perfectly scaled model piles would have been viscous, in accordance with the much reduced Reynolds number. By collecting many piles together into a smaller number of $\frac{1}{2}$-in.-dia. piles, which together presented the same total frontal area, the Reynolds number was raised to the transition region between viscous and turbulent flow. In this transition region the coefficient used for obtaining the force between the water and the piles is not much different, say 20% different, from the coefficient used when the flow is turbulent. This may seem to be a considerable error, but if on the other hand the piles had been perfectly scaled down the coefficient might have been excessive by a factor of two or more.

Experiments to determine protection afforded

Little information was available on the heights and periods of waves that reach
ə inner harbour. This was not a serious disadvantage. Whatever had been
ɔorted to be the most common period of the waves, it would still have been neces-
ry to test the harbours against waves of all periods capable of disturbing shipping.
is is on the assumption that waves of all periods are generated at sea, and it is
ly a matter of time before waves of any chosen period arrive at a stated point.
e upper and lower limits of wave period for which the harbours were tested were
nin and 9 sec respectively. Four minutes was selected as the upper limit because
ə fundamental mode of oscillation of any harbour likely to be built would then be
luded, and since the 3–4 min band of periods that is reported to be dangerous to
ipping would also be included. A 9-sec wave was the shortest wave, bearing in
nd the scales of the larger model, that could be reproduced by the machinery.
On the other hand the height of the particular waves fed into the model, so long
the height is known, is not of prime importance and it is reasonable to suit the
ve height to the convenience of the experiments. It will be seen below that the
haviour of the harbours is presented in terms of "amplification factors," or the
io of the wave height at the berths to the wave height at sea. It is implied in
ə method that doubling the wave height at sea serves only to double the wave
ight at the berths and that consequently the amplification factor remains un-
anged. It is implied moreover that the height of the waves fed into the model can
varied to suit the experiment without affecting the resulting amplification factor.
e alternative new harbours were judged by comparing their amplification factors
th those found for the existing harbour, tested under the same conditions.
though it is not strictly true that amplification factors are not affected by variations
wave height, that assumption was made in order to expedite the experiments, and
is unlikely that the experiments would have all been carried out at one strictly
intained wave height, even if such a wave height had been measured at Port
ttelton.

In order to obtain amplification factors the heights of both the generated wave and
wave inside the model harbour had to be known. The height of the generated
ve was difficult to measure because waves reflected from obstacles like cliffs and
akwaters travelled back and upset the measurements, even close to the generator.
ly under artificial conditions in which all reflexions were suppressed, and only
generated waves were allowed to reach the wave recorder, could the generated
ves be measured accurately. It was necessary to assume that when the model
s operating normally with reflected waves travelling in all directions that the
ldle, moving with the same amplitude and the same frequency, would generate
ves of the same height. Fig. 6 shows the arrangement used for excluding unwanted
exions.

n making the measurements of amplification factors at different frequencies the
erated wave height was not held constant. It was more convenient to leave the
plitude of the paddle motion unaltered and to allow the height of the generated
ve to vary with variations of frequency. Tests were made in both models under
artificial conditions described above to determine how the height of the generated
ves varied with frequency and the wave heights so recorded were used later to
ide into the heights of waves measured in the harbours to determine the amplifica-
1 factors.

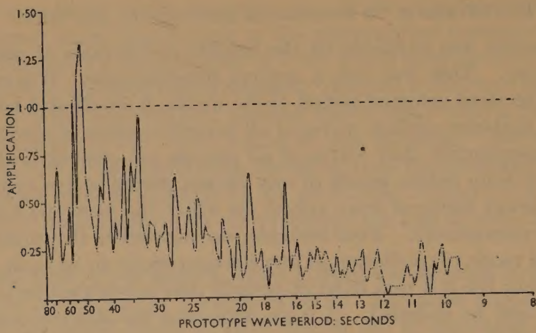


Fig. 7.—Response curve for position (2) in the inner harbour

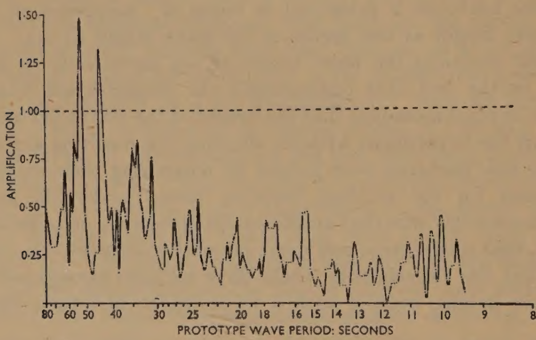


Fig. 8.—Response curve for position (3) in the inner harbour

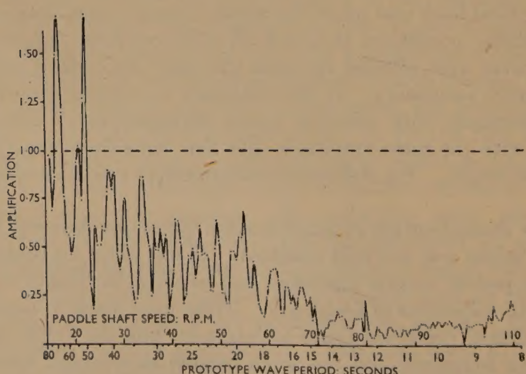


Fig. 9.—Response curve for position (20) in the KIII harbour

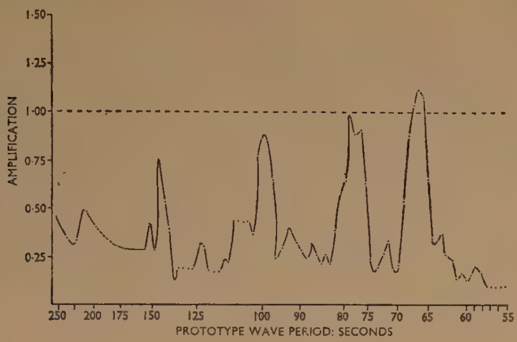


Fig. 10.—Long-wave response curve for position (2) in the inner harbour

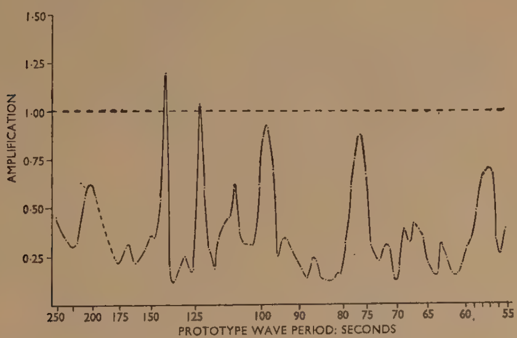


Fig. 11.—Long-wave response curve for position (3) in the inner harbour

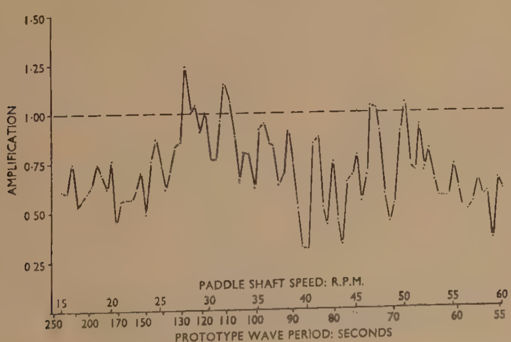


Fig 12.—Long-wave response curve for position (20) in the KIII harbour

The response curves

Figs 7, 8, and 9 are typical response curves, two for a point in the existing harbour and the other for a point in the KIII harbour. The curves relate amplification factors to the period of the waves. A different curve is required for each point in the harbour, for each height of tide, and for each direction of wave approach. The work required to assess the performance of each harbour tested was kept within bounds by taking measurements at only one level of tide (half-tide level) and using only one direction of wave approach. Even so a very large number of measurements had to be made. Fig. 7 represents the results of about 180 measurements and yet contains only half the information required to assess the protection afforded at that one point. Fig. 7 relates only to protection from waves in the 9-sec to 80-sec band of wave periods, this being the band that could be reproduced in the larger model. For assessing the protection afforded against waves having longer periods of up to 4 min the smaller model was used, in which another whole set of measurements were made. Figs 10, 11, and 12 are the response curves for the same three points, as determined by measurements in the smaller model.

It is seen immediately from the response curves that the amplification factors vary widely for quite small variations of wave period, and there is no reason to believe that in this respect the prototype differs from the model. It would, however, be impossible to verify this behaviour in the prototype because there the waves are continually varying in height from one wave to the next and, instead of being of one precise period, the waves have a spectrum of wave periods.

When these irregular response curves were first obtained it was thought that the peaks of amplification were caused by resonance and that every period at which a peak was found corresponded with one of the natural periods of oscillation of the harbour. Some further experiments, however, indicated that resonance was seldom responsible; the experiments indicated that maxima were found when two or more waves coming from different directions coincided in phase, and that minima were found when the waves travelling from different directions cancelled each other out. The experiments are described in a later section of the Paper.

It will be readily understood that it is difficult to make simple direct statements comparing the protection afforded by two harbours when the comparison has to be based on sets of response curves like Fig. 7, especially when there is a considerable difference between the six curves within each set as, for example, between Figs 7 and 8 and between Figs 10 and 11. In the report to the Lyttelton Harbour Board on the KIII harbour the following statement was ventured: "the harbour offers better protection than the existing inner harbour against waves in the 8–16 sec group. The protection against waves in the 16–80 sec group is less good; the waves being about half as high again as in the inner harbour." Such statements necessarily omit many exceptions and in this respect are not accurate.

It is rather easier to compare the protection afforded by the various harbours against very long waves, because of the different form of the response curves. The curves in Figs 10, 11, and 12 and most of the other response curves relating to very long waves fluctuate on either side of a mean that is much the same at one end of the period scale as at the other. The mean is also much the same from one point to another in each harbour. This has enabled us to compare the harbours by comparing the mean amplification factors for each one. The mean amplification factors were found to be:—

About 0·4 for the existing inner harbour,
About 0·8 for the scheme A harbour,

About 1·0 for the scheme B extension.
About 0·7 for the scheme KIII harbour.
KI was expected to be similar to scheme A and was not tested for very long waves.

A second method of describing the wave heights inside the harbours, a method mplementary to the method of response curves, was that of iso-wave-height plans. ese plans show pictorially areas of high and low amplification in the harbours under e set of wave conditions, and are necessary if one is to ensure that particularly bad eas in the harbours have not been overlooked by a fortuitous choice of points at ich response curves have been obtained. For instance in Fig. 13, which is an iso-ve-height plan for the scheme A harbour, two regions of high waves can be seen.

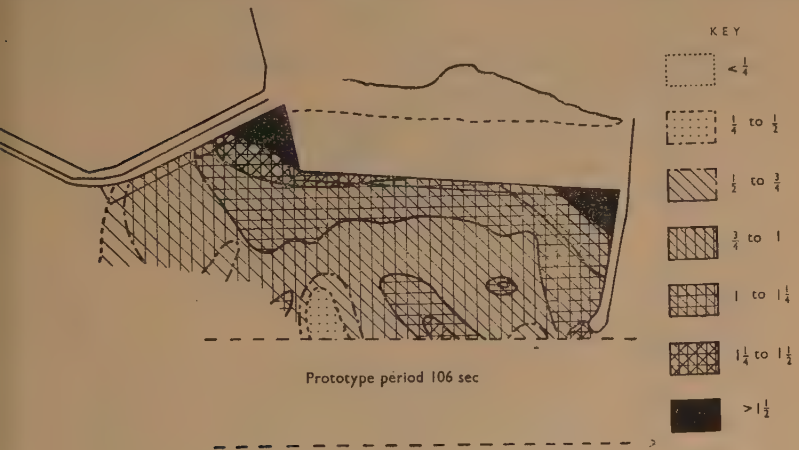


FIG. 13.—AN ISO-WAVE-HEIGHT PLAN OF SCHEME A.
(All wave heights are expressed as a fraction of the generated wave height)

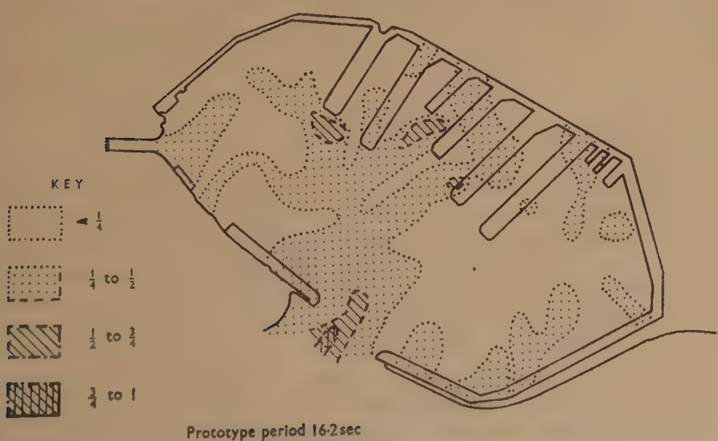


FIG. 14.—AN ISO-WAVE-HEIGHT PLAN OF THE INNER HARBOUR

The high waves in the re-entrant nearer to the existing harbour had previously avoided detection because the apex of the re-entrant had not been chosen as one of the six measuring points. Yet it was found that the iso-wave-height plans, relating to many different conditions of long waves, consistently showed high amplifications here, and the existence of this region of high waves was one of the reasons for rejecting the scheme A harbour.

About ten iso-wave-height plans were prepared for each harbour, half of them by experiments in the larger model and half in the smaller model. They were obtained by traversing a wave-height recorder across the model harbours on a number of parallel lines. The recorder, travelling at a speed of 2 ft/min, made a continuous trace of the fluctuations of the water surface and simultaneously left marks on the record from which the position of the recorder along its traverse could be deduced. In this way a complete picture of the wave height in every part of the harbour could be built up. The heights of waves inside the present harbour and the heights of waves inside the KIII harbour, when the wave periods are 16·2 sec and 19·1 sec respectively, are shown in Figs 14 and 15.

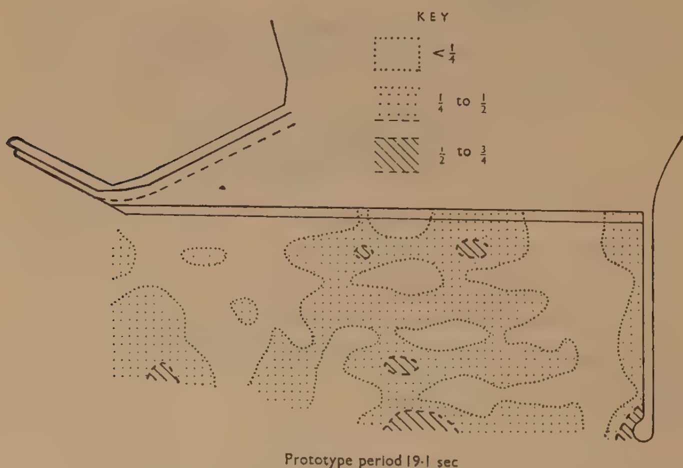


FIG. 15—AN ISO-WAVE-HEIGHT PLAN OF THE KIII HARBOUR

An unusual iso-wave-height plan is shown in Fig. 16. In the investigation of the scheme B extension the response curves showed the harbour to be very unsatisfactory, and consequently further work to establish a number of iso-wave-height plans would not have been justified. It seemed a pity, however, to miss the opportunity of recording the iso-wave-height pattern in so simple a shaped harbour, because only if the shape of the harbour was very simple was there any chance of the pattern being predictable. It must be admitted that none of the patterns obtained up to that time had been predicted; nor when the patterns were obtained could they be explained. In a rectangular harbour like scheme B the disturbance was expected to be built up by resonance whenever the length of the waves was a simple fraction, namely, 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and so on, of twice the distance between the breakwaters.

ıree maxima in the wave-height pattern would be expected, one at the centre and ıe at each end, when the wavelength was exactly equal to the distance between e breakwaters. This condition, which could be known as the first harmonic, is the ndition under which the pattern shown in Fig. 16 was obtained; it will be seen that does agree fairly well with the pattern that was expected.

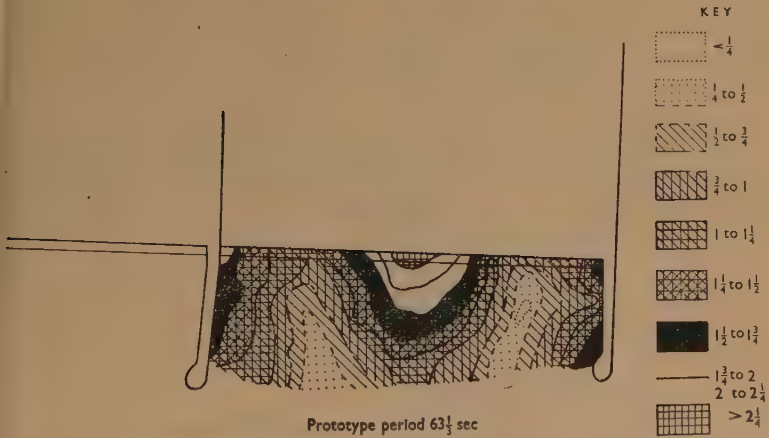


Fig. 16.—Iso-wave-height plan of scheme B. The first harmonic

The defects of the scheme B harbour can be readily appreciated on observing that re are waves more than $2\frac{1}{4}$ times as high as the waves at sea.

erpretation of peaky response curves

ıpart from information on the suitability of the various projected harbours, what erges from the experimental work described above is the extreme sensitivity of he harbours to minute changes in wave period. They were found to be similarly itive to small changes in the level of the water at which experiments were made, nges in water level of as little as $\frac{1}{100}$ in. in the models being sufficient to render lts unrepeatable.

ome experimental work was done in the 600 : 100 model of the scheme A harbour letermine its sensitivity to small changes in wave period and small changes in er level. The resulting iso-wave-height diagrams are compared in Fig. 17. h diagram shows the heights of waves in an area a little greater than the new that would be dredged for scheme A, the height of the waves being represented he thickness of the lines. Reading from left to right one can see the effect of ng the water level by 0·02-in. intervals (2 in. in the prototype), whilst reading top to bottom one can see the effect of increasing the wave period by increments %.

the original assumption is adopted, that maxima on the response curves are ed by various parts of the harbour resonating in phase with the generated wave, it follows that all the experimental results are of little value because they were d under inappropriate conditions. With the period of the waves in the proto- forever changing, and with the water level varying continually with the state

of tide, conditions in the harbour might never be steady long enough for resonant modes of oscillation to become established.

Some experiments were obviously needed in which the tide level was varied according to the correct model time-scale, and other experiments were needed in which the period of the waves was varied continuously.

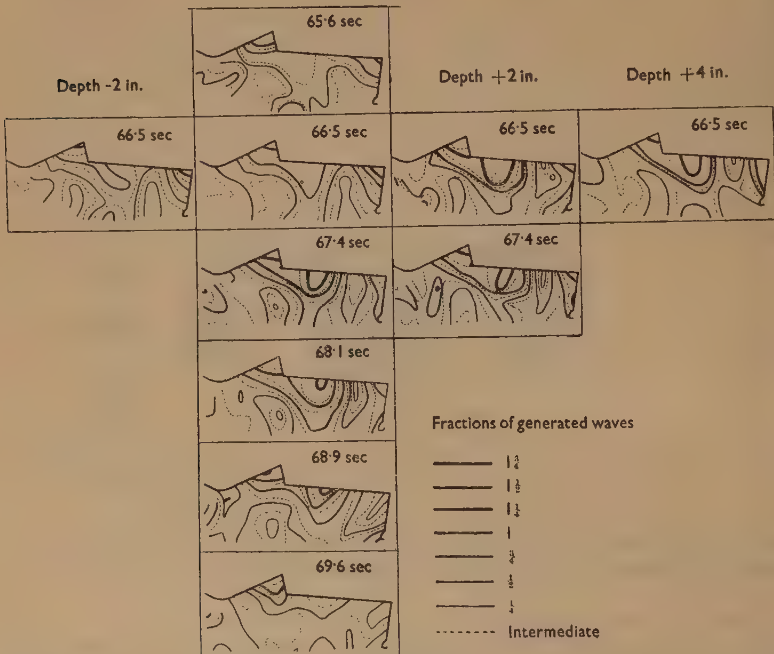


FIG. 17.—TEN ISO-WAVE-HEIGHT PLANS OF SCHEME A ARRANGED TO SHOW THE EFFEC OF SMALL CHANGES IN WAVE PERIOD AND SMALL CHANGES IN DEPTH OF WATER

Experiments with the scaled Lyttelton tide led to a response curve that was ju as peaky as the response curve for the same point, found with a stationary water leve It showed that the resonances—assuming that the peaks were caused by resonances— were not suppressed by the scaled tide. The phenomenon was further investigate by making the same experiment with an accelerated tide, having a period of on one-sixth of the correct tidal period. The response curve found by plotting tl wave height that was measured at the instant of half-tide on the rising tide w different from, but just as peaky as, the curve plotted for a constant water level.

Whether or not resonances are suppressed by a fluctuating water level must deper upon three factors; the time that a mode of oscillation takes to become establishe the least change in water level giving rise to a change in mode of oscillation, and t rate of rise of tide.

The time that the modes of oscillation took to become established is indicated the records reproduced in Fig. 18. They show the movement of the water surfa starting from rest at the instant when the first wave entered the model harbour, a

give some indication of the time taken for an oscillation to become steady. The time is about 20 min in the prototype.

The least change of water level giving rise to a change in mode of oscillation is shown by Fig. 17 to be 2 in., prototype.

The maximum rate of change of water level in a 7-ft sinusoidal tide is $\frac{7\pi \times 12}{12{\cdot}4 \times 60} \approx 0{\cdot}35$ in. per min, prototype.

At this rate of rise a change in level of 2 in., sufficient to alter the mode of oscillation, would occupy 5·7 min or much less than the 20 min that are required to establish

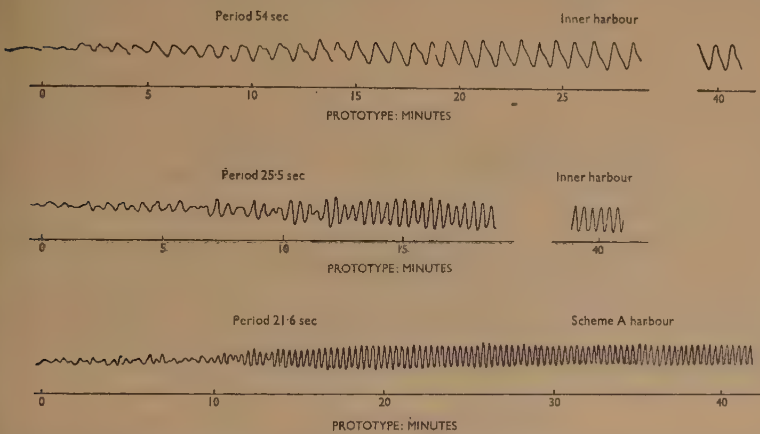


FIG. 18.—WAVE RECORDS TAKEN IN THE 180 : 90 MODEL; SHOWING THE GROWTH OF WAVE HEIGHT FROM THE INSTANT WHEN THE FIRST WAVE WAS SEEN TO ENTER THE UNDISTURBED HARBOUR

the mode. With the tides accelerated six times, there would be even less chance of resonant modes of oscillation becoming established. The continued existence of a peaky response curve, even when the tide was accelerated, suggests therefore that the resonances were not responsible.

Experiments were also made to see if fluctuating the period of the waves would suppress what appeared to be resonances, giving smoother response curves. In the 180 : 90 model the speed of the wave generator was fluctuated over a range of 4 r.p.m. every six waves, and in a later experiment 4 r.p.m. every four waves. The wave heights inside the harbour were found to vary from one wave to the next, repeating every six, or in the case of the second experiment every four waves, as shown by the wave records in Fig. 19. The response curves found in these experiments were different from, but as peaky as, the corresponding response curves for constant wave periods. Since in both experiments resonances can have had little or no chance of becoming established, the conclusion had to be drawn that peaky response curves were not caused by resonance.

One is forced to the conclusion that small changes in wave period and small changes in water level give rise instantly to changes in the pattern of nodes and antinodes; that maxima on the response curves are found when there happens to be an antinode at a measuring station, and that minima are found when there happens to be a node

at that station. In the meantime the maximum amplitude of the oscillation m
remain unaltered. Further reference to Fig. 17 confirms that sharp fluctuations
amplitude for small changes in wave period or water level are adequately explain
in terms of changes in the pattern of nodes and antinodes.

There is no doubt that certain harbours do resonate and, in particular, that t
model of the scheme B extension did resonate, when the length of the disturbi
waves was equal to the distance between the breakwaters. The Author wish

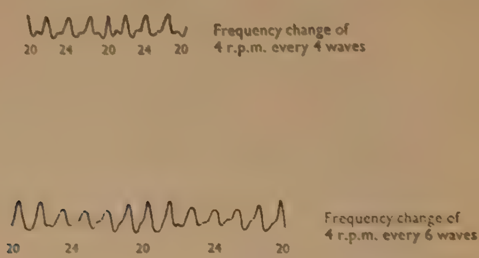


Fig. 19.—Wave records taken when the period of the generated waves was continuously fluctuating

however, to make the point that peaks on a response curve are not evidence t
the harbour in question will resonate.

Errors due to unequal scales

Errors introduced by vertically exaggerated models

One of the errors introduced by the unequal scales lay in the imperfect reproducti
of the waves viewed in plan. Under certain conditions the shape of the mo
waves, viewed in plan, changed as they travelled up the model in an exactly simi
manner to the prototype waves; but this was not usually the case. When the wa
were very long indeed compared with the depth of water—so long that the sp
of the model waves was $\sqrt{g \times \text{model depth}}$ and the speed of the prototype wa
was $\sqrt{g \times \text{prototype depth}}$—there was a constant ratio between the speed of
two waves, and they would have refracted similarly in travelling over simila
shaped sea-beds. When, however, the waves were not so long compared with
depth of water, and their speed was dependent on both the depth of water and
length of the wave; then in the distorted Lyttelton models there was no const
ratio between the speeds of the model and prototype waves. The error was negligi
in the larger model for all waves of 20-sec period or longer, but the error began to
important when the waves were of 10 sec or less. For example, if a prototype wa
of 10 sec were reproduced by a model wave of period $10 \times 180/\sqrt{90}$ sec in the 180
model, the velocity scale in any very shallow water was $1/\sqrt{90}$ or 1/9·48. But
10 ft of water the velocity scale would have been 1/10·1, in 20 ft of water 1/1
and in 36 ft of water 1/12·3.

A typical effect of the inconstancy of the velocity scale is that on the shape o
wave crest which enters the inlet as a straight line when viewed in plan. As it tra
up the inlet the part of the wave travelling over the dredged channel moves ah

of the parts travelling in shallow water on either side of it, so that after a certain time the plan view of the wave crest has become a substantially straight line with a hump in the middle. In the model the same changes occur but because the velocity scale in 36 ft of water, which is the depth of the channel, is smaller than the velocity scale in 20 ft of water, the depth on either side of it, there is less difference between the wave speed over the channel and the wave speed elsewhere; so that the bump that develops in the plan view of the crest is smaller. The effect on the heights of waves inside the harbour is not known.

Although the smaller model had a greater distortion, namely, a distortion of six, the errors introduced were smaller because the model was used only for a study of extremely long waves. The shortest wave that was studied had a prototype period of 55 sec.

Variability of time scale

Similarly, except when the waves are extremely long, there is no constant ratio between the period of the model wave and the period of the prototype wave that are of correctly scaled wavelengths. For example, a 1-sec wave in the 180 : 90 model is a scale model of a $180/\sqrt{90}$ or 18·97-sec wave in any very shallow water; but it is a scale model of a 17·58-sec wave in 44 ft of water. The model wave represents a slightly different prototype wave in every different depth of water.

The difficulty here is mainly one of definition; what prototype wave should be said to be similar to a 1-sec model wave—the one which is similar in very shallow water, or similar in very deep water, or in some other depth of water? It was decided to use the depth of water at the wharves for defining the period of the waves.

Reflexion and absorption of wave energy by sloping boundaries

There was uncertainty concerning the errors introduced by too great or too little reflexion of wave energy at sloping boundaries.

There is experimental evidence [3] to show that in the case of long waves in shallow water the important parameter determining reflexion coefficients is the ratio:

$$\frac{\text{the horizontal length of the slope from bed to water level}}{\text{wavelength}}$$

and consequently that that parameter should be the same in a distorted model as in the prototype. It involves steepening all slopes, correctly according to the horizontal and vertical scales of the model, and this is what was done in the construction of the 600 : 100 model.

On the other hand short waves in deep water require different treatment. They are not affected by the position of the bed and cannot be affected by "the horizontal distance from the bed to the water level." The reflexion coefficient of a slope acting on deep water waves can only be dependent on the steepness of the slope and the steepness of the waves. On the assumption that the slope was the more important parameter, the slopes in the 180 : 90 model were reproduced undistorted.

Perfect reproduction of reflexion coefficients in distorted models probably requires a different slope for each type of wave, varying both with the depth/wavelength and wave height/wavelength ratios. However, it is doubtful whether, even if the subject were completely understood, it would be worthwhile going to this trouble. It would be more convenient to recognize the error and make a calculated allowance for it. At the present moment we cannot do this.

THE INVESTIGATION OF SILTING

One of the objects of the investigation was to determine how much dredging would be required to keep the new harbour and its approach channel at the prescribed depths.

At first sight it looked as if these open types of harbours would silt up at a considerable rate. The Author's view was that when there were any waves coming up the inlet from the sea material would be put into suspension everywhere except in the sheltered region to the west of the main breakwater, and that water flowing through this sheltered region would deposit its suspended load there. Treating the problem as purely one of wave action one would expect the area to the west of the breakwater to be one of heavy accretion, because storm waves in that region would be losing height most rapidly. Alternatively, treating the problem as purely one of tidal flow, one would expect a clockwise eddy to form on the flood tide in the area to the west of the breakwater and this eddy to give rise to accretion at its centre.

However, Mr J. A. Cashin (Engineer-in-Chief to the Lyttelton Harbour Board) considered that the material on the bed was so fine that predictions based on the normal mechanisms of silting and bed movement might not apply. For example the rate of fall of individual particles through water might be so slow that few particles would have time to settle out of suspension in the period when the water containing them was passing through the harbour. Mr Cashin held that after a storm the bed of the harbour was covered with a thick layer of liquid mud, which only slowly consolidated. The flow of this liquid mud was at the heart of the problem.

In the model the material that was used to simulate mud had similar characteristics to the material on the bed of Port Lyttelton, and it is believed that the mechanism by which the bed built up was the same in the model as in the prototype. There was evidence that the flow of a heavy muddy layer along the bed played an important part in the changes that took place in the model.

Forecasts based upon model experiments predicted that the dredging that would be necessary to keep open the various alternative harbours would be a little greater than the dredging now being done. The harbours compared as follows:—

> The present depths in the harbour and its approaches are maintained by dredging a total annual quantity of 604,100 tons.
>
> The scheme A harbour, which among other modifications involves increased depths in all existing dredged areas, would require dredging at the rate of 1,057,400 tons annually.
>
> The corresponding figure for the scheme A harbour with scheme B added would be 644,300 tons.
>
> The corresponding figure for the KIII harbour would be 886,800 tons.
>
> The corresponding figure for the KI harbour was not found by experiment because it was thought to be insignificantly different from scheme A.

Reproduction of the mobile material

The investigation of silting was carried out in the smaller model, using the same scales of 1/600 horizontally and 1/100 vertically that were used for studying the behaviour of very long waves. The choice of a mobile material was made bearing in mind the scales of the model, the rate of fall of model particles in water, and the rate of fall of prototype particles in water. It can be shown that particles in

model that are being carried along by a stream, as they settle out of suspension, can deposit in the correct place only if the speed of fall of the model particles is $x/y^{3/2}$ of the speed of fall of the prototype particles; where $1/x$ is the horizontal scale and $1/y$ the vertical scale of the model. It would, therefore, be desirable for particles in the Lyttelton model to fall at 0·6 of the speed of fall of the prototype particles.

A great many measurements were made by Mr Bushell of the Lyttelton Harbour Board's engineering staff on the particle size of bed samples taken from many different places in the inlet. The analysis showed naturally a random variation from place to place, but in addition there was a consistent difference between the samples taken from the dredged channel and the samples taken elsewhere. The latter were on the whole ten times as coarse as the channel samples. All were very fine.

Some typical results of the analysis are shown in Fig. 20, where they can be

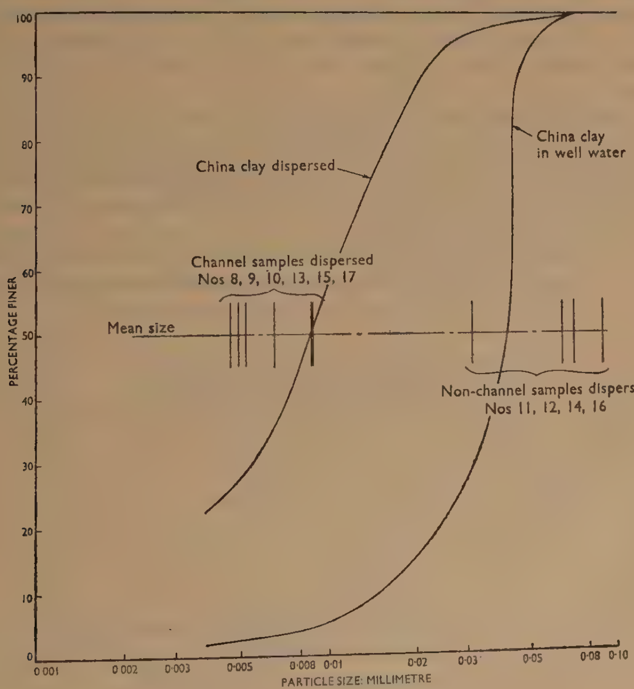


FIG. 20.—PARTICLE-SIZE DISTRIBUTION CURVES FOR CHINA CLAY AND MEAN PARTICLE SIZES OF LYTTELTON SILTS

compared with two size distribution curves relating to the model material. So as to avoid confusion only the mean sizes of the Lyttelton samples have been plotted, but these are quite adequate to show the difference between the channel samples and the other samples. Whilst the mean size of the channel samples is grouped around 0·006 mm, the mean size of the other samples is grouped around 0·06 mm.

These mean sizes relate to the material when dispersed. The rate at which particles actually fall in the water of Port Lyttelton depends on the particle size of the material when flocculated in sea water; just as the rate of fall of the model material depends

on the particle size when the material is flocculated in water from a well. The method of analysis used by the Lyttelton Harbour Board, the A.S.T.M. hydrometer method, was not suitable for an analysis of flocculated material; but some idea of its behaviour was obtained by another method. A sample of the mud was shaken up with sea water in a measuring cylinder and observed while it settled. After a few seconds observation revealed a thin layer of clear water at the top of the fluid, which became thicker as time went by. The rate of fall of the interface between clear and turbid water was recorded.

It was found that the interface fell at a rate that depended on the concentration, when the concentration was high; but that at low concentrations, below 20 g in 1,000 cu. cm of water, the interface fell at a rate that was independent of concentration. It was assumed that the rate at which the interface fell at low concentrations was the rate at which the smallest flocculated particles fell through clear water. The same test, performed on the material used in the model, would not enable a comparison to be made between the mean sizes of the two flocculated materials but a comparison could be made between the finest fractions of the materials. This was of some value.

The material that was used in the model was a fine china clay for which the size distribution curves dispersed and flocculated are shown in Fig. 20. It was fortunate that china clay is a material that is available graded according to particle size for this made it possible to select the most suitable grade, rejecting some as too fine and others as too coarse.

Some results of the tests on the settlement of china clay in well water are shown in Fig. 21 for comparison with a typical curve relating to the settlement of mud from

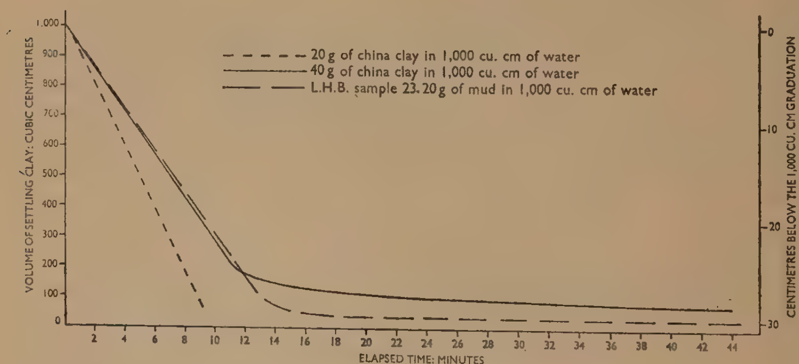


FIG. 21.—THE SETTLEMENT OF SUSPENSIONS IN A 1,000 CU. CM MEASURING CYLINDER

Port Lyttelton. The ordinate on the curves records the graduations on the side of a 1,000-cu. cm measuring cylinder down to which the clear-water turbid-water interface had fallen after certain intervals of time. The sample from Port Lyttelton is one from the dredged channel. Although ideally particles of china clay should fall rather more slowly than particles of mud from Port Lyttelton, it will be seen that in fact, when the concentrations are the same in both cases, the china-clay particles fall rather more quickly. Both speeds are of the order of 2 cm/min. Fig. 21 also shows that the speed of fall of the china clay could be reduced to that of the Lyttelton

mud sample by doubling the concentration. The material was as close to the ideal material as any that could be found, and in view of the large variation in particle size from place to place in Port Lyttelton the material was thought to be as close to the ideal as was necessary.

An important property of the Lyttelton material was well reproduced by the china clay. It was the property of very slow compaction to the solid state, after deposition from suspension. When the settlement test on the mud sample No. 23 was done—i.e., the sample to which Fig. 21 relates, but the property was common to all samples—it was found that the deposit at the bottom of the cylinder behaved as a fluid for 1 hour after the start of the test, although it appeared to have finished settling after 20 min. When the test was repeated with 50 g of mud instead of 20 g, the deposit at the bottom remained fluid for 4 hours.

This fluid mud would be capable of flowing down gentle inclines and could well be responsible for silting in the dredged areas of Port Lyttelton, as was suggested by Mr Cashin.

Method of operating the model

It was found early on in the trial experiments that china clay, laid with a trowel, was not eroded by model waves of any reasonable height, nor by the tidal currents, nor by the two combined. The maximum velocity available for eroding the bed was about ½ ft/sec. For example, 10-sec waves in the prototype, 10 ft high, would scale down to 1-sec waves, 1·2 in. high, and the maximum velocity of the bed beneath these waves would be 0·52 ft/sec in water, the model equivalent of 25 ft deep. The tidal currents in the model were very slow, of the order of 0·1 ft/sec.

However, it was found that model waves would move the material if it had been deposited from suspension. It would remain mobile for 2 days after deposition, by which time compaction had gone far enough to make it inerodable.

The rigid model was converted into a mobile-bed model by depositing upon it a uniform layer of china clay, only 0·25 in. thick, and then raising the water level by the same amount. Eventually a technique was evolved whereby a complete bed could be laid, accurate to 0·05 in., in a period of 3 hours. Changes that took place during the experiments were measured by probing the thickness of the clay layer before and after each experiment. The thickness of the layer was recorded in hundredths of an inch, which was not unreasonable since it was found that the readings of a practised experimenter were repeatable to $\pm \frac{1}{100}$ in. The tides in the model were a somewhat idealized version of the tide in Port Lyttelton; being of constant 7-ft range, midway between springs and neaps, and of sinusoidal shape. They were generated by pumping continuously ½ cusec into the model while simultaneously allowing a variable quantity, sometimes more than ½ cusec and sometimes less than ½ cusec, to flow back from the model into a sump. The varying flow was controlled by a balanced weir that was raised and lowered by a cam, the cam having been cut to a calculated profile. It was rotated by a tiny constant-speed electric motor at a speed of one revolution in 12 min, which was thought to be insignificantly different from 12·4 min—the theoretically correct tidal period for the chosen scales of the model.

The characteristics of the waves fed into the model were found by trial and error. They were chosen so that silting in the model agreed as well as possible with the silting of the prototype. For scaling up the model wave so chosen, not only the height of the waves and the depth of water but also the length of the waves was assumed to scale up according to the vertical model scale of 1 : 100. Only if this were done

could the water-particle velocities in the waves scale correctly, and scale similarly to the velocities of the tidal currents. When the vertical scale was used throughout all velocities in the model were 1/10 of those in the prototype.

Nineteen proving experiments, each under different conditions, were made with the harbour in its present form. The object of the proving runs was to find the best wave height, the best wave period, the best number of model tides for which the experiments should run, and to find the best way of replacing in the model the material lost from it during the ebb tides. The model was judged to have reproduced the silting correctly when the right proportions of the total silt deposited were found to have deposited in the right places.

The five sectors of the inlet for which the quantities of material were known are shown in Fig. 3. They are the inner harbour, the turning basin, and three parts of the straight approach channel. The quantities of material taken from each of these sectors between October 1947 and September 1951 were taken to be typical, and these quantities were converted into figures that showed the average thickness of the deposits that would accumulate in each sector in 1 year, if they were left undisturbed. In Fig. 22 the thicknesses of the deposits are shown by the full line to be 0·32 ft in the inner harbour, 0·79 ft in the turning basin, 1·48 ft in the upper reach of the approach channel, 1·18 ft in the middle reach and 0·63 ft in the lower reach of the approach channel.

Also shown in Fig. 22 (by the broken line) are the thicknesses of the deposits that were found after completing the most successful of the proving experiments. The

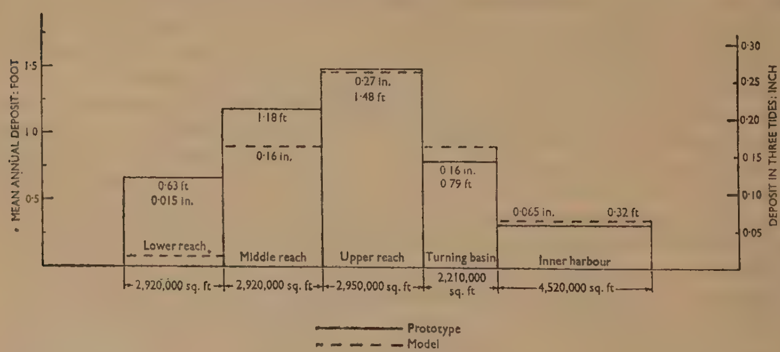


FIG. 22.—SILTING IN FIVE SECTORS OF THE MODEL COMPARED WITH SILTING IN THE SAME FIVE SECTORS OF PORT LYTTELTON

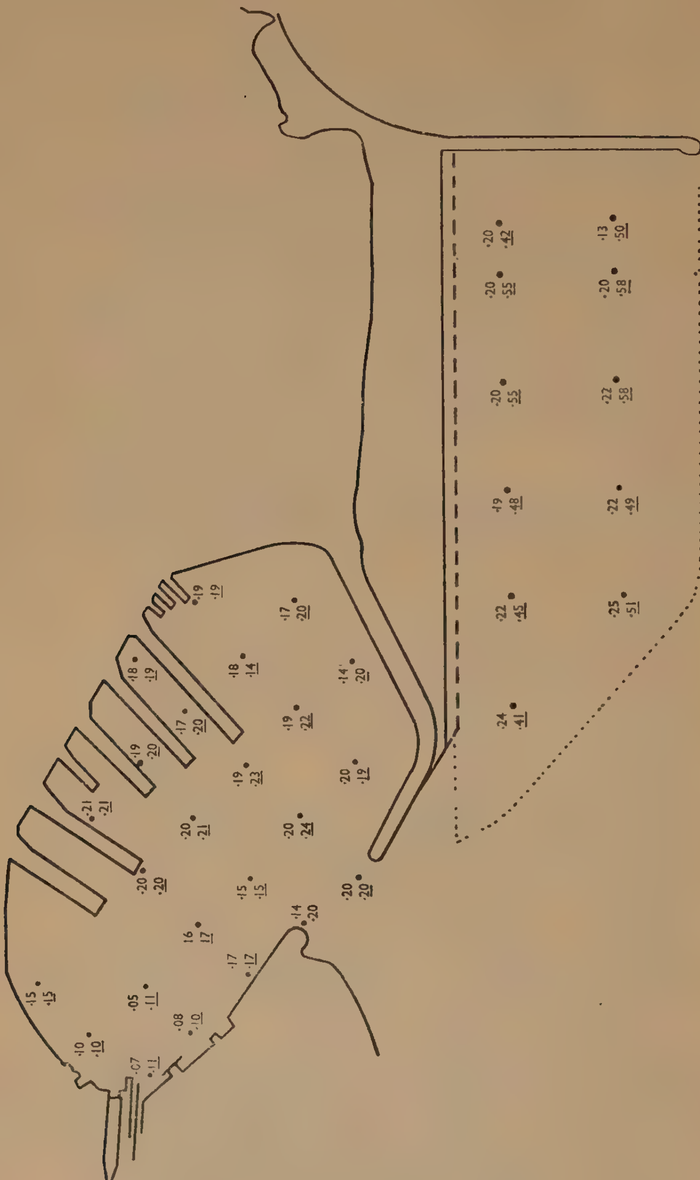
model deposits are plotted to a scale which was intentionally manipulated so as to bring the model deposits up to the size of the prototype deposits. The best scale was found to be:

$$\frac{\text{The deposit in inches occurring over 3 tides in the model}}{\text{Annual deposit in feet in the prototype}} = \frac{1}{5 \cdot 5}$$

This was the scale that was used in all subsequent experiments and the one used to scale up the quantities found by experiment in models of the new harbours.

TABLE 1.—THE PREDICTION OF THE DREDGING REQUIRED TO KEEP THE KIII HARBOUR AND ITS APPROACHES AT SPECIFIED DEPTHS. BASED ON MODEL EXPERIMENTS, BUT CORRECTED FOR MODEL ERRORS

Tons per year	KIII harbour	Inner harbour	Turning basin	Upper reach	Middle reach	Lower reach	Total
Dredging known to be done at present (T)	—	68,700	81,700	205,000	161,800	86,900	604,100
Predicted by model for KIII harbour (T_K) = 5·5 (Model deposit in inches) × (Prototype plan area) × $\frac{62 \cdot 5 \times 1 \cdot 68}{2,240}$	339,000 = 5·5 × 0·307 × 4,280,000 × 0·0468	−3,500 5·5 × − 0·003 × 4,520,000 × 0·0468	14,800 5·5 × 0·026 × 2,210,000 × 0·0468	178,000 5·5 × 0·186 × 3,707,000 × 0·0468	176,400 5·5 × 0·234 × 2,920,000 × 0·0468	83,600 5·5 × 0·111 × 2,920,000 × 0·0468	788,300
Predicted by model for present harbour (T_p) = 5·5 (Model deposit in inches) × (Prototype plan area) × $\frac{62 \cdot 5 \times 1 \cdot 68}{2,240}$	— —	75,800 = 5·5 × 0·065 × 4,520,000 × 0·0468	93,500 5·5 × 0·164 × 2,210,000 × 0·0468	203,000 5·5 × 0·267 × 2,950,000 × 0·0468	122,000 5·5 × 0·162 × 2,920,000 × 0·0468	11,300 5·5 × 0·015 × 2,920,000 × 0·0468	505,600
Corrected prediction for KIII harbour $T_{KIII} + T - T_p$	339,000	−10,600	3,000	180,000	216,200	15,920	886,800



SILTING IN THE INNER HARBOUR AND K III HARBOUR

(Enlargement at A)

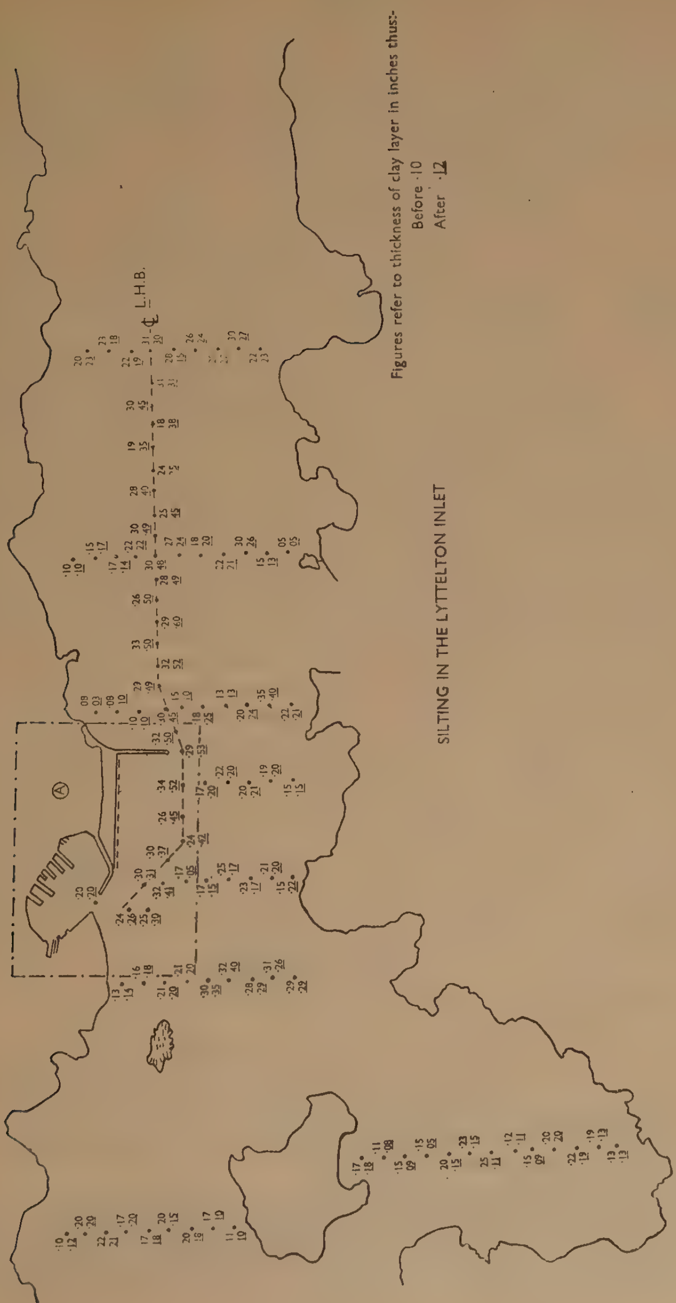


FIG. 23.—A TYPICAL CHART SHOWING ACCRETION AND EROSION IN THE MODEL OF KIII.
(Silting experience No. 2. Comparison of surveys before and after the experiment)

The conditions of the most successful experiment and the conditions under which the new harbours were tested were as follows:—

The slurry of china clay in water from which the beds were laid was of 1·35 specific gravity.

The beds were left for 1 day in which to settle and partly consolidate.

Tides were run without any waves until the range of tide had settled down tc 7 ft. This did not cause movement of the bed.

Waves were then generated for three tidal periods starting at high water. Waves and tides were stopped simultaneously.

The period of the waves was 1·2 sec, 12-sec prototype scale.

The waves were 0·11 ft high off Stoddart Point, 11 ft prototype scale.

To replace the material that was lost from the model on the ebb tides materia was injected on the flood at the rate of 5 gal of slurry, of 1·16 specific gravity each tide.

Two or sometimes three similar experiments—they were repetitions as far as wa known—were made on each of the proposed harbours, and the average was taken o the results. The changes that took place all over the model area were recorded on a chart, of which Fig. 23 is an example relating to the KIII harbour. Althoug changes in depth in every part of the inlet were of some interest, in that one coul speculate endlessly as to whether or not they represented changes that really too place during storms in Port Lyttelton, only changes in the dredged sectors wer employed in subsequent calculations.

From two charts, Fig. 23, and the chart relating to the repetition of the experiment the average accretion in each of the dredged sectors of the model was found an presented in the form shown in Fig. 24. Then, using the 5·5 scale relation derive

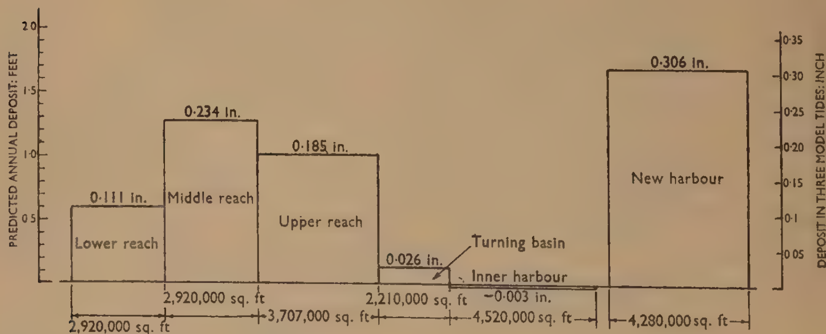


Fig. 24.—Silting in the various sectors of the KIII harbour, according to the model

earlier, it would have been easy to get a figure for the silting of the prototype. practice an adjustment was made at this stage.

Reference back to Fig. 22 reveals that in the proving experiments with the harbo in its present form there was a consistent trend in the difference between the silti of the prototype and the silting of the model when scaled up. The silting of t model was more than it should have been at the upper end of the channel and w increasingly deficient towards the lower end of the channel. For this reason pr

dictions were not based simply upon the scaled-up results of the model experiments; but instead on the results of the model experiments, assuming them to be in error by the same amount, as they were found to be in error by the proving experiments. In Table 1, the quantities of material deposited in various sectors of the inlet, if KIII were built, have been calculated upon this assumption. The information required for the calculation is the dredging now done in Port Lyttelton, the dredging erroneously predicted by this model as now being necessary, and the dredging predicted by the model as being necessary to keep KIII open.

The corrected figure for the annual dredging quantity in KIII is 886,800 tons.

The mechanism of silting in the model

There were several indications that the material in the model moved from place to place largely in the form of a heavy fluid or turbidity current. The first piece of evidence was the behaviour of a settled suspension of china clay at the bottom of a measuring cylinder. The clay flowed like a fluid for some hours after settling and there was no reason to suppose that clay settling out of suspension in the model would have behaved differently.

Secondly, there was the extreme flatness of the bed of the model wherever material had deposited during an experiment. For example, it can be seen from Fig. 23 that in the area to the west of the main breakwater the slope of the bed is no more than 0·1 in. per ft. Even at the tip of the breakwater where, had the bed been composed of sand a scour hole would have been found, the bed had every appearance of being horizontal. This is consistent with the filling up of the scour hole by an inflow of fluid clay from the surrounding area as soon as the wave action stopped.

It was not possible to observe whether there was a flow of fluid clay along the bed because, as soon as waves were started up all the water in the model became milky-white and opaque. The third piece of evidence is, however, fairly direct; an attempt was made to determine what proportion of the material that was deposited in the inner harbour came in along the bed and what proportion was deposited from suspension. For this purpose, a small scale-pan was suspended at a point ½ in. above the bed throughout a silting experiment; it was expected to collect most of the material that settled on it from above, but none of the material that came along the bed. When the scale-pan came to be examined a smear of clay so thin that it was transparent was all that was found on it, whilst beneath it material was found to have deposited in a layer 0·15 in. thick. This was taken to be good evidence that material entered the inner harbour as a turbidity current along the bed.

References

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3. H. J. Schoemaker and J. Th. Thijsse, "Investigations of the reflection of waves." Proc. 3rd Int. Ass. Hyd. Struct. Res., Grenoble, 1949.

The Paper, which was received on 5 August, 1955, is accompanied by twenty-one sheets of drawings and three photographs, from which the half-tone page plates and the Figures in the text have been prepared.

Discussion

The Author introduced the Paper with the aid of a series of lantern slides.

Mr J. A. Cashin (Engineer-in-Chief to the Lyttelton Harbour Board), in a writte contribution read by the Secretary, suggested that it might be of some value to harbou engineers faced with problems at all similar to those at Lyttelton to know something o the situation and events which had led to the hydraulic model investigations describe in the Paper.

The province of Canterbury, New Zealand, had been settled about 1850, and the existin harbour at Lyttelton had been built between 1862 and 1867 to accommodate the sma vessels of that day. The port was separated from the plains of Canterbury by steep hill about 1,000 ft high; those had been originally part of the rim of a volcano of which Por Lyttelton was the centre.

The harbour [4] had been enclosed by two rubble breakwaters totalling 3,200 ft in length and as trade developed wharfs and jetties had been constructed until at the present tim the length of deep-water berthage was 11,000 ft and the harbour shell was full. Th volume of trade was still increasing and it was obvious that further extensions of berthag had to be built outside.

In 1936 a scheme of enlargement had been prepared—essentially a repetition of th pattern of the existing moles and so placed outside them as to enlarge the enclosed area The length of proposed moles was 7,000 ft.

When the scheme had been devised the knowledge of wave behaviour and the technique of investigation by hydraulic model had been much less advanced than they were today It had been felt, no doubt, that to repeat the forms of the existing breakwaters would wit safety cover the need to protect the harbour from storm waves. That scheme had bee considered by the Board on many occasions subsequent to 1936, but, on account of th great expense involved, it had never been put in hand.

Since its inception the Board had been able to provide extensions of berthage, addition shipping facilities, and so forth without difficulty, but any additional berthage no involved the building of new and expensive protection against storm waves.

The existing channel, which had a length of about 2½ miles and a low-water depth o 32 ft, required about 25 years' dredging to cut it to its present depth and its maintenanc at constant depth now cost about £30,000 per annum. The 1936 scheme would hav necessitated the abandonment of that channel and the cutting of a new one by intensiv dredging over as short a period as possible, for the existing channel would have had to b maintained until the new one was ready for use.

When Mr Cashin had become Engineer-in-Chief, it had appeared to him to be of th utmost importance to review the whole question of extensions with a view to reducin the cost to the minimum.

The most urgent need of berthage had been for the longest vessels, and having in min that the average registered tonnage of vessels using the port in 1862 was 810, it had seeme likely that protection against waves from the south-west would not be necessary for moder deep-sea vessels. Consideration of the fetch in that direction and the wind velocity an duration had shown that the waves, observed to have a maximum height of 4 ft, woul be of such short wave-length as not seriously to inconvenience large vessels. It had there fore been decided to abandon the proposed western mole and to use the new berths f only the larger vessels during the infrequent period of south-westerly gales.

There was no doubt that protection was needed against waves from the east. Althoug the present harbour was 5 miles from the sea, long swell waves, occasionally up to 10 high, reached the western breakwater. Measurements of wave-heights by an ech sounder in a launch had shown that at times waves travelling up the inlet from the hea actually increased in height as far as Purau Bay, where some energy was abstracted b refraction. The maintenance of wave energy during the run up the harbour was due,

tated in the Paper, to the extreme flatness of the bed in cross-section; slopes rarely xceeded 1 : 5,000, except at the extreme edges. The longitudinal slope was 1 : 1,000.

If the length of a new eastern breakwater was such as not to interfere with the existing hannel, i.e., if it projected from the northern shore only as far as the northern edge of the xisting channel, then a very considerable amount of work and money would be saved. t was decided that the new wharf should be built about 500 ft from the existing shoreline o allow sufficient room for wharf apron, shed, roadway, open space, and railway marshalling acilities.

Knowledge of wave action against a wharf and breakwater of the type thus envisaged ad been until comparatively recently a matter of empiricism and experience on the part f the designer, but recent researches (largely connected with war-time requirements on he North African and Normandy coasts) now enabled an assessment to be made on a heoretical basis. Reference to information and Tables contained in United States Beach Erosion Board Special Issue No. 1, 1st July, 1949, had indicated that, with the arrangement nvisaged (Scheme A), wave conditions along the berth would be satisfactory from the reakwater to a position about 2,000 ft west. However, the radical departure of the ew proposals from the existing form of harbour, and from anything hitherto envisaged, ad made it highly desirable to obtain confirmation of the behaviour to be expected. Furthermore, a saving of about £2,000,000 of the £2,500,000 hitherto thought necessary or protection had appeared to be a possibility well worth the expenditure of an estimated £10,000 for site and model investigations. Apart from any consideration of reduction of ost, where large expenditure was involved it was essential to take all possible steps to nsure that the completed scheme would function satisfactorily. However, concerning he particular proposal for Lyttelton Harbour, if, in the light of experience gained in peration, increased protection was thought desirable, it could be gained by extending the nole to any desired extent, as for instance, from KI to KII. That would necessitate novement of the channel to the south, a work which could be carried out easily (and very heaply when compared with the cost of cutting a new channel) by increasing the rate of redging and concentrating on the southern side.

Mr Cashin had first considered having a model of Lyttelton Harbour constructed in Canterbury University College (N.Z.) a project which the Professor of Civil Engineering vould have welcomed, but upon looking into the matter further he had realized that the cience and practice of that subject had advanced since he had been engaged in it, to such n extent that it was desirable to enlist the services of a specialist. He had therefore onsulted Sir Claude Inglis, who was then building the Hydraulics Research Station at Wallingford, and a mutually satisfactory arrangement had been concluded. Putting that art of the work in the hands of a specialist did not permit the harbour engineer simply to it back and await results. It was important that engineers contemplating having their roblems investigated should be aware of the amount of preliminary and contributory vork which had to be carried out on the site under investigation.

For a model to operate correctly it should faithfully represent the prototype in all elevant respects, and only the harbour engineer could supply the necessary information o enable that to be done. He should, therefore, some time before enlisting the services f the hydraulic model specialist, make extensive and intensive investigations into the istory and behaviour of the prototype, a project which might take years to complete.

Referring to the model under consideration, in addition to the charts and drawings of iled structures, pitched slopes and so forth, the required data had included wave heights, irections, and periods, and the length of waves at the mouth of the sea inlet and at places equiring protection; also whether seiche or range was prevalent at the berths. In conexion with dredging it had been necessary to determine and tabulate over as long a period s possible, the amount of dredging carried out in the harbour and channel, and to show ow it varied in different parts of the channel, also where the spoil had been dumped. Concerning the regime of the harbour it had been necessary to know the type of material orming the bed in the different areas, where the material had come from (sea or land) and vhether the regime had been maintained by tidal action, wave action, or both. The

methods adopted to determine those factors were outlined briefly in the Paper. Th work had been extremely interesting and although given every assistance by the depar ments of engineering, geology, and biology, at Canterbury University College, and l the N.Z. D.S.I.R., it had covered a period of 2 years continuous work and had been start more than 12 months before the construction of the model had been put in hand.

Whilst, as the Paper showed, it was possible so to adjust scales and so forth as to obta reasonable predictions of wave behaviour in a projected harbour, he had always had son reservations concerning the accuracy of predictions of erosion and accretion, because the diversity and number of the factors concerned. Obviously the correct scale repr sentations would not be maintained for all those factors and it was necessary, therefor to effect the best possible compromise. Unless a model had been shown, during t proving period, to be able to repeat a known change of bed regime following some know change, such as the introduction of a harbour work, he would have no faith in its abili to forecast bed behaviour in a projected harbour. He had, therefore, asked that, addition to reproducing the channel-siltation pattern, an experiment should be carri out to repeat an occurrence of some years previously when, owing to a reclamation w slipping outward, a large mound of mud had been raised about 15 ft above the prevaili bed level. It had been restored to its original level by natural forces in the course of few weeks. Accordingly the experiment had been carried out—with complete success.

The model experiments on Scheme A had indicated that protection against storm wav would be rather better than the theoretical investigation had suggested. It had appear also that the embayment at the western end was a bad feature. Thus Scheme K h been evolved as a modification of Scheme A, KI being that with the minimum length breakwater, and KIII that with the breakwater extended if found desirable for t reasons given earlier.

Mr Cashin felt it no exaggeration to say that the present state of knowledge and t available techniques of hydraulic model investigation were such as to enable engineers set out maritime schemes which until recently would have been deemed to be taking undue risk in the interest of economy.

He paid tribute to the happy relations which had existed throughout the investigatio between the Research Organization and the Harbour Board, and ended by quoting Osbor Reynolds, the originator of hydraulic models, who after making but two models in 18 had said:—

> "This method of experimenting seems to afford a ready means of investigating a determining beforehand the effects of any proposed estuary or harbour works; a mea which, after what I have seen, I should feel it madness to neglect before enteri upon any costly undertaking."

Mr Gerald Lacey (Consultant, Drainage and Irrigation Adviser, Colonial Offi said that although 40 years ago the Paper would have been regarded as highly theoretic today, owing to the very welcome spread of knowledge of model behaviour, it was i mediately recognized as a valuable and eminently practical Paper.

In carrying out model experiments of the type described by the Author it was frequen the practice to have two models, one large and one to a smaller scale, and to refer to t smaller model as a "pilot" model. That was hardly a correct description of the Autho models, because they had been devoted to two different purposes, the larger being clusively a short-wave model and the smaller a long-wave model. In the first model th had been short waves and since the depth did not enter the wave equations it had be desirable to make the model as geometrically similar as possible. On p. 6 the Author h stated that if the experiments had been made with an undistorted model the waves wo have been excessively damped by friction, whilst calculations had shown that with t distortion actually used—i.e., with a horizontal scale of 1:180 and a vertical scale of 1:9(the damping of the waves had been just tolerable. The first model was nevertheles short-wave model, and the exaggeration of the vertical scale had had no significance ot than its function in reducing the damping of the waves to reasonable proportions.

The case of the second model was quite different. In a tidal model with long waves exaggeration was not only permissible but desirable; all experience of tidal models with mobile beds had shown that exaggeration was essential if silt movement was to be reproduced. In the model under discussion the scales had been 1:600 horizontally and 1:100 vertically, and in his introduction the Author had referred to a simple relationship in terms of the scales of the model, which determined what the terminal velocity of the silt particle in the model should be. On p. 18 the Author had condensed a large subject into three or four lines by saying: "It can be shown that particles in a model that are being carried along by a stream, as they settle out of suspension, can deposit in the correct place only if the speed of fall of the model particles is $x/y^{3/2}$ of the speed of fall of the prototype particles; where $1/x$ is the horizontal scale and $1/y$ the vertical scale of the model." Reference had been made to that equation some years previously by Professor Gibson[5] and more recently by Professor Jack Allen.[6] Referring to exaggeration in the vertical scale of tidal models, Professor Allen had stated that according to the theory of Mr Lacey,[7] y should be equal to $x^{2/3}$, and in an earlier chapter in the same work, had referred to the complication which inevitably arose when the vertical scale of the model was exaggerated. Professor Allen had quoted the ratio $x/y^{3/2}$ and had stated that the speed of the particle would have to be made "faster" according to that ratio (it was possible that he had severe exaggeration in mind), whereas in the case of the present Author's model the Author had had to make it "slower."

From the way in which Professor Allen had put forward that equation it might be inferred that whenever the vertical scale was exaggerated the same complication arose, but it did not arise if $x/y^{3/2}$ were made unity. In that case it was possible to use the same material as in the prototype, and since $y = x^{2/3}$ the required exaggeration was uniquely determined from the equation:

$$\frac{x}{y} = \frac{x^{2/3}}{x} = x^{1/3}$$

It was of interest that the equation in question gave exactly the same value for the exaggeration of the vertical scale as Mr Lacey had suggested 25 years ago. It had not occurred, apparently, to Professor Allen or to the Author to consider the case in which that ratio $x/y^{3/2}$ was made equal to unity. In the Author's tidal model, with a horizontal scale of 1:600 and a vertical scale of 1:100, if the horizontal scale had been made 1:1,000 it would have been possible to employ sea-water and the same material in the model as in the prototype. If the relation quoted in the Paper was correct it applied to much more than the mere deposition of silt during a slack period; it applied to all the mobile particles at any time when they were in suspension or saltation in the water. Since the Author had referred to that point, Mr Lacey could not resist the temptation to diverge a little from the subject of the Paper and to suggest that the implications of making the ratio $x/y^{3/2}$ equal to unity certainly deserved examination.

In the matter of the correct reproduction of silt movement there was a little difficulty, in the long-wave model. It was true that the Froude number was the same in the model as in the prototype; but in the one case, however, there was associated with that Froude number a rather finer silt than in the other, and therefore, although the Author had arranged for his rate of fall to be correct, Mr Lacey thought that the finer material might possibly be moved more readily, and therefore in greater quantity. He did not say that that was a criticism of the results, because great care had been taken to prove the model and compare, by applying a suitable ratio, what had happened in the model with what had actually occurred in the prototype.

As for the rest of the Paper, in the first model, with short waves, there was very little distortion. The Author was to be congratulated on the results which he had obtained, and one could only conclude that the harbour eventually decided on was undoubtedly the best one in all the circumstances.

Mr E. I. Loewy (Senior Engineer, Sir William Halcrow and Partners, Consulting Engineers) made a strong appeal to all harbour authorities to keep the most comprehensive records possible of local data concerning waves. A model investigation such as

that described in the Paper could offer little hope of accurate predictions unless such information was available. Such an investigation also required a great deal of time patience, and experience if it was to give useful results.

The type of basic site information required was difficult to get and called for a great dea of patience; the instrumentation was still rather crude. It was, moreover, very easy t be misled (particularly if the theoretical knowledge was incomplete) by subjective visua impressions. For works with which Mr Loewy had recently been concerned a ver interesting instrument had been acquired. The measure of its success could not yet b stated because it had only just been installed, but he understood that it had been use successfully abroad. It was made by the French National Hydraulics Institute and con sisted of a self-contained pressure-type recorder which was sunk in the sea at the desire location; after a predetermined time it was brought up and the record developed an analysed. The use of such an instrument should be encouraged as much as possible. I was not cheap, but it was robust and quite objective.

The Author was fortunate not to have been bothered with the measurement of wav direction, which was usually important. There was apparently as yet no instrumen which would measure it, so that one had to rely on photographs and observations; but great deal of patient looking at the sea and making notes was possible for engineers in th field and ought to be done. In the present case the Author had been able to exclude al directions but one. That was rarely the case, and it was usually necessary in such mode work to decide what wave directions should be chosen, with very little field informatio upon which to base the decision.

Mr Loewy had been very surprised to learn that the short-wave model described in th Paper was to a distorted scale. He had always understood that for energy models, o wave models other than very long-period models, scale distortion should be avoided a all costs. It seemed always that the range of reasonable horizontal scales tended, with th size of the average harbour, to produce insufficient depth of water, in an undistorted mode A method of reducing the viscosity of water (other than by heating it), or the use of som other cheap fluid which had a reduced viscosity would avoid that critical problem. Per haps there was some chemical substance which might help to reduce viscosity as easily a surface tension could be reduced by the introduction of detergents. Even without scal distortion there were enough uncertainties.

The existence of those uncertainties had been well illustrated by the Author, for at th beginning it had been felt that resonance was the explanation of the peaky nature of th response curves, whereas later it had been realized that the real reason was the haphazar coincidence of waves reflected in many different directions. It seemed to Mr Loew therefore, that every method other than distortion should be considered, and if necessar the client should be told that the model work had to take longer. In the present case might have been possible—he put that forward as a suggestion, not as criticism—tha instead of running the two models together one could have been made first and the othe afterwards, so that the scale might have been perhaps about 1:120, which might just hav been acceptable in both directions. On p. 16 the Author had mentioned a particul aspect of uncertainty arising from scale distortion which presumably he had had to leav as there stated, because it was beyond the present range of knowledge. It seemed a pit

With regard to the third paragraph on p. 17, it would be interesting to know what th effect would be on the relevant response curves and iso-wave-height lines of taking different definition of the prototype wave. Mr Loewy wondered whether the "amplif cation factor" should not be called an "attenuation factor," because it was to be hope that the height of the wave in the sea would be attenuated in the chosen harbour rath than amplified.

The Author referred to the limitations of the iso-wave-height lines, which was ve interesting. It was clearly a very fruitful way of showing the results, but, as he had sai hundreds of such patterns were needed to define the conditions of the harbour complete Since there was thus as yet no complete answer to that "presentation," Mr Loewy wonder whether some kind of statistical evaluation might not be possible, for picturing t

state of the harbour as a whole. At present, even if one was prepared to carry out hundreds of those tests, there would still be the problem of interpreting hundreds of charts. The commissioning engineer was at a loss when such a mass of data was presented to him and looked for some way of regimenting and simplifying it.

The Author had mentioned that because of the limitations of the apparatus the wave-heights were not made constant for the different periods for any one run of tests. It would be interesting to know whether it was possible to be completely satisfied that that did not introduce unknown errors, because otherwise one would feel inclined to say that somehow the apparatus should be devised—it was largely a matter of mechanical devices—which would make it possible, by setting dials rather than by the laborious method of unfastening cranks and eccentrics, that for every change of period there was a corresponding change in the throw of the wave-maker, so that wave-heights did remain constant. Mr Loewy wished to ask whether, among the various tests, at least one should not be carried out with the maximum known wave-height in the prototype; in fact, even further, a test, using waves as near as possible to the known maximum wave-height, and with the complex prototype character, to calibrate the response curves against the complex conditions which applied in reality.

Could the Author give some particulars of the apparatus used for measuring wave-heights?

Mr F. H. Allen (Assistant Director, Hydraulics Research Station, D.S.I.R.) had been greatly interested by the Author's remarks about silting and turbidity currents. In certain portions of the Thames Estuary a similar material had been found on many occasions. The main body of water in the area in question, between Barking and Tilbury, had perhaps 500 to 1,000 parts per million by weight of solids in suspension; but under certain conditions near the bed a fluid mud layer was found, and an analysis of samples taken from that layer had shown that the concentration of material in it might range from 30,000 to 100,000 p.p.m. by weight. An interesting point was that in the Thames that material could be detected by an echo-sounder under certain slack-water conditions. Not a great deal could be said about the fluid mud itself yet, but apparently under slack-water conditions the upper boundary was fairly clear-cut and was capable of giving a subsidiary echo, which showed on the echo-sounder chart as a faint shadow above the hard line of the bed. That had been apparent for some years in the Thames, and it would be interesting to learn from the Author whether or not similar traces had been found on the echo-sounder charts presumably used at Port Lyttelton.

Mr Allen's second point concerned the behaviour of the turbidity currents in the model. He presumed that the density scale was 1:1, and in that case he would be interested to know whether any figures were available for the concentration of solids in the fluid mud layer in nature, and similar figures for the concentration in the model. It might not be possible to give that information because he realized that relatively little information was available on the turbidity current in nature.

Turning to the measurement of wave-heights, he had heard of a method known as "the starry sky" for recording wave disturbances in models, though he did not know very much about its use. The method used by the Author was, it was understood, somewhat tedious, taking a very long time and requiring a great deal of patience. It involved the measurement of wave-heights at a great many points and along a large number of sections. The "starry sky" method involved suspending a grid of small lights above the model. If the water-surface were still, the reflexion of the grid was undistorted, but when waves were present there was a highly distorted reflexion. If photographs were taken of that reflexion when waves were present, the photographs could be analysed to reveal wave-height information of a kind which he understood the Author wanted in connexion with Lyttelton Harbour.

On p. 6, when dealing with the collecting of piles together, the Author had stated:—

"Whereas the flow around the piles in the prototype would be turbulent, the same flow around perfectly scaled model piles would have been viscous, in accordance with

the much reduced Reynolds number. By collecting many piles together into a smaller number of $\frac{1}{2}$-in.-dia. piles, which together presented the same total frontal area, the Reynolds number was raised to the transition region between viscous and turbulent flow."

Could he be more explicit on that point and explain exactly the lines on which he had calculated the enlargement required in the piles?

On p. 12 it was stated:—

"In a rectangular harbour like scheme B the disturbance was expected to be built up by resonance whenever the length of the waves was a simple fraction, namely, 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and so on, of twice the distance between the break-waters."

Would the Author explain why twice the distance between the breakwaters had to be introduced there, rather than the actual distance?

Dr L. J. Murdock (Manager, Central Laboratory, and a member of the Board of Management, George Wimpey & Co. Ltd) said that in reading the Paper he had tried to place himself in the position of a harbour engineer to whom the Author was trying to sel the idea of building a model. At first he had tended to be lulled, by the able and concise way in which the Author had dealt with the subject, into the belief that a model would perhaps give him a very good picture of the conditions that were likely to occur in a harbour which was to be built, but then he had become rather worried by the large number of uncertainties which seemed to exist in model-making for hydraulic work, and, like a previous speaker, he had been particularly struck by the apparent lack of data from Lyttelton harbour itself by which the model could be corroborated.

One point which had struck him especially was that with the distortion at present necessary in model-making it might be possible to distort the model so that, while it gave wave-heights and wave conditions corresponding to the actual harbour at certain points such correspondence might not apply generally, leading to misleading interpretations.

Dealing further with those apparent uncertainties, there was an exaggerated vertical scale, a time scale, the extreme sensitivity to prototype period, and the use of well-water instead of sea-water. There were also the questions of the bed material, with its possible different deposition characteristics, scale effects, flow characteristics, and so on, wave reflexion from all the points in and around the harbour, absorption of wave energy, and possibly others which the Author had not mentioned, but which according to other investigators might have some influence, e.g., surface tension effects and the effect of dus on the water surface. Would the Author say whether he agreed with others on tha point?

All those uncertainties had been mentioned, and possible errors of 10 or 20% had been indicated at various points in the Paper, so that one wondered just how large the cumulative error could be. It would obviously be wrong to add all those percentages together but presumably if a statistical analysis was made one might get a figure of 60% or more.

Were the models likely to be substantially correct in seven cases out of ten, or something of that order, or was it more or less? Some assessment of the risk of failure would be valuable knowledge, because more damage could be done to model-making in future by unexpected failures than by the acknowledgement of difficulties in that respect and of the fact that the right answer would not always be given.

Were the problems which had arisen during the model study of Lyttelton harbour being made subject of further research, in order that future models would benefit from the results of the present one?

The time required for such model investigations might be a source of worry to the harbour engineer; it seemed that the scientists might require anything from 6 months to 2 years to get results while his ships and harbour were waiting; so that there was certai to be some pressure to get the work done. A result would be wanted even if some accurac had to be sacrificed for the sake of speed. In view of the other uncertainties, there migh not be any harm in a certain amount of "short-cutting" to speed up the work.

In spite of his apparent doubts, Dr Murdock felt that model-making would prove of increasing value in the future, and he wished the Author and the Hydraulics Research Laboratory success in their work.

Mr A. M. Hamilton (Consulting Engineer) observed that the Author's model was not the first model of Lyttelton harbour. An earlier model had been made by himself and Mr White-Parsons, and they had been in the happy position, unlike the present model-makers, of being able to examine the real harbour as they did their model experiments. The work had been done at the request of the late Mr Cyrus Williams,[4] then Chief Engineer and one of the great harbour engineers of modern times, who had believed that by means of models he could find out what was necessary for his harbour. The present model was the result of the ideas which he had passed on to his successors. In the earlier model they had tried every period there could be, with regard to both wave-height and wave-length, and had done the same class of complete investigation as had been done by the Author, though they had not had the same accurate means of finding out the movements at the different parts of the model harbour.

Under Mr Williams the work on that earlier model had been done in a very thorough way, though in a very short time. The scale of the model had been much the same as that of the present one, 1:600 horizontally and 1:120 (instead of 1:100) vertically. He exhibited photographs of the earlier model in action, which were so similar to those which the Author had shown that it would be difficult to distinguish between them; yet the Author on the one hand and he and Mr White-Parsons on the other had tackled the difficulties of the model in different ways, and both seemed to have succeeded to a certain degree in overcoming them. He spoke particularly at the moment of the relative heights of the two scales. Various speakers had referred to them and said that perhaps it was impossible to get the exact answer if the difficulty of the scale height could not be overcome. The question had been raised of getting a fluid which would act correctly at a true scale height.

The reflexion of the waves from the sides of the model had been mentioned. He and Mr White-Parsons had seen that the reflexion of the waves was greatly disturbing the surface. They were in the happy position of being able to climb a hill and look down on the harbour when certain wave-lengths were coming in, and could say "We must get it like that" and had tried to do so. Their answer had been to attach wire netting all round the sides of the model and put ends of rope tow into the water; that had had a damping effect. Whether it had been completely effective or allowed a certain amount of reflexion he would not like to say, but it certainly gave nearly the correct answer, and the photographs which they had obtained were, as he had already said, almost the same as those which the Author had shown.

The object which Mr Williams had had in view had been precisely the same as that envisaged at the present time to study harbour modifications and reclamations. They had tried the effect of a mole in different positions, but Mr Williams had not had so much money to spend in those days, and he had wondered if he could more cheaply lessen the effect on the ships and anchorage when certain seas were coming in at certain wave-lengths which caused damage to be done and moorings to be broken, so that some ships would not use Lyttelton. He had carefully aligned his main jetties to face the opening. The Author had shrewdly referred to that, although he had not seen Lyttelton. The object of aligning the jetties in that way was to have the motion of the water always longitudinal and not transverse to the jetty. Mr Williams had been so certain of that principle, proved both in the model and in the actual harbour, that he had given Mr White-Parsons the task of making a deep-sea jetty at Tolaga Bay,[8] which was one of the longest jetties leading out into open water in a very open harbour; for 25 years that jetty had been usefully employed by deep-sea ships without any of them suffering damage. They had been able to enter and leave in quite rough water. Mr Hamilton had also written a Paper[9] (unpublished) on his researches and was pleased to see that many of the points which had been raised in the present discussion had been investigated in that early model.

He would like to make one contribution to the work which had been done at Walling-ford. When experimenting with his model, Mr Hamilton had observed a rather curiou thing, namely, that the water at certain stages could go in and out through the entranc in what appeared to be a seiche. Upon looking into the story of seiches he had found tha many were known, but the one at Lyttelton was different. He had, however, deduced simple formula $t = K\sqrt{\frac{\text{depth}}{\text{area}}}$ in which t denoted the time of oscillation and K was a con-stant of an average value of 1·97 for the model or actual harbour. As the horizonta and vertical scales were altered, those two factors, area and mean depth had to be brough into it. Upon applying the formula he had discovered that the real harbour should b repeating the phenomenon of the model seiche at a slower speed. The seiche had a perio of 13 sec in the model, and the period in the real harbour should be about 12 min. The had checked by standing at the end of the Gladstone pier and observing the movemen of seaweed. The seaweed period or main harbour period had in fact been 12 min ! I had not affected the action on ships in the harbour, and he merely gave it as an instanc of the almost uncanny accuracy which models could show.

The seiche had not been known of before because it had not been apparent on the tid gauges then in use, but the Harbour Master had told them that some years earlier an ol tide gauge which had a more open scale had been in use. He had found for them som of the old tide-gauge records, which showed the ordinary tidal range going up and dow but also showed clearly that at certain periods there was a definite oscillation in and ou Mr Hamilton displayed one of the old tide-gauge charts, and on it could be clearly see the 12-min rise and fall. There were not sufficient records to indicate whether th oscillation was in any way dependent upon the weather, or what caused it.

Mr Hamilton believed that much could be done with the dynamic effects of water o model structures and thought that the future of model work was very promising; he di not believe that there was any limit to what could be ascertained. Mr Loewy had men-tioned the need for an exact fluid. Mr Hamilton thought that either the exact fluid woul be obtained or methods would be known to be adequately accurate without having to fin some magic fluid to do the job. One could possibly alter g by centrifugal means, surface tension say by vibration and viscosity by temperature, as well as employ way already discussed.

For purely static models and their strengths as compared to full-size structures, similarit methods and theory were even more certain and accurate, and he had used them repeated himself in checking Callender-Hamilton bridge and hangar designs. There the propo tionate loading in exact scale models of the same material in bolts and members, varied the square of the scale reduction; or in bridges the unit deck loading had to be equal model and original after allowance for dead load. Such model tests had been shown Inst. C.E. Conversaziones of 1933–34, and perhaps afforded the first instance of brid design by models.

Thus the absolute value of the Author's model research work on Lyttelton Harbo could not be doubted, and the Wallingford Research Establishment was to be co gratulated in following up the work of Reynolds, Professor Gibson, and others who h such confidence in the practical value of harbour and river models for engineers, a dynamical similarity properly applied.

The Author, in reply, said that it was very humbling to observe what little differen there was between his work and that done by Mr Hamilton in 1927. After the discussi he had looked at some photographs of the earlier model and he agreed that they mig have been shots of the recent one. He had also seen the report written by Mr Hamilt on the uses of dynamical similarity with particular reference to his Lyttelton model a was struck by its modernity. Only in the matter of technique did the Author have t advantage, and that was because of the great deal of experimental work that had be done on wave models in the interval. For example, experimental work had shown wh a large proportion of the energy of long waves was reflected by sloping boundaries a

consequently how necessary it would be to reproduce the southern cliffs of Port Lyttelton in the model. In the earlier model only the harbour side of Port Lyttelton had been reproduced, and an imaginary training wall had been built down the centre of the inlet. Another rather big difference between the models was that whereas the Author's 600 : 100 model had been used to study the behaviour only of very long waves, longer than 60 sec in period, Mr Hamilton's 600 : 120 model had been used for a study of storm waves. Such a distortion would not be acceptable now in a study of storm waves.

There was again some coincidence between Mr Hamilton's and the Author's findings regarding the long 12-min seiche. Mr Cashin had described it as a seiche of 10-min period and the model had shown it to be of 9½-min period; but that change of periodicity was easily—and most likely correctly—explained by deepening of the harbour and entrance that had taken place between 1927 and 1953. 10 min was too long a period for any system of standing waves in Lyttelton Harbour, and the Author had been able to show that it was the period of an oscillation in which the water level in the harbour rose and fell above and below the water level in Port Lyttelton. An experiment had been made in which the harbour entrance was blanked off temporarily and the water level raised inside it: the entrance had then been suddenly reopened, and the subsequent movement of the water surface had been recorded. The natural period of the rise and fall had then been 30 sec in the model, corresponding to 9½ min in the prototype.

Replying to Dr Murdock, he said that it had been far from his intention to lull anyone into false beliefs; rather was it his intention to state as precisely as possible what the models could do and in what way they fell short of perfection. Taking Dr Murdock's origins of uncertainty in the rigid models first: vertical exaggeration, he thought, he had dealt with in the Paper. The existence of a time scale introduced no errors. Sensitivity to prototype period introduced no errors if the speed control on the wave-generators was adequate. The use of well-water instead of sea-water introduced no errors. The existence of surface tension introduced errors in the speed at which the shortest waves travelled, but never with any of the waves that had been employed had the error amounted to as much as 1%, and that error had been neglected. He had not heard that dust on the surface introduced errors. The mobile-bed model, on the other hand, had to be judged by quite different standards. It was well known that not all the phenomena that should be reproduced in such a model could be made to scale similarly, and it was to some extent a matter of opinion which phenomena were the important ones. Confidence concerning a moving-bed model would depend largely upon whether it reproduced known changes satisfactorily.

With reference to the possibility of making short-cuts in the work the Author said that one of the objects of the Paper was to emphasize that a complete analysis of the wave action in a harbour took several months. It was not accuracy that would suffer from short-cuts so much as confidence that the worst conditions in the harbour had been observed. Whereas it was desirable that a model of the harbour, as it was finally to be built, should be tested thoroughly, it was reasonable to test preliminary designs sufficiently only to show whether they were probably better or worse than alternatives.

Some of the problems that had arisen during the investigation were being given thought at the Hydraulics Research Station, and accordingly succeeding models differed in some respects from the Lyttelton model, but it would be inaccurate to give the impression that the problem of how to get waves to travel across a small-scale undistorted model without loss of height would be solved, or that waves would ever be persuaded to follow in a distorted model the paths followed by the waves in the prototype. The biggest advance that he sought was in a method of presenting an enormous mass of evidence on the disturbance in a harbour under many different conditions in some compact and digestible form, so that an unerring choice could be made by the client between alternative harbours. That difficulty had been lucidly described by Mr Loewy in his remarks.

Replying to Mr Allen, the Author said that Mr Cashin had been asked whether any shadow had been found on echo-sounder records which could be attributed to a layer of fluid mud. He had replied that no shadow had been found.

Regarding the density scale for turbidity currents the model could only work correctly if the scale was 1 : 1. But it was not known whether in fact the densities were the same in model and prototype. He had no figures for either. Although he had spoken of a turbidity current, suggesting perhaps a well-defined current of uniformly dense fluid, it was more likely that during a storm in Port Lyttelton and during an experiment in the model there was a continuous variation in solids concentration from almost clear water at the surface to compacted mud at the bed. Because the concentration of solids at a certain point at a certain instant depended partly on the rate at which a fluid mud compacted into immobile material, and because the time scale for that compaction between model and prototype was unlikely to be the same as the Froude time scale, he could not expect the density of turbidity currents to have been correctly reproduced.

The "starry sky" was a very wonderful idea. A photograph of a model with a "starry sky" over it showed the water covered with a large number of "orbits," the size of which was related to the waves in the area. The method fulfilled a need for a quick if not completely correct assessment of the disturbance in a harbour. It suffered from two defects that derived from the fact that the "orbit" size was a measure of the change of slope of the water surface. In the first place, whenever there was a standing wave—and the disturbance in a harbour nearly always consisted of systems of standing waves—the maximum orbit would be found at the nodes, the places of maximum change of slope, where, however, vertical movement was zero. The places of maximum vertical movement were shown on the photograph as places of minimum orbits. One might say that the method gave one as wrong a picture of the disturbance as could be imagined. The other defect derived from the necessity to operate with unrealistically low waves if satisfactory orbits were to be obtained. That introduced important errors in the energy that was reflected from sloping boundaries. The Author accepted the possibility that an experimenter after some experience with the method might become adept at interpreting the photographs and obtaining from them the level of disturbance in the real harbour.

He wished to associate himself with Mr Loewy's remarks on the need for wave records and for data on wave directions. The factor by which waves were reduced was normally of no value unless the height of the storm waves was known. They were fortunate in the Lyttelton investigation to have a harbour there already, the performance of which was known to be satisfactory, or almost so; by studying in the model both the existing and proposed harbours the proposed harbour could be assessed without the need for accurate wave data. A further advantage of having an existing harbour lay in the way it obviated the need for data on what waves in the harbour were tolerable. The subject of permissible wave-heights was a complicated one, relatively unexplored except theoretically,[10] and to the Author's knowledge there was no accepted data on it.

Mr Loewy had hit upon an important point in suggesting that some uncertainty was introduced by operating the model with one wave-height for each value of the same period; but he did not agree that working with a constant wave-height would have been any improvement. It was known that the longer waves—those longer than, say, 25-sec period—were lower than the storm waves and that was partly taken care of by the characteristics of the wave generator. Although the results were presented in the form of amplification factors, perhaps thereby suggesting that the factors were not affected by the incident wave-height, it was known that incident wave-height did affect amplification factors. It was probably adequate to operate with one wave-height only, provided it was close to the maximum that was expected; and that was the practice at the Hydraulics Research Station. The thought of having to duplicate or triplicate all the experiments in order to deal with several wave-heights was horrifying.

The wave-height recorder consisted essentially of a shallow circular float 4 in. in diameter, fixed by a ball-joint to a vertically sliding shaft. There was a pin-hole in the shaft through which light was directed to fall on a photographic paper. As the paper was drawn past the shaft and as the float rose and fell a curve was drawn on the paper. The advantage of such an instrument was that it was simple and that the amplitude measured off a record was without doubt and with no possibility of error the amplitude of the float's

travel, but there was a small error involved when measuring short waves of the order of 1 ft long because the float tended to span the troughs and sink into the crests. An electric-resistance wave recorder was now being used at the Hydraulics Research Station. It had the property of permitting the record to be magnified or reduced at will—something of a mixed blessing—and it overcame the difficulty that the float had in following short waves; but above all it had the advantage of allowing the record to be examined while it was being made.

In reply to Mr Lacey's remarks on the choice of scales he said that in the main there was no disagreement. It was agreed that the speed of fall of model particles should be $x/y^{3/2}$ of the speed of the prototype particles, and it was further agreed that if $x/y^{3/2}$ were unity then the prototype material could be used in the model. The Author was not certain whether Mr Lacey thought that $x/y^{3/2}$ should be unity because of the convenience of using the prototype material in the model, or because it resulted in a more satisfactory model. He would not have willingly used Lyttelton mud in the model because of its lack of uniformity from layer to layer. Analysis of core samples had shown that the surface was often ten times as coarse as a sample taken from a depth of 1 ft; and that as big a change was often found between depths of 1 ft and 2 ft. China clay, on the other hand, could be obtained in a consistent grade. Only if 1 ton of the material was similar to the next could experiments be repeated and the results of successive experiments compared.

The movement of sand and shingle by flowing water had been studied for many years, both experimentally and by observation of rivers and canals; and that work had enabled Mr Lacey and others to formulate empirical laws governing the shapes of large rivers and of small rivers that could be said to be scale models of them. Even accepting for the moment Mr Lacey's contention that the prototype material should be used in the model, the Author did not think that rule could reasonably be transferred to models where turbidity currents played an important role. The mechanics of material movement by turbidity currents had barely been touched experimentally—indeed he thought that the Lyttelton model was the first three-dimensional model in which they had been reproduced—and there was no experimental evidence for suggesting that the prototype material should be used in the model. Nor could he admit that there was a theoretical reason for the practice.

Mr Cashin's opinion on the investigation, as one who had in the past operated hydraulic models and who was now a harbour engineer, was greatly to be valued.

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The closing date for Correspondence on the foregoing Paper was 15 March, 1956. No contribution received later than that date will be printed in the Proceedings.—Sec.

PUBLIC HEALTH ENGINEERING DIVISION MEETING

8 November, 1955

Mr C. A. Risbridger, Member, Chairman of the Division, in the Chair

The Chairman said that in 1953 the Trustees of the Chadwick Trust had offered a Chadwick Medal in silver gilt and a prize of the value of £5 to a Lecturer, or to the Author of a Paper, adjudged by the Council of the Institution to be the best submitted upon a civil engineering aspect of Chadwick's "Sanitary Idea." On the recommendation of the Council, the Chadwick Trustees had awarded the Medal and prize to Dr Albert Parker, C.B.E., for his Lecture on "Atmospheric Pollution: Causes, Effects and Prevention" delivered to the Public Health Division in 1953.

Mr E. M. Rich, C.B.E., B.Sc., who was Chairman of the Chadwick Trustees, was present that evening to present the Medal and prize to Dr Parker, and the Chairman invited him to be good enough to do so.

Mr E. M. Rich said that when Sir Edwin Chadwick had died in 1890, at the advanced age of 90, he had left a very considerable sum of money, and by his will a trust had been created for the purpose of assisting in the promotion of his "Sanitary Idea." Those objects were to be secured by the promotion of public lectures, by grants to educational institutions, and by the presentation of Chadwick Medals and prizes to persons who had shown exceptional merit in regard to subjects coming within the scheme of the Trust. Under the last provision, the award of Medals, the Trustees had for many years given the Medal to a student who had done very distinguished work in the Chadwick Engineering Department at University College. Under the Trust also every third year a Medal was to be given in succession to someone in the Army, the Navy, or the Air Force who had done most to promote the health of the men in the Service concerned, according to the year. The Trustees had also given Medals to other societies. Three years ago Mr G. M. Binnie, M.A., a Member of the Institution, had suggested that the Institution of Civil Engineers was a very appropriate body with which the Trust could be associated, and that, if proper arrangements could be made, a Medal might be given on the recommendation of that Institution.

Dr Parker had given a Lecture to the Institution on "Atmospheric Pollution: Causes, Effects, and Prevention." Dr Parker had given a Chadwick Lecture on the same subject very many years ago, and the Chadwick Trust had also sponsored two Lectures on the subject, one by Mr C. J. Regan, who had been a member of the Beaver Committee, and another, at the Royal Institution, by Sir Hugh Beaver himself, a Lecture attended by more than 400 people. Mr Rich had had the pleasure of reading Dr Parker's Lecture, and thought that the Institution had been right in recommending the award of the Medal to Dr Parker.

Mr Rich then presented the Medal and prize to Dr Parker.

Dr Albert Parker, in acknowledging the award, said he greatly appreciated the honour of being the recipient of what he understood was the first Chadwick Medal and prize presented by the Institution of Civil Engineers. He had long been interested in environmental health, including, in addition to air pollution, water supply and water pollution, and he appreciated very greatly that recognition of the little that he had been able to do by giving his Lecture.

He would treasure the Medal and it would always be a reminder to him of the Institution and of the Chadwick Trust and of the interest taken in the subject of his Lecture.

The following Paper was presented for discussion, and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Public Health Paper No. 13

THE DEVELOPMENT OF SEWAGE TREATMENT IN THE CITY OF COVENTRY

by

* Granville Berry, M.I.C.E., and Cyril R. Deeley, D.F.M.

SYNOPSIS

The Paper traces the development of sewage treatment in the City of Coventry since the first sewerage system was laid down more than 100 years ago, and gives some indication of the problem which the disposal of sewage and trade effluents presents in a rapidly expanding modern industrial city.

As a result of the introduction of new and highly specialized industries the major growth of the City has taken place in the first half of the present century; during that period the population has grown by more than 200,000 and the area of the City has increased from 4,147 to 19,167 acres.

The importance of preceding major extensions by installation of small-scale pilot plants and the undertaking of experimental work was recognized by the City Council as long ago as 1910, and the Paper deals not only with some of the earlier work in connexion with experimental filters and activated-sludge units but also with more recent research carried out in conjunction with the Water Pollution Research Laboratory, and the Gas Council with regard to recirculation and alternating double-filtration processes and the treatment of sewage containing gas liquor, the results of which may have very important consequences on the design and operation of sewage-treatment works.

The post-war extensions saw the adoption of the partial purification system with heated sludge digestion and gas utilization, and the works carried out are described in some detail. Information is given on the data on which the extensions have been designed and the mechanical and electrical plant provided on the works.

Historical

The main purpose of this Paper is to trace the development of sewage treatment in a modern industrial city, and to review the results of the experimental work carried out during the period that has seen that city's most rapid development.

In 1900 the City of Coventry had 65,000 inhabitants, but by 1955 this had increased to approximately 270,000 and it is estimated by 1971 it will have risen to 336,000.

This four-fold increase in population in the past 55 years has presented the Corporation with many difficulties and problems in connexion with provision of essential public services.

In none of these services has the problem of keeping pace with continued rapid growth been more acute than in the sphere of main drainage and sewage treatment.

* Mr Berry is City Engineer and Surveyor, and Mr Deeley is Chief Assistant Engineer (Main Drainage Department) Coventry City Council.

In addition to the increase in population, there has also been a very considerable expansion of Coventry's industrial capacity during the past 50 years.

Factories have increased in number and become impressively larger; a wide range of new industries and industrial processes has also been established, largely as a result of the development of the rayon, motor-car, and aircraft industries.

Contemporaneously with these expansions considerable developments in sewage-treatment methods and practice have occurred, and the continuing need for extending the City's sewage-treatment facilities has enabled the Corporation to keep abreast, to some extent at any rate, of the general advance in purification methods.

During the same period the adoption of any new process has invariably been preceded by the installation and operation of an experimental unit, in order to determine whether new processes, some of which had been successfully adopted elsewhere, would prove satisfactory if applied to Coventry's sewage.

Unfortunately, the Corporation's plans for dealing with its sewage-treatment problems have been interrupted by two world wars, and consequently proposals for extending the capacity of the treatment plant have often been seriously delayed on account of the national situation.

It has, therefore, been necessary, on many occasions, to improvise to rearrange, and to adopt temporary expedients in order to obtain the maximum benefit from existing plant.

Owing to the "mixed" character—containing as it does a wide variety of trade wastes—Coventry's sewage has always been rather difficult to treat.

Coventry's first sewage scheme

The sewering of Coventry was first undertaken in 1852, and for about 20 years the sewage was merely discharged into the River Sherbourne at a point about one mile downstream from the then existing southern boundary.

Referring to conditions in about 1870, a contemporary writer said:—

> "this sewage is extremely foul, and, coloured by refuse dye thrown into the sewers from the numerous silk-dyeing and varnish works, etc., soon converted the river into a large, open and extremely offensive sewer, which finally entered the River Avon and contributed towards the supply of drinking water to the town of Warwick."

Although the Corporation, according to a report of 1872, "did fulfil what was considered a generation ago to be all the requirements for draining the town and conveying the sewage to the nearest natural carrier in the shape of a running stream," a conception was already emerging of the need to maintain the purity of Britain's rivers and this led in 1874 to an Injunction, issued by the Court of Chancery, restraining the Corporation from continuing to discharge untreated sewage into the river.

First sewage-treatment plant

It is interesting to note that Coventry's first sewage-treatment plant was constructed in 1874 by a private company, which undertook to purify the city's sewage—which at that time amounted to 2,000,000 g.p.d.—without cost to the Corporation.

The Company, known as the General Sewage and Manure Co. Ltd, was optimistic enough to suppose that the manufacture and sale of artificial manures, using sewage sludge as a base, would prove to be remunerative.

This assumption was, of course, quite erroneous and after operating at a loss for

2 years the arrangement was terminated and the treatment plant was taken over by the Corporation.

The method of treatment consisted of chemical precipitation, using lime, copperas, and sulphate of alumina, followed by irrigation on 8 acres of underdrained land at Whitley.

It is also interesting to find that towards the end of 1876 the Town Council appointed a Committee to investigate various methods of dealing with the sewage, with a view to some change in the treatment processes; they came to the conclusion that the system of treating the sewage by precipitation, which had then been in use for nearly 3 years, was a "sanitary success" and the "Rivers Purification Association Ltd" (possibly an early forerunner of present-day River Board) was informed accordingly.

This plant was subsequently converted into a storm-water treatment plant and is still being used—more than 100 years later—for this purpose.

Baginton sewage farm

By 1900 the requirements of the city had far outgrown the capacity of the Whitley Works and the Baginton Sewage Farm was established.

The total area of land purchased by the Corporation was 480 acres, 380 of which were actually available for the treatment of sewage by broad irrigation methods.

Whitley pumping station

In order to convey the sewage to the Baginton Sewage Farm, the Whitley Pumping Station was constructed on a site adjoining the original Whitley Works, now used for storm-water treatment.

This plant comprised four triple-expansion steam engines operating three 21-in.-dia. ram pumps, having a total theoretical capacity of $11\frac{1}{4}$ m.g.d., and operating against a total head of about 70 ft.

This plant was started on the 3rd February, 1901 and has been working continuously day and night ever since.

Experimental work

Experiments with a small bacteria bed

By 1910 Coventry's population had increased to 106,000, the dry-weather flow of sewage was about $3\frac{1}{2}$ m.g.d. and it again became necessary for the Corporation to consider further extensions to its treatment plant.

About this time the bacteria-bed system was beginning to receive considerable attention from chemists, bacteriologists, and engineers. After inspecting pilot plants at Manchester and Birmingham the Committee decided to conduct an experiment to determine whether Coventry's sewage could be satisfactorily treated by this process.

An experimental filter was accordingly constructed at Baginton, 90 ft in diameter and 5 ft deep with a capacity of 1,180 cu. yd.

The tank serving this experimental filter was converted into a septic tank giving an average retention period of about 12 hours.

The experiment was continued for more than 2 years and during this period detailed records of the behaviour of the plant at different rates of dosing, etc., were kept.

These showed that a satisfactory effluent could be obtained when this bacteria bed was dosed at 140 gal/sq. yd of filter, i.e., at 84 gal/cu. yd of media. These rates of dosing were the actual rates of application to the bed and do not refer—as is now the general practice—to rates of dosing in terms of the dry-weather flow of sewage.

It is significant that as long ago as 1912 Dr Bostock Hill's report on the effluents from this first bacteria bed included the following comment:—

> "the sample contained too large a quantity of matters in suspension, in the form of humus . . ."

—and this feature of Coventry's effluents has prevailed up to the present time.

Large-scale bacteria bed installation

In view of the encouraging results of this early experiment the Corporation decided to augment the facilities provided by the Baginton Sewage Farm by constructing twelve bacteria beds 117 ft in diameter and 5 ft deep, but, owing to the incidence of the first World War, it was not possible for these beds to be completed until 1919.

In the meantime Coventry continued to grow and, owing to the establishment of munition factories and other engineering works connected with the war effort, the rate of development was accelerated and the City's sewage-treatment problems continued to demand attention.

Experiments with the activated-sludge process

About this time the activated-sludge process was being developed and accordingly in 1920, the Corporation established an experimental plant to gauge the merits of the process in relation to Coventry's sewage.

The detailed records of this experiment were destroyed during the second World War, but it is known that the experimental unit operated on the Sheffield (paddle wheel) system.

The results of this experiment must have shown considerable promise, for, in 1924, a full-scale activated-sludge plant, designed to treat a dry-weather flow of 2,000,000 g.p.d., was constructed at Baginton.

Baginton activated-sludge plant

For reasons which are not apparent the Baginton activated-sludge plant wa designed to operate on the air-blowing system despite the fact that the experimenta plant had not been of this type.

The performance of this unit, operated as a full-treatment plant, proved rathe disappointing in that its effective capacity was found to be not more than 1 m.g.d

The extension of city's boundaries

In 1928, and again in 1932, Coventry extended its boundaries. Its populatio after these two extensions was 182,000, and the dry-weather flow of sewage rose t 6,000,000 g.p.d.

It therefore became necessary to consider a further extension to the sewage-treat ment facilities and, since these Boundary Extension Acts had added to the City larg areas which could not be drained to the Whitley Pumping Station (and thence b pumpted to Baginton), a new sewage-disposal works was established at Finham.

The main sewerage system

The area comprising the extended City is traversed by three main valleys:—

(1) The valley of the River Sherbourne, which runs through the centre of the old City.
(2) The valley of the River Sowe, which lies on the eastern side of the city, and which is joined by the River Sherbourne at a point about one mile below the Whitley Pumping Station.
(3) The valley of the Canley Brook draining the western side of the city, a tributary of the Finham Brook which joins the River Sowe at Finham.

The City's main intercepting sewers naturally follow these three valleys. The Sherbourne Valley sewer is approximately 6 miles long and at its lower end is a 7-ft-6-in. × 5-ft egg-shaped sewer constructed of concrete cast in situ.

The Sowe Valley sewer is mainly a brick-lined concrete sewer, but considerable lengths of steel pipes were used in areas considered to be liable to mining subsidence.

This sewer is about 9 miles long and is 54 in. in diameter at its junction with the Sherbourne Valley sewer.

The Canley sewer is $5\frac{1}{2}$ miles long, consists of concrete tubes, and has a diameter varying from 18 to 33 in.

The final length of the main sewer—after the confluence of the three main intercepting sewers—has been designed to take 6 times the dry-weather flow from an ultimate population of nearly 400,000.

The Finham site constitutes the natural focus to which the sewage from the whole of the extended City could be brought by gravitation.

The establishment of the Finham works

In the light of the experience gained at Baginton it was decided to adopt the bacteria bed system at Finham, and a plant designed to deal with a dry-weather flow of 3,000,000 g.p.d. was installed there and brought into operation in 1932.

Land treatment at Baginton was discontinued but the bacteria beds (capacity 2,000,000 g.p.d.) and the activated-sludge plant (capacity 1,000,000 g.p.d.) were retained.

The original Finham Works comprised screening and detritus tanks, horizontal-flow sedimentation tanks, eighteen 120-ft-dia. bacteria beds, and upward-flow humus tanks.

The sewering of the areas recently added to the City, combined with the continued growth of the pre-extension City resulted, within a matter of 2 or 3 years, in an additional flow of about 2,000,000 g.p.d. arriving at Finham. Further extensions were therefore necessary and by 1936 the capacity of the Finham Works had been increased to provide for a total dry-weather flow of 5,750,000 g.p.d. These extensions included the provision of a further seventeen bacteria beds bringing the total number up to thirty-five, their total capacity is 87,500 cu. yd (see Fig. 1).

Experiments with partial purification

It became obvious, about this time, that if the rapid growth of Coventry was going to continue (and there was every reason to suppose that it would) the increasing quantities of sewage could not satisfactorily be accommodated merely by construction of further bacteria beds. It was, therefore, necessary to seek an alternative

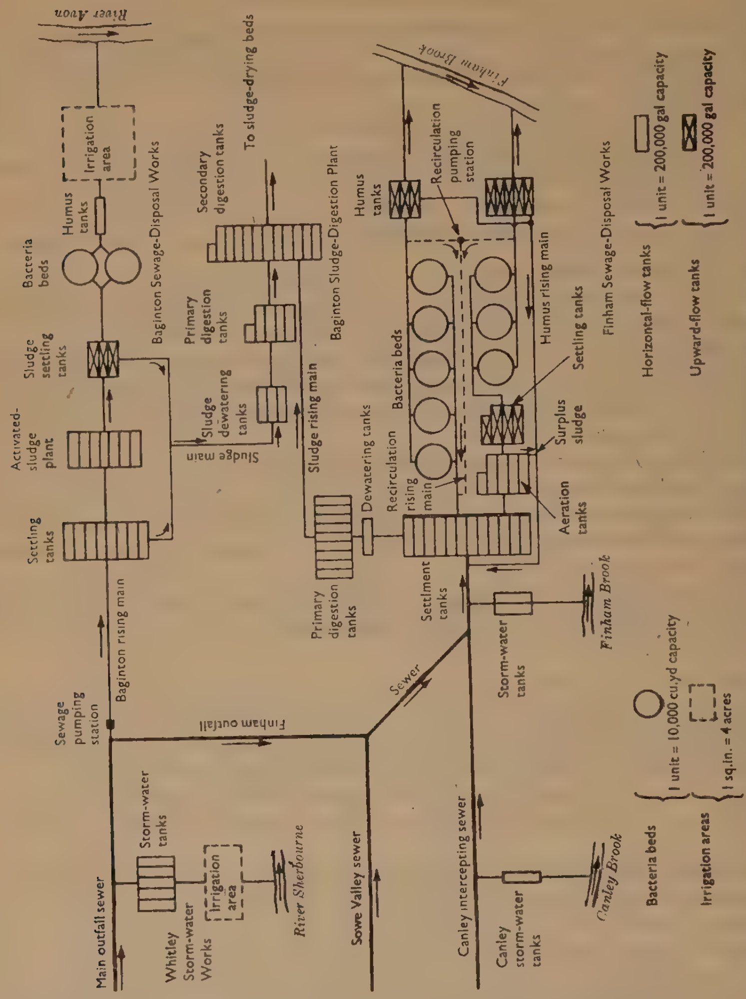

FIG. 1.—GENERAL ARRANGEMENT OF SEWAGE TREATMENT FACILITIES IN THE CITY OF COVENTRY

method of increasing the capacity of the plant without adding to the already lar area occupied by bacterial filters.

It had also been found that none of the existing treatment units—thirty-fi bacteria beds at Finham, twelve bacteria beds at Baginton, and the Bagint activated sludge plant—produced an effluent of a very high quality and it was thoug that in view of the rather complex character of Coventry's sewage, better resu might be obtained if a two-stage process was adopted.

The original experimental bacteria bed at Baginton was still available and, isolating a section of the activated-sludge plant for this purpose, it was a fairly sim matter to provide an experimental unit—comprising partial treatment by the ac vated-sludge process followed by high-rate filtration.

The total aeration capacity of this activated-sludge unit was 140,000 gal and it was followed by a settlement tank holding 105,000 gal and a bacteria bed containing 1,180 cu. yd of granite media.

The partial-purification experimental plant was brought into operation in September 1938, and the investigation was continued for exactly 2 years, thus covering varying weather conditions and seasonal variations, etc.

The results of this experiment showed that a bacteria bed could be expected to produce a satisfactory effluent when dosed at a rate of 150 gal/cu. yd/day with sewage which had received normal sedimentation and 4 hours' preliminary treatment in an activated-sludge unit.

Here, then, was a convenient means of increasing the capacity of the Finham Works merely by interpolating an activated-sludge unit between the existing settlement tanks and the bacteria beds.

A detailed scheme for the abandonment of the Baginton Works and the conversion of the Finham Works to the partial-purification system was, therefore, prepared; this would have been carried out if war conditions had not caused the indefinite postponement of such projects.

Incidentally, a further interesting experiment was carried out before the activated-sludge plant was restored to its normal working conditions.

This plant is sub-divided into five more or less separate units and for several months the whole of the sewage was given partial treatment in the first unit, followed by full treatment in the remaining four units.

It was found that when operated in this way, giving a two-stage activated-sludge treatment, the total capacity of the plant was increased by 50%, i.e., from 1,000,000 to 1,500,000 g.p.d.

War-time difficulties

Besides preventing the carrying-out of a sewage-disposal extension which was both urgently necessary and long overdue, the war added to Coventry's drainage and other difficulties in a variety of ways.

It led to the construction of many new factories and a considerable influx of munition workers and a consequent increase in water consumption and, of course, sewage flow.

Indeed, the strain on the City's water resources became so great that in 1942 the Corporation promoted a Bill in Parliament, as the result of which authority was obtained to abstract an additional 1,000,000 g.p.d. from the River Avon to augment the normal supplies.

The question naturally arose as to how this additional flow could be accommodated at the sewage-disposal works, bearing in mind the fact that a major extension at either Finham or Baginton would not be permitted during wartime.

The success of the partial-purification experiment indicated a very simple and convenient method of dealing with the problem.

Conversion of the Baginton works to partial purification

The Baginton Works already included an activated-sludge plant (capacity 1,000,000 g.p.d.) and twelve bacteria beds (capacity 2,000,000 g.p.d.) with a total capacity of 3,000,000 g.p.d.

Experience had indicated that if these two units could be re-arranged to form a

partial-purification plant, this could be expected to deal quite satisfactorily with a daily flow of 4,000,000 gal.

The various supply and effluent pipes were accordingly re-arranged, so that the whole of the Baginton flow could be passed first through a part of the activated-sludge plant and subsequently be distributed on to the bacteria beds.

A difficulty which arose in connexion with this scheme concerned the humus tanks which follow the twelve bacteria beds.

These were designed to deal with a flow of 2,000,000 g.p.d. and would obviously be inadequate for providing satisfactory settlement at the new rate of 4,000,000 g.p.d.

Fortunately, the Corporation were the owners of a number of fields, lying between the bacteria beds and the River Avon and it was possible to pass the Baginton effluent over this grassland before discharging it into the river.

The area of land used for this purpose is approximately 15 acres, and since it formed part of the original sewage irrigation area, it already contained an adequate system of effluent-collecting channels.

Re-arranged in this manner the Baginton Works successfully treated the additional flow for a considerable number of years.

It was, of course, never intended that this revised arrangement should be more than a temporary war-time expedient but it has been found necessary to continue its operation much longer than originally contemplated and during recent years the efficiency of the final irrigation area has become seriously impaired.

Introduction of recirculation facilities at Finham

For various reasons the practice of pumping to Baginton a more or less fixed quantity of sewage has been adopted, leaving the Finham Works to accommodate the daily fluctuations in the rate of flow.

During dry weather rate of flow to these works fluctuates between a minimum of 3,000,000 to a maximum of 14,000,000 g.p.d., and it was thought that the efficiency of the plant would be increased by the introduction of some arrangements for balancing the sewage flow.

This proposal was, however, eventually abandoned in favour of a recirculation scheme, and this was brought into operation in April 1945.

The recirculation plant comprises three centrifugal pumps having a combined capacity of 5,000,000 g.p.d. and these are used to pump filtered effluent back to the outlet channel of the settlement tanks.

Owing to the limits imposed by the capacity of the existing distributors, recirculation is not normally practised during the middle of the day, but as the sewage flow decreases one, two, and finally three pumps are progressively brought into operation.

The introduction of recirculation facilities did not result in any striking improvement in the quality of the Finham effluents, as judged by the usual chemical tests, but it has certainly led to an appreciable improvement in the condition of the bacteria beds.

The incidence of the usual seasonal discharge has been greatly minimized and surface ponding has been appreciably reduced.

Experiments with recirculation and alternating double-filtration processes

In 1945 a small-scale "pilot" plant was constructed in order to enable the recirculation and alternating double-filtration processes to be investigated and compared.

Fig. 3.—Experimental filtration plant at Finham

Fig. 6.—Partial-purification plant at Finham

FIG. 8.—CLOSE-UP OF DISTRIBUTOR SPEED-CONTROLLING DEVICE

FIG. 9.—GAS-LIQUOR TREATMENT EXPERIMENT AT STIVICHALL

This plant comprises four 12-ft-dia. filter beds 6 ft deep, four upward-flow humus tanks each having an effective surface area of about 20 sq. ft, and ancillary plant for pumping and gauging sewage and effluent. (See Fig. 2, Plate 1, and Fig. 3.)

A small laboratory adjacent to the pilot plant was also provided and from 1946 until 1954 the experimental plant was operated in conjunction with the Water Pollution Research section of the Department of Scientific and Industrial Research. It was first used to compare the recirculation and alternating double-filtration processes as applied to Coventry's sewage, and the method of operation was later changed to enable the effect—in the alternating double-filtration process—of omitting intermediate settlement to be investigated.

These investigations, which are dealt with in more detail in Appendix IV, showed that:—

(1) Coventry's sewage could be satisfactorily treated by the alternating double-filtration process.
(2) The adoption of this process would enable the rate of dosing to the existing filters to be increased from 70 to 140 gal/cu. yd/day.
(3) Intermediate settlement tanks need only be of only nominal capacity, say, ½ hr D.W.F.

A scheme for converting a part of the Finham Works to this process is now in course of preparation.

Mechanical de-sludging of sedimentation tanks

In connexion with the mechanical de-sludging of the main sedimentation tanks which will form part of the next extension scheme consideration is being given to the following alternative ways of equipping the existing rectangular tanks with sludge scraping mechanism:—

(1) The provision of twelve individual scrapers, one to each tank.
(2) The installation of six tandem-type scrapers, each spanning two adjoining tanks.
(3) The provision of one or more transferable machines capable of being moved from one tank to another.

A further alternative would be to convert the twelve existing tanks into three large circular tanks and to equip these with revolving scraping mechanisms.

Preliminary estimates indicate that there would be very little difference between the cost of carrying out any of these alternative schemes and it may be that the final decision will be influenced by the arrangements which it is found possible to make for dealing with the flow of sewage while the necessary structural alterations are being made.

Sludge-treatment plant

The development of Coventry's sludge-treatment facilities constitutes another story of frequent extensions and improvisations in order to deal with the increasing quantites of sludge.

Prior to 1932 the sludge from the Baginton settlement tanks and the surplus sludge from the activated-sludge plant was dried out in large lagoons, supplemented by a number of shallow under-drained drying beds.

When the Finham Works were constructed it was decided that the sludge from the

4

new works should be pumped to Baginton for treatment together with that from the Baginton tanks.

A sludge-digestion plant, consisting of primary and secondary cold digestion tanks having a total capacity of 1,500,000 gal was therefore constructed and at the same time the area of under-drained beds was increased to 6 acres.

In 1936 the digestion-tank capacity was increased to 3¾ million gal and the drying-bed area was extended to 10 acres.

Experiments with sludge-heating and gas utilization

In 1941 it was decided that an investigation into the effects of heating the sludge during the digestion process should be instituted.

A reinforced concrete hood, covering a total area of approximately 1,000 sq. ft was placed over two of the existing primary tanks and arrangements were made for heating the sludge and collecting the gas evolved during the digestion process.

Provision was made for measuring the gas yield from each tank so that two different sets of working conditions could be tried out simultaneously.

The sludge gas was used to run a 14-h.p. gas engine driving a sludge-circulating pump and to operate the sludge heater.

In this way the temperature of the sludge was gradually raised to between 70° and 80° F. with a corresponding increase in the efficiency of the plant.

The surplus gas was used to generate electricity for lighting the works and some adjoining farm buildings.

This experiment was continued for 2 years and led to the following conclusions that:—

(1) The existing sludge-digestion plant would be capable of dealing with twice the normal quantity of sludge if full-scale heating facilities were installed.

(2) A daily yield of about 250,000 cu. ft of sludge gas might be expected if all the primary tanks were covered and heated.

This gas contained about 70% of methane and had a calorific value of about 670 B.T.U./cu. ft.

Adoption of partial purification, heated digestion, and gas utilization at Finham

Between 1950 and 1952 the second major extension of the Finham Works was carried out.

This scheme provided for the conversion of a part of the Finham Plant to the partial-purification process by placing a suitable aeration unit in front of an existing block of fourteen bacteria beds and dosing these beds at twice their previous rate, i.e., at 140 gal/cu. yd.

Heated primary sludge-digestion tanks were also constructed and provision was made for the methane gas to be collected and used to meet the power requirements of the new plant.

The sludge gas is converted into electricity by means of dual-fuel engines and alternators; the electricity is used to drive the air compressors, operate the sewage- and sludge-pumping plant, drive the sludge-scraping mechanism, to light and heat the new buildings, and for outside lighting on the works.

The waste heat from the exhausts of the dual-fuel engines and that from the engine cooling-water system is utilized, by means of suitable exchangers, for heating the sludge, thus accelerating the digestion process and increasing the gas yield.

This scheme also provided for the construction of additional humus tanks (Fig. 4) and for the modernization of the inlet works.

The effect of this extension was to increase the capacity of the Finham Plant (Fig. 5, Plate 2) from 5¾ to 9¼ m.g.d.

The general pattern

It will be seen that the story of sewage treatment in Coventry during the past 50 years is one of repeated extensions in an endeavour to keep pace with the exceptionally rapid and sustained growth of the City.

Owing to difficulties caused by, and ensuing from, two world wars, these endeavours have not been completely successful, and the enlargement of the plant has not always kept pace with the increased flow of sewage.

Indeed, it has generally been the case that the design flow has been reached by the time an extension scheme has been completed. (Fig. 6 and Fig. 7, Plate 1).

Present trends in sewage purification

Sewage disposal is far from being an exact science, new processes are continually being investigated, and it may well be that in 20 years' time present methods will appear as obsolete as some of the earlier processes now seem.

Future developments are difficult to forecast but one or two obvious trends may be observed from the changes which have already taken place.

One of these is what one might call an "intensification" of treatment methods—such as the adoption of very high rates of dosing of bacteria beds and the use of mechanical or other aids of flocculation to increase the efficiency of settlement tanks.

Another tendency is towards the adoption of various forms of two-stage biological treatment, the earlier objections to such methods being largely removed by developments in gas collection and utilization practice.

Increased mechanization and automatic control are being more generally adopted, partly through availability of cheap power and partly because of the difficulty of obtaining labour for carrying out the rather uncongenial tasks which have to be performed at a sewage-treatment works.

There is also an increasing tendency towards the use, by Water Undertakings, of water taken from streams and rivers. In fact it is not unusual, these days, to find a Local Authority discharging a purified effluent into a river from which another Authority, lower down the river, abstracts water to augment its public water supply.

This second Authority then subjects the river water to further purification, after which it is re-used for general consumption.

One wonders whether this practice may not eventually lead to closer co-operation between sewage-disposal and water authorities.

It may be that the sewage works of the future will, because of high-intensity methods, occupy a much smaller area than they do at present. It is possible that purification processes may be carried on to a much more advanced stage than is now generally attained, and one can visualize such a plant being almost completely automatic in operation.

Before this stage can be reached, however, there is still a great deal to be learned about the complex sewage-purification processes, many of which are still very imperfectly understood.

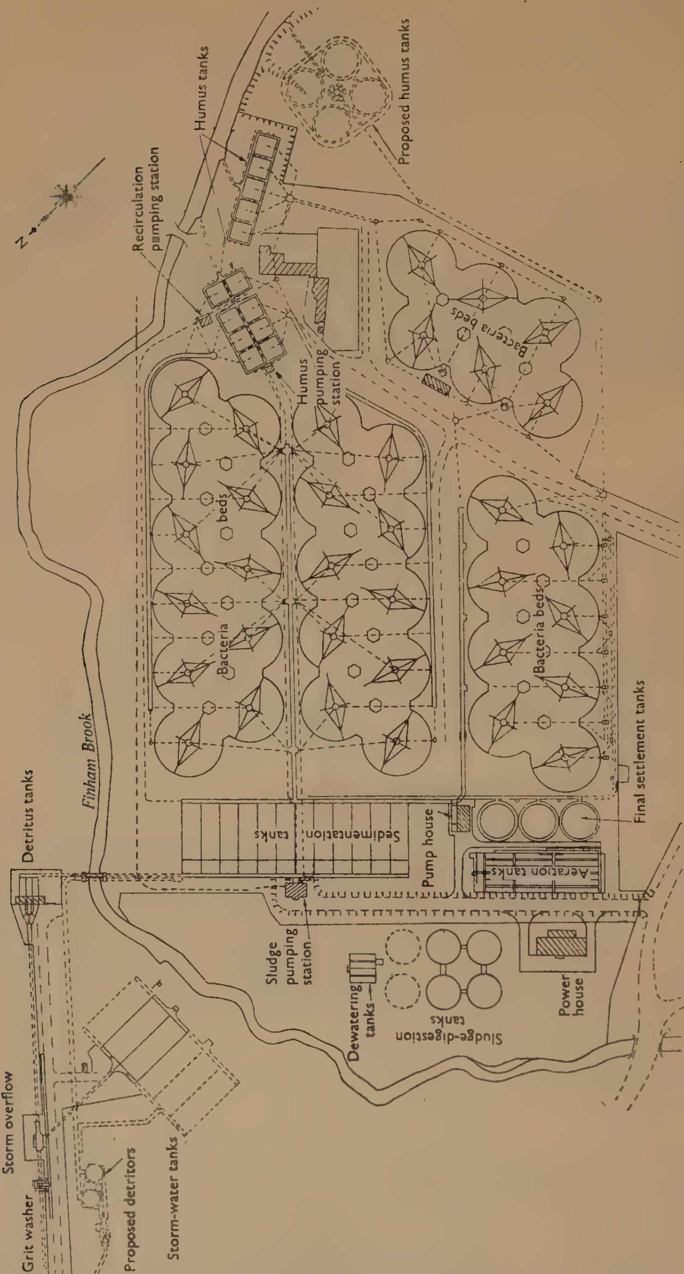

FIG. 4.—GENERAL LAYOUT OF FINHAM WORKS

THE DEVELOPMENT OF SEWAGE TREATMENT IN THE CITY OF COVENTRY

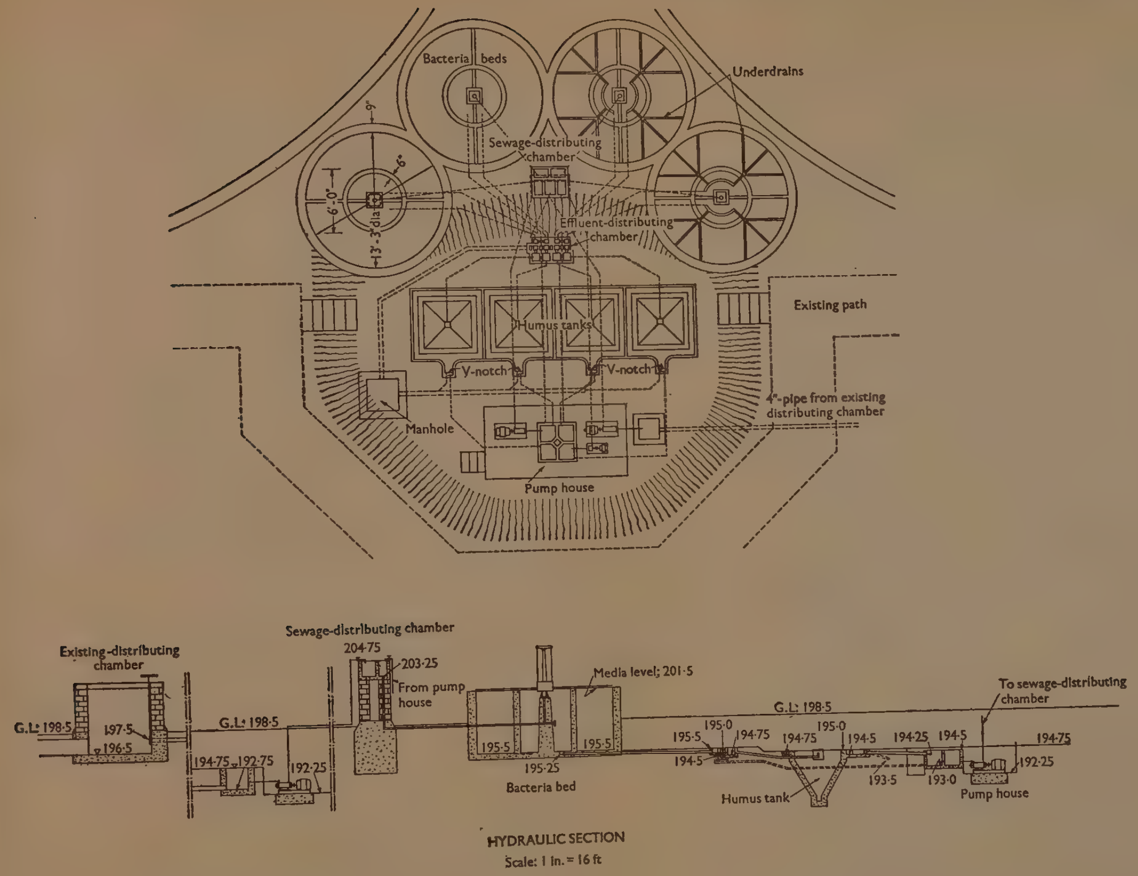

FIG. 2.—EXPERIMENTAL PLANT—RECIRCULATION AND DOUBLE FILTRATION

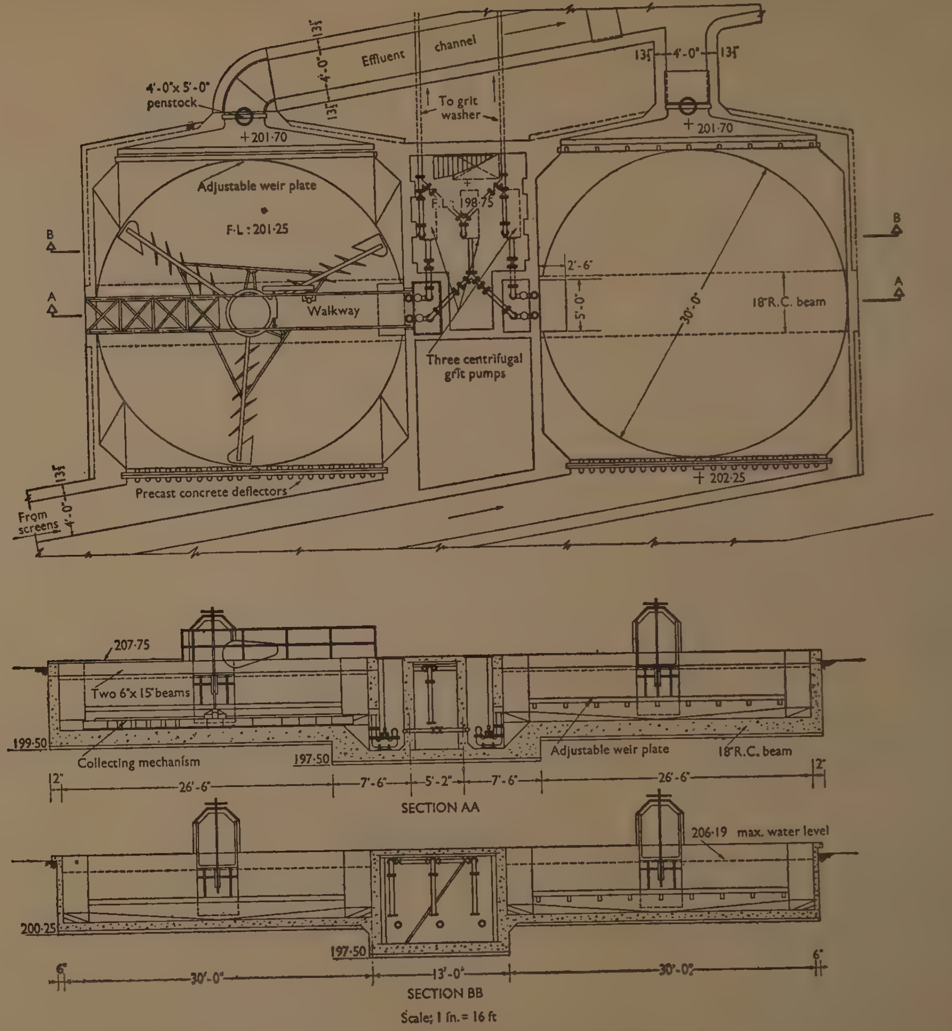

FIG. 7.—DETRITORS

PLATE 2

SEWAGE TREATMENT IN COVENTRY

THE DEVELOPMENT OF SEWAGE TREATMENT IN THE CITY OF COVENTRY

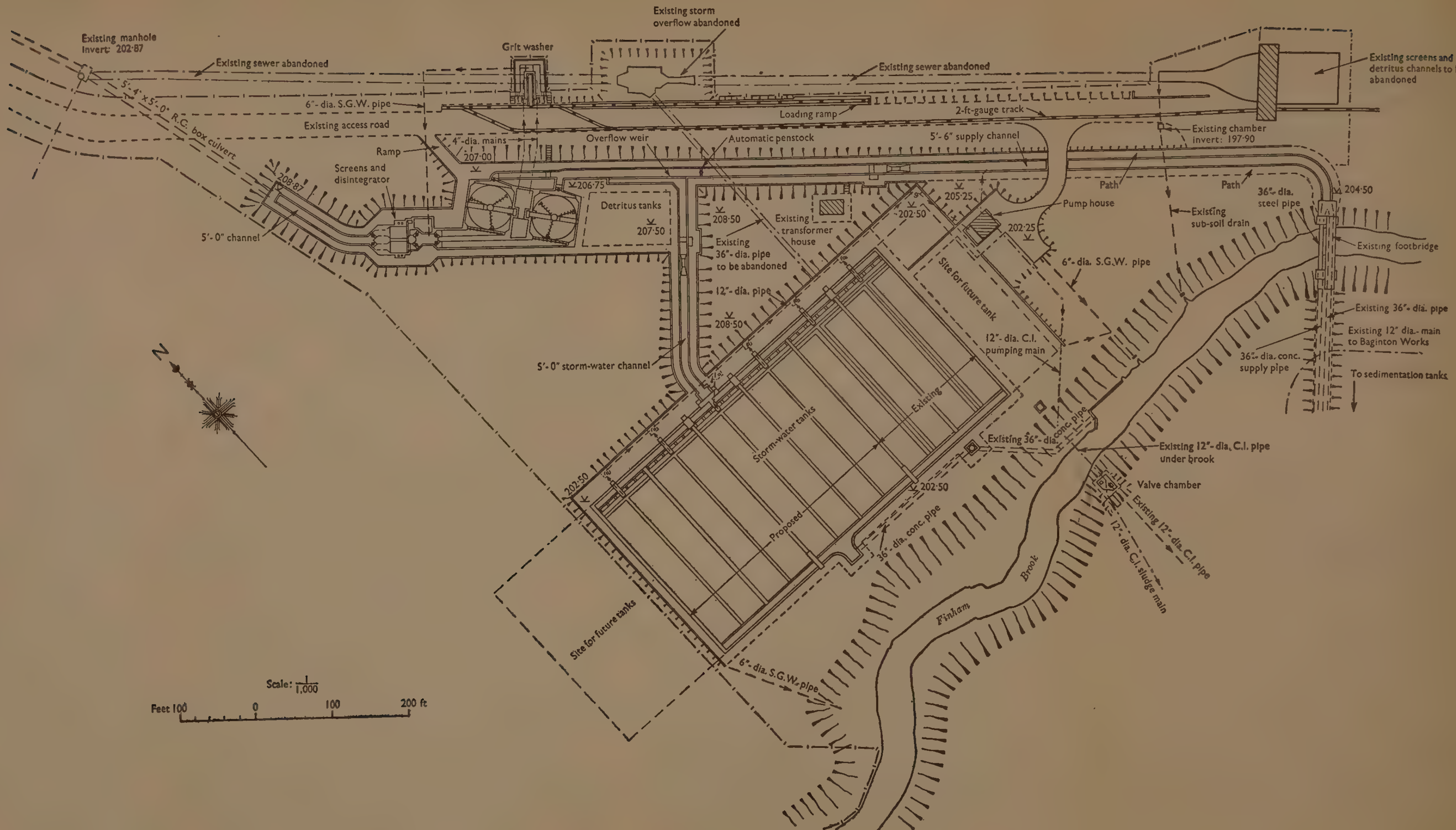

FIG. 5.—GENERAL LAYOUT PLAN OF INLET WORKS

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GRANVILLE BERRY and C. R. DEELEY

William Clowes & Sons Limited: London

Conclusion

It is obviously impossible in a Paper of this nature covering the events of more than half a century, to deal with any of them in very great detail.

All that one can hope to do is to present an overall picture of the sewage-treatment problems which have to be faced, when a city achieves a four-fold increase in population within 50 years, and to describe, in general terms, the attempts that have been made to solve them.

Much of the experimental work referred to here has been described in detail in Papers published by the Water Pollution Research section of the D.S.I.R., and some of these investigations have not yet reached a decisive stage.

It is, nevertheless, hoped that this more general survey of the various factors which have determined the present composition of Coventry's sewage-treatment plant may be of some interest.

The inclusion of a large number of figures in the text has been deliberately avoided, but Appendices are included giving the dimensions and capacities of the various treatment units, etc., at Whitley, Baginton, and Finham, together with machinery details and particulars of the data upon which the design of the Finham plant has been based.

Acknowledgement

The Author is indebted to Dr B. A. Southgate, Director of the Water Pollution Laboratory at Coventry, for permission to publish the imformation given in Appendix IV.

The Paper, which was received on 2 May, 1955, is accompanied by seven photographs, and ten sheets of drawings, from some of which the half-tone page plates, folding Plates 1 and 2, and the Figures in the text have been prepared, and by the following four Appendices.

APPENDIX I

DETAILS OF THE WHITLEY, BAGINTON, AND FINHAM PLANTS PRIOR TO THE 1950–55 EXTENSION SCHEME

Whitley pumping station

Three vertical triple-expansion surface-condensing steam engines.
Six bucket-and-plunger type pumps: ram diameter: 21 in.; length of stroke: 3 ft.
Diameter of rising mains: one 27 in.; one 21 in.
Distance from Whitley to Baginton: 2 miles.
Total static head Whitley to Baginton: 54 ft.
Capacity of each pumping unit: max. $3\frac{1}{4}$ m.g.d.; min. $2\frac{1}{8}$ m.g.d.

Bagington settlement tanks

Eight horizontal-flow tanks, each 150 ft × 30 ft × 7 ft.
Total capacity: 1,575,000 gal.

Baginton bacteria beds

Twelve circular beds, each 117 ft dia. × 5 ft deep.
Quantity of media in each bed: 1,990 cu. yd.
Total quantity of media: 23,880 cu. yd.
Six inverted pyramidal humus tanks, each 29 ft × 29 ft × 12 ft.
Total capacity of humus tanks: 252,300 gal.

Baginton activated-sludge-plant

Five aeration tanks, each 106 ft × 69 ft × 8 ft deep with central re-aeration channels.
Five inverted pyramidal sludge-settling tanks, each 33 ft × 33 ft × 34 ft deep.
Total aeration tank capacity: 1,442,000 gal.
Total capacity of re-aeration channels: 105,000 gal.
Total capacity of sludge-settling tanks: 525,000 gal.

Finham sewage-disposal works

Three detritus tanks, each 45 ft × 9 ft × 8 ft.
Total capacity: 30,000 gal.
Twelve horizontal-flow sedimentation tanks, each 100 ft × 30 ft × 9 ft 9 in. average depth, preceded by inverted pyramidal hoppers 18 ft deep.
Total capacity of sedimentation tanks: 2,542,000 gal.
Thirty-five circular bacteria beds, each 120 ft dia. × 6 ft deep.
Quantity of media in each bed: 2,500 cu. yd.
Total quantity of media: 87,500 cu. yd.
Grading of granite filtering media:—

In the eighteen original beds: 12 in. of 6-in. stone; 5 ft of $\frac{3}{4}$-in stone.
In the seventeen additional beds: 12 in. of 6-in. stone; 2 ft 6 in. of $1\frac{1}{2}$-in. stone; 2 ft. 6 in. of 1-in. stone.

Thirteen humus tanks, each 32 ft × 32 ft, divided below water level into four inverted pyramids 21 ft deep.
Total capacity: 915,000 gal.
Three 8-in. centrifugal recirculating pumps each capable of delivering 1,500 g.p.m.
Maximum combined capacity: 5 m.g.d.
18-in.-dia. C.I. recirculating delivery main 320 yd long.

Sludge-digestion plant

Four dewatering tanks, each 62 ft × 26 ft × 16 ft.
Total capacity: 645,000 gal.
Six primary digestion tanks, each 89 ft × 32 ft 6 in. × 16 ft.
Total capacity: 1,735,000 gal.
Sixteen secondary digestion tanks, each 40 ft × 23 ft × 15 ft.
Total capacity: 1,380,000 gal.
Total tank capacity: 3,760,000 gal.

Storm-water works

At Whitley:—

Eight tanks, each 140 ft × 29 ft × 5 ft.
Total capacity: 960,000 gal.
These tanks are followed by 8 acres of underdrained land.

At Canley:—

Six tanks, each 45 ft × 15 ft × 5 ft.
Three tanks, each 44 ft × 12 ft × 5 ft.
One tank 29 ft × 12 ft × 6 ft.
Total capacity: 192,000 gal.

At Finham:—

Two tanks, each 120 ft × 40 ft × 7 ft.
Total capacity: 435,000 gal.
Total capacity at all storm-water works: 1,587,000 gal.

APPENDIX II

DATA UPON WHICH THE POST-WAR EXTENSION SCHEME HAS BEEN DESIGNED

Pre-aeration unit

(Dry-weather flow 5 m.g.d.)
Aeration tanks: 4 hours D.W.F.
Compressed air: 15 cu. ft/sq. ft.
Settlement tanks capacity: 3·6 hours D.W.F.
Settlement tanks upward velocity: 10 ft/hour at 2 × D.W.F.
Returned activated sludge: up to 50% D.W.F.

Sludge-digestion plant

(Total quantity of sludge estimated to be 780 g.p.d. per million gallons of sewage treated.)
Dewatering tanks—two tanks, each 1 day's capacity.
Primary digestion tanks: 25 days' total capacity.
Estimated gas yield: 0·78 cu. ft per person per day.

Secondary treatment

Bacteria beds D.W.F. rate of dosing 140 gal/cu. yd.
Humus tanks capacity: 4 hours D.W.F.
Humus tanks upward velocity: 5 ft/hour at 3 × D.W.F.

APPENDIX III

DETAILS OF THE PLANT PROVIDED AT FINHAM IN THE 1950–55 EXTENSION SCHEME

Screening chambers

Two channels, each 15 ft × 8 ft with bar screens 8 ft wide × 6 ft deep and automatic raking gear operated by differential float control.
One 10-in. horizontal type disintegrator.

Detritus tanks

Two circular tanks, each 30 ft dia. × 6 ft 6 in. deep.
Total capacity: 57,360 gal (4 min retention on ultimate D.W.F.) with mechanical grit-collecting and washing mechanisms.

Aeration tanks

Six tanks, each 200 ft × 10 ft × 12 ft deep.
Total capacity: 900,000 gal (4 hours D.W.F.).

Settlement tanks

Three circular tanks, each 60 ft dia. by 24 ft deep. Total capacity: 750,000 gal (3·6 hours D.W.F.).

Sludge-dewatering tanks
Two tanks, each 45 ft × 25 ft × 14 ft deep. Total capacity: 200,000 gal (2 days' storage).

Primary sludge-digestion tanks
Four circular tanks, each 60 ft dia. × 36 ft deep. Total capacity: 1,800,000 gal (25 days' storage).

Gas holders
Four circular floating gas holders, each 59 ft dia. × 10 ft deep.
Total capacity: 120,000 cu. ft.

Humus tanks (Additional)
Four tanks, each 60 ft × 8 ft 6 in. average depth.
Total capacity: 590,000 gal.

Storm-water tanks (Additional)
Four tanks 120 ft × 40 ft ×7 ft.
Total capacity: 870,000 gal.

Pumping station
Four 16-in. Vickers-Gill horizontal-propeller sewage pumps. Capacity: 1,785 g.p.m., driven by 14·5 b.h.p. motors.
One 6-in. Lee Howl centrifugal sludge-return pump, capacity 250 g.p.m., driven by 7 b.h.p. motor.
Two 3-in. Lee Howl centrifugal surplus sludge pumps, capacity 80 g.p.m., driven by 3 b.h.p. motors.

Power house
Sludge-circulating pumps
Three A.C.M. centrifugal pumps:—pumps 300 g.p.m.; motors 6 b.h.p.

Gas engines
Two English Electric dual-fuel engines, 220 b.h.p.; 600 r.p.m.

Alternators
Two English Electric rotating-field type alternators, 150 kW; 600 r.p.m.

Air-compressing plant
Three Holmes-Connersville blowers. Capacity of each: 1,200 cu. ft/min at 6 lb/sq. in.
Three English Electric slipring motors: 65 b.h.p.; 975 r.p.m.

Heat-exchangers
Two Ames Crosta "Simplex" heaters. Capacity of each: 44 million B.T.Us. per day.

APPENDIX IV

THE WORK OF THE WATER POLLUTION RESEARCH LABORATORY AT COVENTRY (1945–54)

Biological filtration
Experiments carried out by the Laboratory at Minworth from 1938 showed that the rate of treatment of sewage in percolating filters could be increased by 2 to 4 times if the sewage was treated in two filters in series and the order of the filters was reversed periodically (alternating double filtration). It was also found that a similar increase in

[illegible]ate could be achieved by mixing final effluent with the sewage applied to the filter [illegible]recirculation).

The experimental plant completed in 1945 at Finham was put at the disposal of the [illegible]Vater Pollution Research Laboratory under the direction of Dr B. A. Southgate, C.B.E., [illegible]).Sc., with the object of testing these processes with a different sewage and of exploring [illegible]he possibility of adopting them on a large scale at Finham. It comprised four circular [illegible]ilters, each 6 ft deep and 12 ft dia., with water-wheel distributors, four upward-flow [illegible]umus tanks, each with an effective surface area of about 20 sq. ft, and ancillary plant [illegible]or pumping and gauging sewage and effluent.

In the first experiment one filter was operated by single filtration at 60 gal/yd/day, one [illegible]with recirculation of effluent) at 120 gal/yd/day, and a pair were operated at 120 gal/yd/ [illegible]lay as alternating double filters with a weekly period of alternation. The quality of [illegible]he effluents produced by all three processes was worse than had been expected from the [illegible]Minworth experiments. There were two main reasons for this—first, the strength of the [illegible]sewage fluctuated considerably, and secondly, the use of water-wheel distributors gave [illegible]rise to heavy growths of fungi and algae on the surface of the filters which caused prolonged [illegible]periods of ponding.

In November 1948 the water-wheel distributors were replaced by jet-driven distributors [illegible]and the filters were operated as two pairs of alternating double filters at rates of 90 and [illegible]120 gal/yd/day, but the final effluent from both pairs of filters continued to be of rather [illegible]poor quality.

At the end of 1949 electric drives were fitted to the distributors, each of which consisted [illegible]of a single rotating arm with nine jets, making one revolution in about 2½ min. This [illegible]caused a considerable improvement in the condition of the filters; surface ponding dis[illegible]appeared and the quality of the effluent improved.

From 1949 to 1952 the effect of omitting sedimentation of the effluent from the primary filter of alternating double filters, operated at rates of 140 and 180 gal/yd/day, was investigated. Unfortunately during the first half of 1950 the frequent occurrence of flashes of oil in the sewage seriously affected the performance of the filters. From October to December 1949, the two pairs of alternating double filters, operated at 140 gal/yd/day, produced final settled effluents having, on average, biochemical oxygen demands of 12·0 and 10·5 parts per million, and permanganate values of about 17·0 p.p.m. and containing about 25 p.p.m. oxidized nitrogen. The final effluent from the pair without intermediate settlement had the slightly higher biochemical oxygen demand.

The results of the latter part of this experiment when both pairs were operated at 180 gal/yd/day, showed that although the final settled effluents were on average slightly worse than the Royal Commission standard, only a slight reduction in quality resulted when intermediate settlement was omitted; no deterioration in the condition of the filters was apparent. During five successive periods from 1950 to 1952 the average biochemical oxygen demands of the final settled effluent, with no intermediate sedimentation, ranged from 14 to 29 p.p.m. compared with 12 to 33 p.p.m. from the filters operated with intermediate sedimentation.

The work carried out at Finham as well as experiments at Minworth had shown that the efficiency of a circular filter could be increased by slowing down the speed of rotation of the distributor. An experiment was begun in 1953 with all four filters operated as single filters at 360 gal/yd/day but with the single-arm distributors rotating at speeds of one revolution in ½, 2, 12, and 26 min. This filter loading was twice that used in the previous experiments on alternating double filtration. Whereas at the two highest speeds the surface of the filters became ponded, at speeds of one revolution in 12 and 26 min the condition remained satisfactory during the critical winter months. A speed of one revolution in 12 min has so far given the best results. Effluents were of good quality for the rate of treatment, i.e., they had about the same biochemical oxygen demand as effluent from a similar filter at Minworth operated at 200 gal/yd/day.

The progressive improvement in performance of the small-scale filters since 1946 may be attributed partly to improvements in distribution and partly to regulation of the amount of gas liquor admitted to the sewer and the absence during recent years of sudden flushes of gas liquor. This work continued the results of experiments carried out elsewhere on alternating double filtration and demonstrated the possibility of dispensing with extra humus tanks for intermediate sedimentation. It also emphasized that varying the dosing cycle has an important effect on the efficiency of percolating filters.

Removal of suspended matter from filter effluent

A marked characteristic of effluents from both large and small-scale percolating filters

at Finham is the presence of finely divided humus which does not settle readily in humu tanks and which has an appreciable biochemical oxygen demand in the settled effluent Experiments were started in 1948 on the removal of suspended matter by filtration throug a bed of sand. Results of earlier work with two small-scale sand filters, each 1 ft dia and 6 ft deep, showed that about 80% of the suspended matter could be removed b filtration through sand at rates of 100–200 gal/sq. ft/hour and that the biochemical oxyge demand of the effluent was thereby reduced by about 55%.

This work was continued in 1952 with two pilot-scale sand filters each 4 ft sq. an also with a micro-strainer filtering the effluent through a Mark 1A woven stainless stee fabric. One sand filter was used to treat effluent from the main works before settlemen and the other to treat the effluent after settlement in a humus tank; in each filter the rat of treatment was 100 gal/sq. ft/hour and at the end of each run the sand was cleaned b backwashing. About 80% of the suspended matter was removed from the unsettle effluent and the biochemical oxygen demand was about halved. The filter treatin humus-tank effluent removed about 60% of the suspended matter and reduced th biochemical oxygen demand by about 30%. The filter treating unsettled effluent had t be backwashed every 13–16 hours whereas that treating settled effluent could run fc more than 24 hours. The performance of the microstrainer was much poorer; at rates c 100–160 gal of humus-tank effluent per sq. ft of submerged fabric per hour only 30–40% of the suspended matter was removed, resulting in a reduction in biochemical oxyge demand of about 8%.

The result of experiments with the small-scale plant show that alternating doubl filtration is a successful process for the biological treatment of sewage at Finham. The also suggest that it might be possible to achieve some saving in cost by omitting inte mediate sedimentation of the primary effluent. Results obtained so far from the exper ments on periodicity of dosing indicate another way of increasing the purification capacit of existing filters (Fig. 8). One feature of this small-scale work has been that effluents better quality than those from the largo-scale plant at lower overall rates of treatment ha been obtained.

The presence of relatively large amounts of fine suspended matter, difficult to remove i humus tanks, in the final effluent is partly responsible for its poor quality. Treatment effluent either before or after sedimentation by mechanical filtration in sand filters or in micro-strainer would remove much of this suspended matter.

Treatment of gas liquor in percolating filters at Stivichall

The most important industrial effluent, so far as the treatment of sewage at the Finha works is concerned, is gas liquor. It was decided to investigate the effects of gas liquo and of different constituents of gas liquor, on the treatment of sewage by biologic filtration and for this purpose the sewage works at Stivichall was placed at the dispos of the Water Pollution Research Laboratory in 1947. One part of this investigation w concerned with the testing of different concentrations of liquors from the Coventr Leamington, and Hinckley gas works when added over periods of several months to t sewage treated under normal works conditions in the two full-scale filters. In the oth part the effects of different constituents of gas liquors on the biological filtration of sewa were followed in a number of small filters 1 ft dia. and 6 ft deep housed in a hut adjace to the large filters. In this second part the gas liquor was submitted to processes extraction at the Gas Research Laboratory at the University of Leeds and solutions of t "fractions" obtained were then added to the sewage treated in the filters and the effluen were compared with that from control filters treating sewage only.

The large-scale experiments (Fig. 9) showed that a normal spent liquor added in concentration of 0·5% of the volume of sewage treated caused a marked deterioration the quality of the filter effluent. At Hinckley gas works, however, where tar fog removed from the hot gas in an electrostatic precipitator before condensation begi retort-house liquor is kept separate, and steps are taken to limit the amount of thiocyana formed, a spent liquor is produced which has a much smaller effect on biological filtratic At most gas works the retort-house liquor, which may constitute 10% of the total ma of liquor, is combined with other liquor. The composition of the spent liquor fr Hinckley differs in three main respects from a normal spent liquor—it contains few polyhydric phenols, less fixed ammonia, and less thiocyanate. Hinckley retort-hou liquor had a marked effect on biological filtration in concentrations of only 0·1%.

The small-scale experiments on constituents of typical crude vertical-retort gas liqu showed that monohydric phenols are fairly readily decomposed but dihydric phenols

more resistant to treatment. Certain other constituents—carboxylic acids and material similar to humic acids—have a distinct effect. The liquor residue after solvent extraction had the greatest effect of all the constituents tested separately. This contained all the ammonia and thiocyanate as well as small quantities of unidentified substances. Fixed ammonia was an important constituent of the residue.

The different constituents of Hinckley retort-house liquor were also tested in the small-scale filters. The main effect of this liquor may be attributed to polyhydric phenols and ammonia.

It may be concluded that if electrostatic precipitation of the hot gas is practised and the retort-house liquor is disposed of separately or is treated for the removal of higher tar acids and ammonia a liquor far more amenable to treatment at a sewage works is likely to be obtained.

Discussion

The Authors introduced the Paper with the aid of a series of lantern slides.

Mr C. D. C. Braine (a Partner in the firm of G. B. Kershaw and Kaufman, Consulting Engineers) said that the Authors' interesting historical Paper sketched broad outlines only and did not deal with detail. It was not easy to discuss because so many important pieces of information had been omitted from the text. For instance, not a single sewage analysis or effluent analysis was given. He suggested that even in an historical Paper that was an important omission, because it would have been interesting and instructive to compare the sewages of 1910, 1930, and 1950—taking 20-year intervals—and to study the effluents from the works at the different times in the light of the designs adopted to meet the circumstances at those particular times. In any case it would have been a valuable record of changes in sewage over a given time.

One of the most striking omissions was the failure to mention the increase in the daily sewage flow *per capita* over the passing years, yet that was an extremely important trend, the end of which was still uncertain. He inferred from the Paper that in 1932 the flow had been about 33 gal/head/day, whereas it was now more than 50 gal/head/day, which represented a 50% increase. In 1932 the population had been about 180,000, whereas today it was about 270,000, so that the total increase in the flow to be handled over the period was approximately 2¼ times the original, and not 1½ times, as might have been gathered from a casual reading of the Paper. If the suspended solids content of the sewage had kept pace with the flow of sewage, as he suspected it had, the sludge-disposal problem, about which almost nothing had been said, would have given a good deal of trouble. The problem of storm-water still seemed unsettled, because, if his interpretation of the information given was correct, the storm-water tankage was still only about 3 hours dry-weather flow, which was low. It might be assumed, therefore, that the Severn River Board had been markedly tolerant, to say the least. If so little storm-water reached the works that the existing storm-water tanks proved more than adequate, was it to be assumed that the sewers in the town were now completely inadequate and that a good deal of sewage escaped to the river?

Perhaps the most important issue which the Paper brought out was the great weakness which existed in many large authorities, namely, the lack of specialist design experience in the field of sewage disposal. That might be the reason why few large authorities had themselves constructed within a reasonable period of time successful sewage-disposal plants. Usually the process was so painfully slow that increases in population and water supply overtook the new extensions, which were out of date or under-sized before they were even completed and in operation. It should not be thought that he was critical of Coventry itself; he was merely critical of the existing municipal system in the United Kingdom.

A good deal had been said about experimental work. He was a tremendous believer in it, but he believed—he might be completely wrong—that many of the larger authorities

undertook experimental work which could be considered unnecessary. The Authors h described a number of such cases. It was stated that, some time after 1910, the Covent Council decided to conduct an experiment to determine whether or not Coventry sewa could be satisfactorily treated on bacteria beds, whilst next door, so to speak, the Ta and Rea District Drainage Board had 30 acres of bacteria beds treating some of t strongest sewage in England, and an industrial sewage, a metallic sewage, very akin that at Coventry.

The same situation arose in respect of the experiments carried out at Coventry on part treatment, i.e., bio-aeration followed by filters. At the time that that particular expe ment had been started at Coventry, the bio-aeration plant at Minworth had been in ope tion for 10 or 12 years. After 2 years' work, the Coventry experimenters had confirm results which specialists in that field had accepted for nearly a decade.

In 1941 the same administrative process—he did not blame individuals, it was t process which was to blame—had once again been repeated, when experiments to det mine the effect of heating digested sludge had been carried out. After another 2 years work, conclusions had been reached which were given in the Paper, namely, that it pa to heat sludge-digestion tanks and that a useful yield of sludge gas could be obtained fr heated tanks. At that stage, as was well known, there were heated sludge-digestion tan in England, in Europe, and in America, and for 6 years there had been at Mogden (whi was the largest disposal works in the world outside the U.S.A.), a system whereby the wh of the power used there was obtained from sludge gas from heated digestion tanks. O could not help wondering whether all the experiments had been necessary at Covent any more than were similar experiments which had been carried out elsewhere.

Mr Braine was always a little astonished that the Ministry permitted those repetiti experiments to be carried out. If new ground was to be broken, as was frequently t case, he was wholly in favour of experiments which were essential, but to go over and o the same ground was another matter.

At the present time, with acute shortages of staff in every office, it seemed essential t repetitive experimental work should be sternly discouraged, whilst helpful experiments new fields should be assiduously encouraged. The Ministry had surely a duty there, a Mr Braine felt certain that Dr Southgate and his department, who had done first-cl work at Coventry, would welcome the opportunity offered for such work.

Towards the end of the Paper the Authors had referred to sundry trends in sew purification. With those he agreed, but one of the most important trends had not be mentioned, namely, the degree of specialization that was now required if a purificat plant was to be properly designed and thereafter economically operated.

Many large authorities had their own water engineers, but, so far as Mr Braine kn few had a comparable organization for dealing with drainage.

He believed that the time had come when in large towns that state of affairs should changed. He thought it likely that shortage of staff would bring it about, and that sm regional boards would be set up to look after a dozen or more works, depending on s and conditions, and the work centralized, as it was in many cases with a water departme

In speaking as he had done, he in no way decried the first-class work done by the t Authors. He had merely dealt with what he considered was perhaps the most import of the broad issues raised by their Paper.

Dr B. A. Southgate (Director, Water Pollution Research Laboratory) said that might be able to throw some light on one of Mr Braine's problems. The Water Polluti Research Laboratory had worked for some years at Minworth developing a system alternating double filtration, which had not originated in their laboratory but had b taken over from Birmingham, and they had shown that it was possible to treat betw two and three times the volume of sewage which could be dealt with by single filtrat per cubic yard of medium. Almost everyone in the profession had then said that it co be done at Birmingham only because it was a peculiar sewage. The Water Pollut Research Laboratory had therefore been extremely glad to accept the offer which been made at that time by Coventry to carry out confirmatory experiments at their wo

He would like, on behalf of the Laboratory and of his Department, to thank the City of Coventry and Mr Berry for all the help which they had given. It was a great pleasure to hear that at any rate something of practical utility had come out of the work. He agreed that for a new system of the kind in question at least one set of confirmatory experiments should be made, and he thought that it had been a wise decision to do that at Coventry.

It was now being said that although quite a number of such plants had come into service little had been written about their operation, and that seemed to be true. The Water Pollution Research Laboratory had therefore examined one of the plants and hoped to look at others. From the first one which they had examined it seemed that the original conclusions reached from the work at Minworth and at Coventry were being reproduced.

It was commonly said—he heard it often in his Department—that the British were slow to put the results of research into practical use. He had always said that that did not apply to improvements in sewage treatment. The present Authors, for example, had shown a picture of a prototype machine for regulating the speed of rotation of jet-driven distributors. The experimental work on which that had been based was not at all complete—though it had shown, he thought, that there was an optimum rate of distribution—yet the idea was already being tried in large-scale practice.

Two points had occurred to him when reading the Paper. When the Water Pollution Research Laboratory had been working at Coventry he had been struck by what happened at the Baginton Works, where sewage effluent flowed over grassland. He remembered mentioning it in a Paper[1] which he had read to the Public Health Engineering Division. He had been struck by the tremendous improvement which had occurred in the quality of that effluent merely by passing for quite a short distance over the surface of grass. It was something that the Laboratory was often asked about, and he thought that it was a very useful final polishing process, particularly for a small sewage works.

The other point was that in the present trends of treatment referred to at the end of the Paper he would have included the probability that many sewage effluents might in the future have to be treated to remove at any rate a large proportion of the suspended organic matter from them. He believed that that would have to come about in the future because quite a large part of the oxidizable organic matter resided in the suspended matter. Unlike the material in solution, it was not carried down the stream, thus being oxidized slowly over a distance of many miles; on the contrary, it was deposited in quite a small length below the outfall, and a large part of the oxidation was therefore concentrated in that small distance. That was one of the reasons, he thought, why an otherwise inexplicable high de-oxygenation of a stream often occurred below an outfall. He was interested to see that Coventry was beginning to have success with micro-filtration of their effluent, a process which, with sand filtration, he thought would become more widespread.

Dr Arthur Key (Senior Chemical Inspector, Ministry of Housing and Loca lGovernment) observed that the very rapid growth of Coventry had created grave difficulties, and some of the difficulties in relation to sewage disposal had been described by the Authors. The same rapid growth had, however, one advantage, in that it provided the designing engineer with a good answer to those who said that his designs were either too small or too large! If it was suggested that he had provided too much capacity, he could reply that he did not like to spoil the ship for a ha'porth of tar, and that any spare capacity would be taken up almost before it was provided by the needs of the growing community. If, on the other hand, it was suggested that he had provided too little, he could reply that he was of an economic turn of mind and tried to do things as cheaply as possible, and that if he had made an under-estimate it would be put right in a couple of years when the next extension took place! Seriously, such a situation did provide an opportunity for the engineer, because over a period of years it meant that there was a large-scale plant which he could investigate, and the performance and capacity of which he could determine far

[1] B. A. Southgate, "Pollution of Streams: Some Notes on Recent Research." Public Health Paper No. 1, Instn Civ. Engrs, 1951.

more accurately than in any other way. He could then use that information to provid the final stage (if there ever was to be a final stage at Coventry) accurately designed an computed—far more so than if the original scheme had to be the ultimate scheme and i was necessary to work without information of that kind.

On p. 47 the Authors had stated "Here, then, was a convenient means of increasing th capacity of the Finham Works merely by interpolating an activated-sludge unit betwee the existing settlement tanks and the bacteria beds." Dr Key would not have commente on that sentence had it not been for the word "merely," which seemed to suggest that t interpose an activated-sludge plant between the settlement tanks and the bacteria bed was the simplest thing in the world to do; he did not think that it was. Later on the sam and subsequent pages the Authors had described how that had been done at Baginton, an the implication was that the two plants, by working in series instead of in parallel, turne out a better effluent. So far as Baginton was concerned, he did not intend to disput that at all, but the discussion was concerned with generalities, and if it was intended t express a general opinion that a two-stage process was better than a one-stage process h did not know whether he could go all the way with the Authors. One of the points o which he would want information before coming to a decision had been mentioned by th Authors. They had said that the humus tanks serving the bacteria beds were in thos circumstances too small. That seemed natural, because the beds were treating a fa greater volume of liquid than they had been designed to treat, and the humus tanks ha to remove the humus from that greater volume. Fortunately at Baginton there had bee an area of land which served the purpose of humus tanks. Had that not been the cas the cost of the extra humus tanks which would have had to be provided would have to b set against the advantage of the two-stage treatment.

The second of the points concerned the activated-sludge-settlement tanks, which i those circumstances would also have been too small, he thought, because they, too, ha to treat the whole of the sewage instead of merely part of it. He would like to have th Authors' comments on that, but he certainly thought that in those circumstances it woul be necessary to provide the activated-sludge plant with larger settlement tanks than if i had been merely treating part of the sewage, and the extra cost of those tanks also woul have to be set against the possible advantages of two-stage operation.

Speaking of two-stage operations, he had often wondered why no one had ever invest gated methodically the possibility of using activated-sludge following and not precedin filters. Perhaps they had done so, but if so he had not noticed it. On paper at any ra it would seem that that system offered some advantage. The sewage would first reac the filters, and only secondly the activated sludge. It was commonly supposed that filte were better able to withstand sudden shocks, caused either by a phenomenally stron sewage or by the presence of toxic materials, than was activated sludge. Secondly, t final tanks would have to be activated-sludge-settlement tanks, and it was usual, thought, to find that effluents from activated-sludge-settlement tanks contained le suspended solids than effluents from humus tanks. In his experience, effluents were unsatisfactory quality more often because of a high content of suspended solids than fro any other single cause.

Thirdly, it was quite possible that, with activated sludge following filters, it would found that the humus from the filters could be allowed to pass through to the activate sludge plant and then be removed in the final tanks, avoiding the duplication of settleme tanks which he had previously stated to be a disadvantage of dual treatment.

He was not asserting that the system which he had described was a good one, becau he did not know whether it was or not; but on paper it seemed to have some advantag and might be worth further investigation. He had reason to suppose that he was not t only person who was thinking along those lines at present. Perhaps one's though travelled in that direction automatically, having regard to the recent results of the Wat Pollution Research Laboratory, which showed that provided filters were not expected carry out a great percentage of purification they could carry out a great amount purification.

Mr G. W. Bennett (Manager of Coventry Sewage Works) said it was stated in the Paper that in 1874 an Injunction had been issued against the Corporation of Coventry. He believed that the conditions were very much better downstream now, not necessarily because of Coventry's methods of sewage disposal but possibly because the towns lower down had found alternative sources of drinking water.

Sludge disposal had not been dealt with fully in the Paper. Some members of the Institution had visited the Finham Works recently and had seen the heated sludge-digestion tanks. As stated in the Paper, Coventry had the other works at Baginton, and all the sludges were pumped to the sludge-disposal area on the site south of the airport at Baginton, where there are unheated secondary digestion tanks. At present, owing to the acute labour shortage in Coventry, the existence of which must be emphasized, it was impossible to carry out the normal sludge drying process, but at present they were dealing quite adequately with the sludge by the Refuse Section of the City Engineer's Department carting refuse there and making suitable lagoons which Mr Bennett filled up with sludge. That did not sound a very scientific way of dealing with the sludge problem, but it was cheap, and they were very fortunate in having enough suitable land to be able to continue to use that method for many years to come. It should be remembered that Coventry was a manufacturing town, so that there was a large proportion of trade wastes, including many toxic substances; the sludge was, therefore, totally unsuitable in its present condition for agricultural purposes. Experiments had been carried out with the sludge, and it had been found that some of the toxic metals created problems in agriculture, which was one reason why they did not press its use for agricultural purposes.

The reason why the activated-sludge plant and bacteria beds had been combined during the war, a matter to which Dr Key had referred, was that Coventry had taken in great numbers of war workers, hostels had been built, and it had been necessary to take more water from the River Avon—about another million gallons per day. It had been found that Coventry's sewage disposal works were not large enough, but by combining the two plants they had been able to give better treatment to a further million gallons, and the conversion at that time had cost a mere £2,000. Mr Bennett did not think that even the experts could build an extension to a sewage works for one million gallons for £2,000. The grassland was now giving a little trouble, as was to be expected.

There had been trouble in the past owing to gas liquor, and work was going on at the present moment to take some of the colour from the gas liquor. They had had in the past a good deal of ponding on the bacteria beds, and during the work carried out by the Water Pollution Research Board there had been troubles due to ponding during the winter months. It was mentioned in the Paper that various types of distributors had been used, but when the electric controls had been attached and the distributors operated so that the interval between doses was approximately 2½ min, some of the ponding had been relieved and the conditions improved. The Water Pollution Research Laboratory, as a result of work which they had been doing at Birmingham, had then put on controls and controlled the four distributors at different speeds. It had been found that the best results were obtained in the winter with the distributors operated at a given dosing of approximately 12-min intervals. In Coventry they were following that up, and experiments were still continuing by controlling two of the larger distributors on the works. They had not given any figures yet, but one of those controls had been in operation for about 12 months and was showing very good results. It was hoped that before long they would be able to control a block of eight filters. That block happened to have its own humus tanks, and they would be able to compare it with a block having the same type of effluent but without controls. He had no doubt that the City Engineer would report on the work in a few years' time; experimental work took time to carry out.

Dr F. Wormwell (Head of Corrosion Group, Chemical Research Laboratory, Teddington) referred to possible trouble arising from corrosion of sewage-treatment plant, and the means of prevention. Any system of the kind in question involved the use of metals, and he would like to know whether any appreciable trouble was experienced with such equipment because of corrosion and, if so, whether any attempt was made in designing the

equipment to take care of it. For example, he assumed that the metals used were probably mainly steel and cast iron, but possibly non-ferrous metals were also employed, and any information on the subject would be of interest.

He had received an inquiry recently from an engineer who had apparently experienced corrosion in part of the sewage-treatment system and who had asked if cathodic protection would be useful. His reply had been that in principle it should be, i.e., if a metal was immersed in water in conditions conducive to corrosion, it was theoretically possible to control that corrosion by applying the principles of cathodic protection, but it should be remembered that alkali would be produced on the cathode—i.e., on the metal which was to be protected. That alkali could help in the prevention of corrosion of ferrous metals, but it could and sometimes did cause corrosion of non-ferrous metals such as zinc or aluminium alloys.

Another possible difficulty which had been mentioned by one of his colleagues was that the development of alkalinity could interfere with bacterial action in some cases. They had been concerned at Teddington only with the activities of sulphate-reducing bacteria in stimulating corrosion. Alkalinity did affect them and might affect others.

The other point lay outside his own field, but at the Chemical Research Laboratory experiments had been carried out in the development of methods of utilizing the organic material in sewage for the stimulation of the activities of the sulphate-reducing bacteria by which they could convert sulphates to sulphur—ultimately as a possible source of sulphur. He believed that experiments had been carried out, and that more were projected elsewhere. He wondered whether at Coventry the economic possibilities of the production of sulphur from sulphates had been considered.

Mr W. A. M. Allan (Divisional Engineer, London County Council) said that on p. 44 reference was made to the Baginton activated-sludge plant, which would appear to have been installed in 1924 and had not proved satisfactory. It would be interesting to know the reason for that in view of the fact that the original experimental plant was not of the air-blowing type. Could the Authors state what was the type of the experimental plant and whether it had proved to be satisfactory?

Secondly, it would appear that a considerable length of sewers had been constructed in concrete. Mr Allan would be interested to know how old those sewers were and whether they had suffered any deterioration. It had been his experience that where crude sludge had to be carried, concrete was not always the best material to be used.

Owing to the presence of certain trade wastes, the sludge was apparently unsuitable for use as a fertilizer, therefore there was at Coventry, as in many other places, the problem of having to dispose completely of all the sludge, and where fields were available, as they were at Coventry, a hole was dug, filled with sludge and a "hope for the best" method again was adopted. He knew that that was an easy and cheap method of doing the job. In London at present also the line of least resistance was taken, and the outfall works were situated on a tidal river; shipping the sludge to sea was easy and cheap.

The time, however, was coming when other means of dealing with sludge would have to be adopted, and although sewage works managers always had that problem in mind, he suggested that further experiments should be undertaken in the hope that before long a practical and economic method of sludge disposal might be devised.

Mr B. F. P. Babcock dealt with one of the points which had been raised by the Authors regarding the scraping of sedimentation tanks. He said that he had had some experience with tanks of about similar size to those at Coventry, namely thirteen of 110 ft long, 30 ft wide, and 9 ft deep, built about 1890. In that case a single machine scraper had been used, and the form of machine adopted had been one with retractable transferring wheels. The difficulty there had been the adjustment of the machine's interlocking controls, which were very complicated indeed. If he was faced with a similar problem again he would adopt the system using individual scrapers for each tank. Assuming that

the tanks were the old-fashioned horizontal-flow tanks, with hoppers at the inlet end for the reception of sludge, very cheap and light scrapers which could be made suitable for running on the walls of the tanks without great modification were available. Looking at the matter from the point of view of the sewage-works manager, who had to keep the plant going while constructional activities were going on, the difficulties were very great. While Mr Babcock had been working on that scheme he had fully appreciated the difficulties of his "opposite number." He thought it would be found that by using the suggested form of scraper the work of construction would be made very much easier, as compared with having to demolish the existing tanks before constructing three big circular tanks; furthermore, the continued treatment of the sewage would not be materially hampered.

He then referred to Dr Key's mention of the possibility of putting an activated-sludge plant after the filters. Mr Babcock had thought at one time that such a process would show quite a number of advantages, but, in the past few years, a great deal had been heard about synthetic detergents, which he understood, caused the substantial foaming difficulties especially towards the outlet end of the aeration plant. Presumably, therefore, having the activated-sludge plant following the filters would result in enormous foaming difficulties, unless a very high concentration of suspended solids was retained in the liquor or some other method discovered for preventing that condition.

The Chairman (Mr C. A. Risbridger, Chief Engineer, City of Birmingham Water Department) referred to the section of the Paper entitled "Present Trends in Sewage Purification," and to the paragraph which read: "One wonders whether this practice may not eventually lead to closer co-operation between sewage-disposal and water authorities." He himself had often wondered that, but during the past 2 or 3 years there had been some rather disappointing experience. It had been thought that with the passing of the Rivers Boards Act and of the Rivers (Prevention of Pollution) Act, and with the publication of the Report of the Rivers Pollution Prevention Sub-Committee of the Central Advisory Committee, there would be some improvement in the condition of rivers from which potable supplies, or supplies which had to be made potable, were drawn, and in the Report reference was made to the desirability of making by-laws—which the River Boards now had powers to make—differing according to the different uses to which rivers were put. That inferred that a rather higher standard might be expected for rivers from which water was extracted for potable purposes than that recommended by the Royal Commission, which had been designed, he believed, more to keep rivers in an ordinary good condition for general purposes than for potable supplies.

During the past 3 years he was aware of a case where, after a water-supply undertaking had used a river for about 70 years for water-supply purposes, a local authority had decided to build a sewage-disposal works. That was an excellent decision because it was on the gathering ground and the sewage effluent had to get to the river in any case. There had been an inquiry by the Ministry, and the water-supply undertaking had represented that the effluent from those sewage-disposal works ought to be to a somewhat higher standard than that recommended by the Royal Commission. Unfortunately, the Ministry did not appear to agree, as no promise of a higher standard of effluent from those works had been obtained than if the river were merely used for general purposes.

He suggested that either the authorities concerned did not have the powers which they ought to have or, if they had them, they had not the courage to use them. He suggested very strongly that something would have to be done to ensure that where a river was used for the abstraction of water for potable purposes, it should be protected by by-laws calling for a higher standard of effluents discharged thereto than those which might be adequate in respect of effluents discharged to a river not so used.

Mr Deeley, in reply, said the merit of a Paper, he had heard, could be gauged by the quality of the discussion it provoked. If that were true the Authors were entitled to feel a little flattered, because the present discussion had been both comprehensive and stimulating.

It was true, as Dr Key had said, that rapid growth helped the sewage-works designer not, Mr Deeley suggested, just because it provided a "get-out" if he had designed wrongly, but because frequent extension made it possible to take advantage of the latest developments.

He was sorry that the word "merely" had been used in referring to the interpolation of the activated-sludge unit before the bacteria beds. What the Authors had had in mind when that sentence was written was not that it was something easy to do, but that it was something which could be more readily done and which would cost less money than any other method which they could devise, for increasing the capacity of the Baginton works by 1,000,000 g.p.d.

Dr Key had perhaps misunderstood the Baginton proposition a little, no doubt to some lack of clarity on the part of the Authors. When that plant had been converted tp the partial process (by putting the activated-sludge plant in front of the bacteria beds) the *whole* of the aeration plant was not used. Only two of the five aeration units were incorporated but the whole of the activated-sludge settlement tanks were used so that it was not necessary for the capacity of the settlement tanks to be increased.

Dr Key suggested that serious consideration should be given to the suggestion for a plant in which the activated-sludge unit followed the filters, instead of preceding them. It might therefore be of interest to mention that that had been tried in a rather preliminary way at Finham. It had not been in any sense a controlled experiment; they had used a tank of a certain size simply because it happened to be available, and the investigation had not been, in any way, quantitative. Filtered effluent had been fed into the tank and some sludge circulated, but that experiment was abandoned after several months because it had never succeeded in producing any flocs at all. Mr Deeley was unable to say whether that was attributable to the crude methods which were adopted or to some oeculiarities in Coventry's sewage. He did not suggest that it had been an experiment in any real sense, but merely a preliminary try-out which had proved to be far from promising.

Mr Allan had called attention to the fact that the activated-sludge plant at Baginton had been designed to operate on the air-blowing system, despite the fact that the experimental plant had not been of that type, and pointed out that no reasons for the presumed comparative lack of success of the earlier plant were given. Mr Deeley mentioned that the whole of the records relating to the work done in Coventry prior to November 1940 had been destroyed in the Coventry "blitz," and that detailed information relating to the first activated-sludge experiment was not available. It was known, however, that the experimental plant had been on the paddle-wheel system, and that must have shown some promise, since the Corporation had decided to go ahead with the process. It was still rather a local mystery why the full-scale plant, which was constructed in 1924, was of the air-blown type.

With regard to the suggestion that the performance of that plant had proved to be disappointing, the disappointment lay in the fact that it had been hoped that the plant would accommodate a dry-weather flow of 2,000,000 g.p.d., but that in practice it had been found that it would produce an acceptable effluent only when used to treat 1,250,000 to 1,500,000 g.p.d. It had not been disappointing in the quality of the effluent produced but in the quantity of sewage which the plant would accept.

In reply to Mr Allan's question as to whether there was any evidence of damage to concrete from crude sewage Mr Deeley said that there had been no experience of damage to concrete either at the sewage works or in sewers. In brick-lined sewers dealing with sewage containing a large proportion of trade effluent there had been trouble with the cement-jointing mortar of the brickwork, but no evidence had been found of trouble with concrete.

The question of the best final means of disposing of sludge was one which Mr Deeley could not answer. He was glad that it had been raised, because it was an aspect of sewage treatment which was too often neglected. An enormous amount had been written about the biological and settlement processes, but they still had a long way to go before they found a complete solution of that problem of the ultimate disposal of sludge.

The Authors were grateful to Mr Babcock for giving his experience in connexion with the mechanical de-sludging of tanks. They had given that matter a great deal of thought, and realized that there were several factors which entered into it. It was not just a question of economics; the condition of the existing tanks had to be considered and arrangements made to deal with the flow while any major improvement scheme was being carried out.

Mr Granville Berry, who also replied, said Mr Braine had dealt with the broad issues involved and had also said that not a single sewage analysis was given in the Paper. Mr Deeley had dealt with that so far as the past was concerned by referring to the complete destruction in the "blitz" of all their records. Mr Berry suggested that in any case it would not necessarily be of great value to give such information at the present stage. Were they to give a detailed analysis of the position today, when the works were passing through a transitional stage, or were they to give the detailed analysis for Baginton, for Finham, and for the other small works which they still had in the city, and which would eventually be absorbed into the great Finham works of the future?

Mr Braine had referred to an increase from 32 to 50 gal per head during 20 years, but it was necessary to take account of the war years. During the 1939–45 war there had been considerable damage to main sewers in Coventry, and they were still getting a considerable amount of infiltration into the sewers which they could not trace. One length of trunk sewer which they were now re-laying had been affected considerably and had been subject to subsidence. As a result of mining operations it had sunk about 5 ft 6 in. All those factors affected the figures to which Mr Braine referred. The Authors agreed that the storm-water provision at the moment was insufficient, and at present they were very considerably increasing it and at the old Whitley Works, which was 100 years old, they were now reconstructing the storm-water tanks which had been there all that time. It was a matter of which they were fully aware, and they were anxious to increase the capacity as quickly as possible.

On the question of lack of experienced engineers and of the collaboration which was needed, Mr Berry said that the general organization of a large municipal engineer's office called for special staffing considerations where a large and varied programme of engineering works had to be carried out. In the larger offices there was a high degree of sectionalization and specialist staff employed on capital works and that arrangement did ensure closer co-operation than was otherwise possible. They had been the "guinea-pigs" about 18 months previously for an "O and M" investigation of their organization by the Treasury. One thing which stood out in the "O and M" report was that the work which they were doing on capital schemes was not costing, so far as staff salaries were concerned, more than 3% of the capital cost. In addition, "O and M" indicated that in their considered opinion the most satisfactory method was not to have a large number of self-contained departments, which was what Mr Braine seemed to have in mind, but that one department should be responsible for engineering services generally, so that the closest collaboration was possible.

Mr Berry felt that Mr Braine, in the remarks which he had made about research, was very much out of date and still did not appreciate the real value of the research and investigations which had gone on in various activities during so many years. It was impossible to shut down all external research and concentrate it all at Stevenage or elsewhere. Dr Southgate had made very clear, in following Mr Braine, the value of having confirmatory experiments carried out at other places. The sewage at Coventry, for example, was vastly different from that at Luton, and quite different results were obtained from similar experiments. It was impossible to base the design of future sewage works on one centralized series of experiments.

The Authors would like to thank Dr Southgate for his very kind remarks. They had greatly appreciated working with him and meeting him regularly to discuss their results. The necessity of confirmatory experiments had been shown very clearly. The illustration on one of the slides of the speed-controlling mechanism indicated that the Water Pollution

Research Laboratory could get good results quickly, and Mr Berry was certain that that applied to all the laboratories of the D.S.I.R.

So far as Baginton was concerned and the improvement in quality obtained by passing over grassland, about 20 years ago Mr Berry well remembered Mr Sandford Fawcett, who had done such a grand job in the United Kingdom for sewage treatment, saying, in answer to some highly technical question about sewage treatment, that land treatment was still the best. It might well be that the best treatment today was still land treatment, but it had to be realized that it was too expensive and too great a luxury; they could not afford to devote 1,000 acres of land to that purpose.

Dr Southgate had emphasized the importance of the humus problem, which was one of those which had to be tackled in the future. As the Authors had tried to point out, it was and always had been the great problem in sewage-treatment at Coventry. In Mr Berry's experience of about seven local authorities, Coventry was unique in that respect; he had never met the problem to the same extent elsewhere. Greater attention had to be given at that stage, particularly in relation to the quality of the rivers into which the effluent was discharged.

Mr Deeley had dealt with some of the points which Dr Key raised in regard to Baginton and the possibility of introducing activated-sludge treatment after the bacteria beds. They had made some inquiries but they had been unable to find in the United Kingdom or in American practice any worthwhile experiments which had so far been carried out. As Mr Deeley had said, the experiment at Coventry had not been very encouraging, possibly as a result of the type of equipment used. Mr Berry felt that further experiments should be carried out along those lines.

He was glad that Mr Bennett had joined in the discussion, since someone with managerial experience qualified on both the mechanical and the chemical sides of sewage treatment was invaluable on any large works. Mr Bennett had referred to the use made of refuse, which was an example of co-operation within the department. Whilst Mr Berry agreed that it was not necessarily a permanent solution it was one that was available at the moment and a means of eventually bringing back into productive use areas of land which would otherwise be sterilized, while helping with the disposal of domestic refuse.

Dr Wormwell raised the question of corrosion and cathodic protection on sewage works. They had not yet tried out any work on those lines at Finham, because they had not had any serious corrosion problems. Usually if they met with trouble it concerned the distributors and the diffusers. The distributors which they had replaced in 1954 had been in use for 20 years and, whilst there had been some crumbling in parts of them, it had not been uniform and was no greater than expected during such a period. The diffusers at Baginton had been renewed about 4 years ago after being in use for 13 years. Mr Bennett had tried various protective paints in order to prolong their life and safeguard them, and the length of life obtained from the equipment was regarded as fairly satisfactory.

The danger, of course, was always with surfaces which were subject to alternate wet and dry conditions, and the secret, as Dr Wormwell would probably agree, was to give protection at the waterline. The corrosion problem in sewage works was far more serious with steel than with cast iron. On the question of corrosion, a series of experiments had been carried out in Coventry about 5 years ago with aluminium alloys. The alloys had been suspended in humus tanks, partly under the liquid and partly in the dry. Mr Berry thought that six different types of aluminium alloy had been used. Some of them just disappeared and some stood up fairly well. He believed that commercial use had been made of the results of the tests which had been carried out at Finham and that they had been used in certain constructional work in the United Kingdom on sewage works built in the past 3 or 4 years.

Mr Deeley had dealt very well with the points raised by Mr Allan. So far as damage to sewers was concerned, there was one works in the city from which they received about 1,000,000 g.p.d. The previous authority with which Mr Berry had served had to deal with a similar type of effluent from another of the firm's works but the problem had been vastly

different. In the one case there had been serious damage to sewers and in the other no trouble at all, largely as a result of the many different types of trade effluent which had to be dealt with and which had some compensating influence.

The Authors agreed entirely with the Chairman that there should be closer co-operation between sewage and water authorities. Mr Berry believed that the day would come when there would be a far more regional conception both of sewage works and of water-supply undertakings and that higher standards would have to apply all round, and not merely where the discharge was into small volumes of river water. At Coventry they had had very helpful co-operation from the Severn River Board, both sides knowing that only by working together could they meet the needs of the city and maintain the Avon in a good state.

The closing date for Correspondence on the foregoing Paper has now passed without the receipt of any communication.—Sec.

STRUCTURAL AND BUILDING ENGINEERING DIVISION MEETING

22 November, 1955

Mr Ralph Freeman, Member, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Structural Paper No. 45

PRESTRESSED CONCRETE AS APPLIED TO BUILDING FRAMES *

by

† **Francis Walley, M.Sc., A.M.I.C.E. and Hugh Campbell Adams, B.Sc.**

SYNOPSIS

The early use of prestressed concrete for building work was largely confined to simply supported beams, acting alone or in conjunction with in-situ slabs; continuity in any form was not attempted. Subsequently it has been applied to building frames of various types.

The Paper describes the progress from simply supported structures, through semi-continuous structures (the Sighthill Stationery Office Store) to the fully-continuous building frame, illustrated by a single-bay portal with prestressed legs and beams; a three-bay portal structure with reinforced legs and prestressed beams, and a fully framed five-storey building with reinforced concrete columns and prestressed beams, the whole being designed as a continuous frame. The design, erection, and the experience gained are fully described. The Paper discusses further possible solutions to the problem of the application of prestressing to building frames, with appropriate illustrations of methods tried in other countries, and points out the difficulties which remain to be solved.

INTRODUCTION

PRESTRESSED concrete in its infancy in Britain was largely used for simply supported beams. Since this was so, the techniques of stressing and the appreciation of the new processes at work could more readily be developed and assimilated without real complication from the design side. Indeed, the designing of simply supported prestressed concrete units, being less complex than reinforced concrete work, enabled attention to be concentrated more on the mechanics of the process than on the drawing office procedure. Although even to-day probably by far the greater effort is put into simply supported units, because construction demanding such units is still a favourite and cheap method of building, little by little prestressed concrete is becoming applied more widely and with more confidence to more complicated structures.

On the building side, the next step after simply supported units is, of course, some form of continuity and the Paper has been written to take stock of the position which has now been reached and to illustrate the emergence from simply supported conditions to fully-framed conditions. It attempts to do this by describing the design and construction of several buildings which the Ministry of Works has erected, not necessarily in the order of their construction, but in a framing sequence from the simple to the more complicated. Reference is also made to other relevant buildings in Great Britain and abroad.

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Partly framed structures

Immediately after the war the Ministry embarked on a large programme for the construction of temporary office buildings. These all had basically the same layout, but their method of construction varied. Among the methods was one using a simply supported prestressed beam approximately 40 ft long spanning from brick pier to brick pier. This replaced a precast concrete scheme employing two internal posts. This, in 1949, was the first use of larger units by the Ministry, although prestressed flooring units had been used in 1948. It is understood that this type of construction was as cheap as the precast concrete system and was without the planning difficulties associated with two internal columns. These units are shown in Fig. 1. This is obviously the simplest and least complicated form of construction and it has also been applied to the building of various storage sheds.

Nevertheless this application is very limited, particularly as most multi-storey buildings of any size are framed in some way, and in 1948 the design of a three-storey stationery storage building was undertaken by the Ministry. At that time it was not considered that sufficient was known about continuous structures in prestressed concrete with its attendant prestressing reactions to embark upon a fully-framed structure, yet some measure of continuity was desirable.

Further, it was doubtful whether much was to be gained from fully continuous main beams since the incidence of loading (an applied load of 3 cwt/sq. ft) could give conditions of heavy reversals of moment. The principle eventually adopted as illustrated in Figs 2 and 3, Plate 1, was that of simply supported main beams supported on bracketed columns and carrying secondary beams on corbels. The latter were stressed by means of one 16-wire cable initially and, after erection and filling-in the end space between the end of the secondary unit and the main beam, by a 12-wire cable 120 ft long passing from back to front of the building through the main beam. This could be done for all secondary beams except those occurring at columns, which were fully stressed before erection. The actual construction procedure varied considerably from that laid down in the design stage. To limit the stresses it was not intended to stress fully the main beam until the secondary beams had been erected, and again it was not intended to stress the second cable in the secondary beams until the floor slab had been cast, yet after the erection of the first bay of the first floor this procedure was dropped; from the constructional side the simpler one of erecting completely stressed units was adopted for the main beams, and complete transverse stressing of the secondaries before casting the floor was adopted.

It is interesting to record that the quality of the concrete was such that although it involved stresses of a little more than 2,700 lb/sq. in. (compression) and 600 lb/sq. in. (tension) with no mild steel in either the compressive or tensile zone, no trouble was experienced; although in the early days, the design was criticized as being heavy. It is doubtful if these stresses have been exceeded in any other job for post-tensioned work.

One purpose in this design was to avoid continuity. The ends of the main beams were coated with bitumen paint to prevent adhesion to the column concrete and, when stressing the secondaries, only the lower part of the gap between them and the main beam was filled and the sides of the main beam were similarly coated with paint. In this way the centroid of the prestressing force coincides with the centroid of the cable. Had the whole space been filled, it would have taken some other position, in this case detrimental to the structure.

For architectural reasons the columns were a little larger than is usual and consequently stiffer. The effect on the columns of the prestress across the building was therefore investigated, and the theoretical force absorbed in deflecting the columns was calculated. In the first few bays of the building erected, the columns were jacked apart to overcome this resistance. Although this was a relatively simple operation, it was discontinued since the force involved was not a large one compared with the prestressing force.

Although it was realized that the form of construction adopted at Sighthill was not a universal method of overcoming the difficulties inherent in prestressing, yet confidence in prestressing as a method of construction was being gained from these buildings.

This method of stressing avoiding full continuity has proved useful in several forms of prestressed construction, nevertheless the full framing of a building still seemed a desirable attainment.

An opportunity to design a building on these lines presented itself when the construction of a new Telephone Manager's Office was under discussion in 1950; the design of this building will be discussed later.

SINGLE-STOREY FRAMED STRUCTURES

To build up the picture in a logical sequence, the easiest fully framed structure to design is the single-bay, single-storey portal. A method of design has been given in some detail by Guyon[1] and by Kee and Jampel.[2] It is perhaps easy to design because the redundant reactions caused by prestressing are readily calculable and can easily be allowed for.

In the example of this type of construction described in the Paper, the design was based on the method of adjusting the cable path so that no relative rotation takes place between the head of the column and the end of the beam at the knee of the portal. The only effect due to stressing is then the elastic shortening and the subsequent creep and shrinkage, and in a pin-ended portal these are small. Typical calculations for a portal are given in Appendix I.

This building comprises five single portals and one double portal where the high portion of the building meets the low portion. The clear span is 41 ft and the clear height 25 ft. The general details of the building are shown in Fig. 6, Plate 1. Fig. 4 shows a detail at a knee and Fig. 5 is a general view. It will be seen that the whole scheme was cast in situ. The option was given to the contractor to precast or cast in situ, and because of the relative smallness of the building, he elected to do it in situ. The Macalloy bars were enclosed in steel sheaths, which were in turn supported by mild-steel reinforcement.

It does not always pay to prestress columns, particularly when the knee moment is small and the column load high. In this instance the load was low and the moment large due to the high column stiffness which resulted from the deep and narrow section required by the architectural treatment. It would have been difficult to produce a reasonable normally reinforced column within the desired dimensions. The eccentrically prestressed column is naturally being "worked" at nearly twice the stress allowable in a reinforced column and has also a greater effective section modulus.

The concrete mix employed was, on the whole, rich, being a 4·5:1 by weight, the

[1] The references are given on p. 83.

water/cement ratio being 0·40. The average cube strengths at 7 and 28 days are shown in Table 1.

TABLE 1

Days	Average strength: lb/sq. in.	Maximum and minimum cube strength: lb/sq. in.
7	7,460	8,700 5,660
28	8,820	10,000 7,000

A minimum cube strength of 6,000 lb/sq. in. at 28 days was specified, the mix proportions being left to the contractor. A maximum water/cement ratio of 0·45 was specified.

Internal vibrators were used in the columns and external vibrators on the beam shutters. The rods in the columns were stressed first, followed by the beam rods. Normally the eccentricity of the column rods at the top would "hog" the unstressed beam and might cause damage. In this case, however, use was made of a linear transformation which has the effect of reducing this eccentricity and preventing excessive, if temporary, loads on the unstressed beam.

This transformation has been described in detail by Guyon,[1] but may briefly be described here. After the tendon paths in both beams and columns have been found to satisfy the structural requirements, and as indicated by the dotted paths in Fig. 7, these paths are moved to the positions shown by the full lines, which are in fact the final positions.

For the beam the parabolic path is simply dropped an equal amount e'_b for all ordinates. The curvature is thus unchanged and the only effect is to reduce the eccentricity at the knee by an amount e'_b, producing a negative moment $F_b e'_b$.

For the column, the path, which is straight, is rotated inwards about the base reducing the eccentricity by e'_c at the knee. Again, the only effect is to produce a positive moment at the knee of $F_c e'_c$.

It follows therefore that if these moments are numerically equal, then the resultant effect on the portal is nil. As mentioned above, it enabled the physical eccentricity of the column cable to be virtually eliminated whilst still retaining the desired effective eccentricity. In other words, the column was given a nearly uniform prestress, the desired eccentric prestress being induced by the beam tendons. The reduction of eccentricity of both beam and column tendons at the knee reduced the ultimate moment of resistance there and set a limit to the extent of the transformation.

At the centre of the beam the eccentricity is increased, but again, a limit is introduced by the need for adequate cover to the rods. The device is, however, useful, and has the further advantage of improving the position of the knee anchorages and the knee detail generally. This may well be decisive in the design of a light portal of this type, where effective eccentricities may require to be large to reduce the

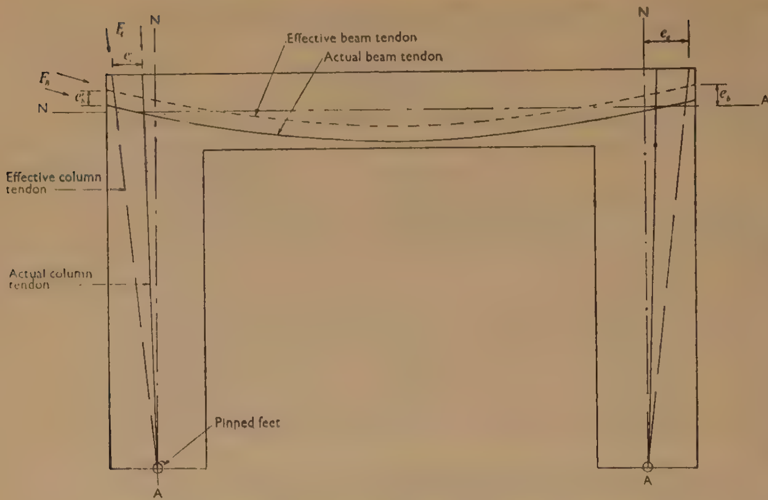

FIG. 7.—LINEAR TRANSFORMATION OF PORTAL TENDONS

number of rods or wires. There seems in fact no objection to having an effective tendon path outside the members, and it may well be the best solution. At least there can be no aesthetic objection.

A small amount of mild steel is incorporated in beams and columns; the longitudinal steel is about 0·14% in each face, and the transverse steel consists of ¼-in.-dia. stirrups at 14-in. centres.

The design was based on placing the roofing units in position before stressing sc as to improve the final stress distribution in the frame. The units themselves were prestressed inverted trough sections made on the long-line system. In-situ concrete with mild-steel rods between the roof units was placed over the beams tying the beams and precast units together. The whole was then assumed to be effective under live-load conditions. Only the solid portion of the roof slab was assumed to act as the table to the beam, though tests indicate that it is difficult to prevent the whole roof from doing so. The weight of this roof including the in-situ concrete is 18½ lb/sq ft. This is heavier than a metal deck, but on the other hand is effective structurally

The sequence of stressing the two-storey portal was similar. The legs were firs stressed, then the lower beam, then the upper. The procedure for the whole job wa simple and straightforward and no real difficulties were experienced.

A similar though much taller building, 51 ft span and 40 ft high, is to be starte shortly. For this building, on account of its height, it was considered uneconomi to use an in-situ scheme and the design was based on precasting. The frames ar spaced at 14-ft-8-in. centres. A typical frame and detail of the joint are shown i Fig. 8. Again Macalloy bars are being used as this has been found to give a con venient detail to the knee.

The single-storey portal, being an entity, presents little difficulty. The applicatio of prestressing to multi-bay single-storey construction is not significantly mor difficult. There are, of course, more conditions to investigate.

An example of this type of construction is to be found in a three-bay worksho built for the Ministry of Supply. This building was designed for the normal roof an

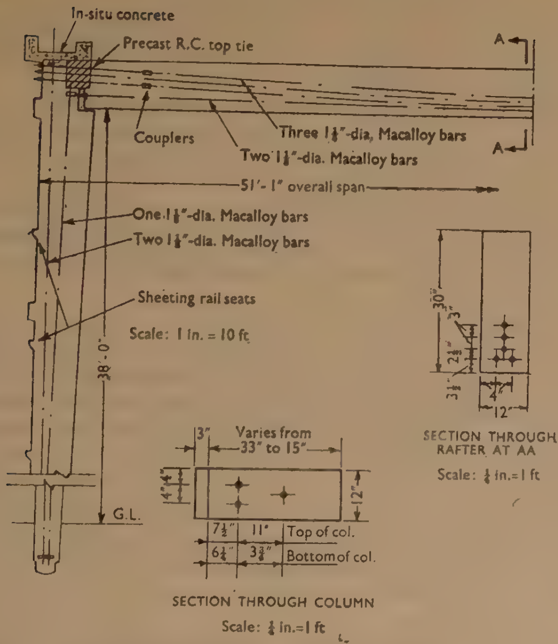

FIG. 8.—TYPICAL FRAME AND JOINT DETAIL

now loadings plus two cranes of 10 and 3 tons capacity, in the centre aisle, and a -ton monorail in the outer aisles. Fig. 10 shows a view down the centre of the uilding, Fig. 11 shows the frame in course of construction, and Fig. 9, Plate 1, ives general details.

The frame was precast and one of the conditions of the design was that there ould be no propping of the beams, because this would interfere with the floor con- ruction. (A similar building using precast reinforced concrete had been built on e same site; this involved propping which had proved a hindrance.)

The chosen solution to this problem was the use of precast reinforced concrete lumns and prestressed beams. The columns were cast to full height with two re- forced concrete brackets on the internal columns, one for the crane rail and the her, a small one, to lodge the roof beams of the outer aisles. A Freyssinet cone was st in the columns opposite the low-level beams.

The roof beams were designed to carry in the first instance themselves and the ight of the precast roof units, and were prestressed by one 12-wire cable of 0·200 in. . A second cable was also inserted in the beam with sufficient length to pass rough the columns. This cable was left unstressed until after erection.

The procedure for construction was as follows. All the columns, beams, and lintel ams, which were in reinforced concrete, were precast. The columns were erected d plumbed and a temporary collar fixed to the top of each outer column, a pre- utionary measure, since, in reality, the beams sat on the outer inch of the column ncrete. The prestressed beams were erected and then the precast roof units. The tel beams were then placed in position and shuttering was erected at the head of

the column where the beams met. The necessary anchorage cones were inserte together with mild-steel reinforcement. The concrete was then cast, together wit the concrete above the main beams. Again, a small part of the roof was assume to act in conjunction with the main beam.

In the case of the lower beams the procedure was slightly different since the junc tion at the middle bay was between two precast faces, the end of the beam and th face of the column. In this case a gap of about 3 in. was left and filled with a higl quality concrete, well tamped and vibrated.

When the in-situ concrete had reached the requisite strength, the second cable wa stressed and the structure was then complete except for the final screeding and felting In the meantime the laying of the floor was proceeding without interference from th main construction.

On the design side of this work, two conditions of framing were considered, firs during construction, when the whole was considered to be simply supported, an secondly, under full loading conditions of wind, snow, and crane loads when the whol was designed as a fully-framed structure. The moments at the connexions at th knees were taken care of by the prestressing cables at the top of the beam and b reinforcing rods taken out from the column and turned into the in-situ concrete ove the beams for the columns. To provide continuity the Authors consider that it desirable to introduce a positive stress across the joint without relying solely o reinforcement. Carrying a beam cable through and anchoring it on the outside of normally reinforced column is a convenient method of achieving this and appears t be fully effective. Valid continuity at the joints of precast reinforced concret frames is not easy to achieve.

In two of the works already described, composite action has been assumed to b effective between the prestressed beam and the in-situ topping. To achieve this is important to get a good key, which has been done by "jetting off" the top of th beam with water while the concrete is green. In addition upstand stirrups in th top of the beam were provided.

In the longitudinal direction no prestressing was attempted, and the lintel bean were all of reinforced concrete.

Two expansion joints were provided in the length. These took the form of tw copper strips with an oiled-paper liner between resting on the edge of one beam; th roof units from the adjacent bay simply rested on the upper strip.

It might be argued that expansion joints are not necessary in a building of th length, yet few data are available to show the limits to which it is safe to go witho such joints for different forms of construction, and the initial shrinkage has to b provided for, in reinforced concrete at least.

It is interesting to compare this building with two previous workshops of the sar shape and size built on the same site. The first one was built in uncased steel co ventionally framed, the second of precast reinforced concrete.

The relative weights of steel and costs are as shown in Table 2.

The method of construction already described is easy, provided the roof membe are at different levels. Anchoring devices can be readily incorporated in preca columns or in the in-situ joint at the top of the column.

Where the roof is, however, all at the same level, quite different problems ari mainly caused by the physical difficulty of anchoring wires or rods satisfactoril For another workshop shortly to be constructed, a number of solutions to th problem were proposed. The workshop is 300 ft long × 100 ft wide, and a colur grid of 33 ft × 30 ft was required. One half has to have a crane running in the cent

TABLE 2

Building	Weight* of steel: lb.	Relative cost of frame
Steel	8·6	100
Precast concrete	3·5	110
Prestressed concrete	1·3	100

* Weights are for a square foot of floor area.

aisle and the other half is divided into six separate transverse bays, each with a 10-ton crane. The roof is of light construction with light welded tubular steel trusses forming monitor lights in each bay. The dead-weight of the frame was therefore one of the important loads. The solution eventually adopted was as follows, as shown in Fig. 12, Plate 2. The whole of the construction is precast, involving only three different columns and two types of beams. Again, as in the above workshop, the stressing of the beams is done in two operations, with one tendon stressed for dead-load conditions and handling, and the second for continuity. For a length at the end of the beams, taper is introduced within the width of the beam. By this arrangement three advantages are thought to accrue; first, the lower tendon can be placed almost flat along the bottom of the beam and the anchorage devices are not in the way of the stressing of the upper tendon. This is possible since the centroid of the cross-section is lowered towards the end, so reducing the eccentricity. No tension is then developed at the ends. Secondly, the second tendon passing over the supports can be placed easily before the in-situ concrete is poured. This is particularly important where more than two beams meet at a column, which is the general case in this building. Thirdly, it enables a number of spans to be stressed continuously without producing impracticably large moments in the end columns due to elastic shortening of the beams, which occurs when a continuous tendon is used.

This system gives effectively two tendons over the supports which, combined with a greater eccentricity there, gives a higher possible moment of resistance than at midspan. By making the midspan section smaller than the support section, which in any case has to accommodate the anchorages, a redistribution of moment is achieved. The midspan moment is decreased and the support moment increased. This effects an appreciable saving in structural weight. In this case the variation in moment of inertia along the beams is 1·62:1.

It was important here, as in all cases where prestressed and normally reinforced members are rigidly joined, that as small an amount as possible of the prestress energy was absorbed by the reinforced columns. This was achieved by arranging that the stressing of the second tendons produced no rotation or deflexion of the beams at the columns. This is simple to achieve in the internal spans, but less easy in the end spans where this requirement tends to conflict with the stress requirements, particularly near the penultimate support.

There have, of course, been other methods of obtaining the reverse moment over the supports. Perhaps one of the most notable is the capping cable method as used in France, which is illustrated in Fig. 13. This has always seemed to the Authors an uncertain method and difficult in operation, particularly so when only small beams,

as in this case, are required. The short length of the cap cables give a correspond-ingly small extension on stressing, which is difficult to measure, and the sharp curvature invites excessive and, in practice, not readily predictable friction.

Care has to be taken, particularly in the method described here, in detailing to ensure adequate bond between the in situ and precast portions of the work.

In this building an attempt is made to stress the building longitudinally as well as transversely. This of course gives rise to stresses in the columns, particularly the end ones, on account of the progressive elastic shortening of the roof beams with consequent redundant reactions in the beams. The calculated total elastic shortening at each end of the building over a length of 300 ft is 0·46 in., and to reduce

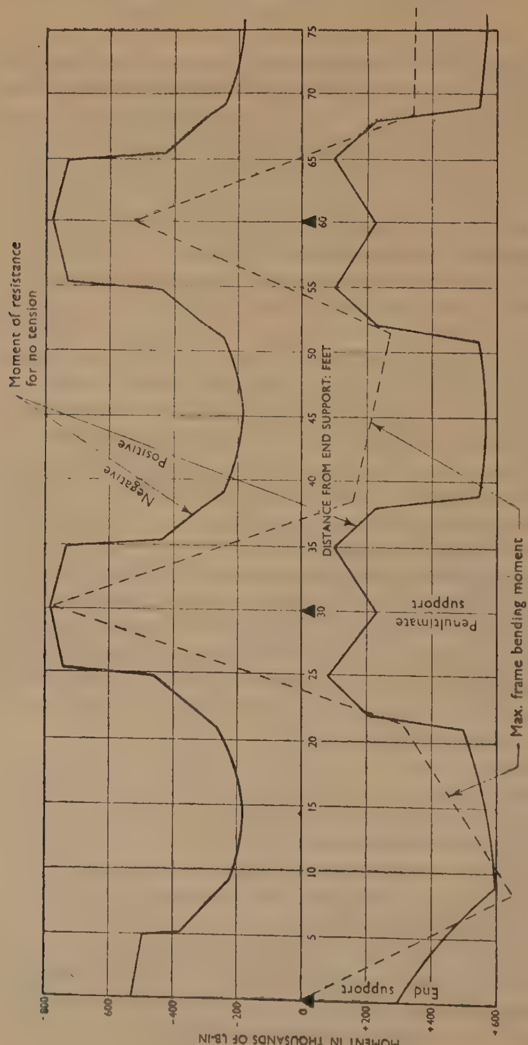

FIG. 14.—BENDING-MOMENT DIAGRAM FOR A MULTI-BAY WORKSHOP

this as well as provide an expansion joint in this length, a double frame is introduced in the workshop at the change of occupancy.

As in the previous workshop, the dead load of the construction is acting at the time of the erection, i.e., in the free-ended condition. All the moments due to wind, cranes and snow loads act when the frame is fully continuous.

The only complication in the design of such buildings is the choice of suitable sections and the Authors know of no easy way to shorten this work other than by using tables to arrive at the approximate values of the stiffnesses, fixed-end moments, and carry-over factors for beams of varying inertia, as published for instance by the Portland Cement Association of America. This assumes moment distribution for the analysis.

Moment areas, using the theorem of three moments and plotting the bending moments due to the prestress graphically, appear to be the simplest method of achieving the condition of no slope or deflexion at supports.

The Authors consider that plotting the prestress bending moment diagram is worthwhile in any case, if only because it gives the designer a clearer picture of what he is "doing" to the member in question than figures can.

The bending-moment diagram for the multi-bay workshop is shown in Fig. 14. In practice, the sharp discontinuities which occur at the anchorages will no doubt be somewhat softened by local stress redistribution. The stress under the anchorages, for instance, may be outside the region of linear stress/strain relation.

For slope and deflexion with members of varying section, the bending-moment diagram has to be converted into an M/I diagram.

Unfortunately, as in many engineering problems, it is experience which ultimately shortens the work. The differences between design in this medium and that of steel and reinforced concrete is that whereas in steel the design is made to a large extent within a framework of given sections and in reinforced concrete the shape of the section is largely immaterial except for the compression flange and stiffness ratios, in prestressed concrete one has the liberty of choice of section and that choice is all-important. It is certainly possible to design very rapidly in prestressed concrete if the choice is restricted, say, to rectangular sections, but if any interest at all is taken in the economical and aesthetic side of design, the design work becomes more complicated.

This digression has been made since single-storey shed construction will probably form a large part of any building programme and it is hoped that what has been said proves that prestressed concrete is not an unworthy addition to the proved methods of construction of these types of buildings.

Multi-storey Construction

In the realms of multi-storey construction, far less has been done on the application of prestressed concrete, perhaps because it has appeared at a time when the fierce light of analysis beats upon all design methods. It is difficult to *prove* (theoretically) that certain methods of construction will work, although in practice they would probably do so and would be perfectly safe. Steel and reinforced concrete constructions built to code requirements are perfectly safe although the calculated stresses often bear little or no resemblance to actual stresses.

For this reason, when the design of an actual building was undertaken the framing was deliberately chosen to be as simple as possible. In this way the design unknowns were reduced as much as possible. The first multi-storey building other than the

Stationery Office Store was an office building, designed for 50 lb/sq. ft plus 46 lb/sq. f for partitions and finishes, of 45 ft clear span and of five storeys, the top one being se back. In the first instance several different schemes were investigated and taken long way in the design; among them was a set of pinned portals, one on top of anothe and π frames with either pinned or fixed feet and pendulum props at the ends of th cantilevers.

The design finally chosen was one with reinforced concrete columns and prestresse beams. This scheme was designed in two ways; first, with in-situ columns and pre cast beams, and secondly, with in-situ columns and beams. The contractor chose th latter method. The two designs are fundamentally the same, although the differ ence in stressing procedures does alter them a little. At the time there was no hel to be obtained from the designs of other buildings, although in fact a four storey building on similar lines was being erected at the same time in the Belgia Congo.

The Magnel-Blaton system of stressing was chosen since it appeared to hav advantages for the detailing of the ends of the beams. To cater for all the moment it was possible to have a single cable of the same width except for one floor, and thi led to simplicity of the detailing of the column reinforcement, particularly as th rain-water pipes were incorporated in the columns.

The framing of the building is shown in Fig. 15, Plate 2. The design had t take into account the various stages of construction—not normally done in steel o reinforced concrete multi-storey buildings. The sequence of construction was a follows: casting the columns to the underside of the beams; casting the beams an a short upstand of the column; placing precast floor units, hollow tiles, and structura screed, and leaving a gap between this screed and the main beams. The beam were then stressed through the columns in two operations and in a definite sequenc designed to reduce secondary bending in the longitudinal edge beams.

A third of the wires in No. 1 beam were stressed, then a third in No. 2, then a third in No. 3. Then the second third in No. 1, then a third in No. 4, and so on. I this way it was hoped to avoid the transfer of stress from the stressing of one colum to the adjacent one. In actual practice this appears to have worked very well an was not difficult to carry out on site.

When the stressing of two beams was completed, the remainder of the floor ove the beam could be placed. In this way the major portion of the dead load was i position before stressing, when the frame consisted of simple rectangular columns an beams. On completion of the floor the beams became tees of the width of the infil (theoretically) to carry the applied loads. The stress distribution around the fram becomes, of course, completely different because of the change of inertia. I practice it was found that the whole of the floor acted under incidental loading as a T-beam, including a portion of that in the adjacent bays. Thus the floor was i practice almost infinitely stiff.

It must be emphasized that in the design of this building the whole of the fram was analysed as a frame, and this gives rise to what at first sight seems an anomalou situation, where some of the columns on the fourth floor in each bay are larger tha the columns below. This is so because large moments are caused in the beam abov from the set-back column in the fifth floor. Using less rigorous methods of desig this column would, no doubt, have been smaller, though perhaps the beam woul have been bigger. Some notes on the detailed design of this building are given i Appendix II. The end frames of this building, the basement, and the service bloc at the back are in normal reinforced concrete construction.

FIG. 1.—PRE-TENSIONED BEAMS, 36 FT LONG, ERECTED IN SINGLE-STOREY GOVERNMENT OFFICE BUILDING

FIG. 2.—PRECAST PRESTRESSED UNITS IN PLACE BEFORE POURING CONCRETE

Fig. 4.—Detail at knee of portal

Fig. 5.—Fully prestressed double portal

FIG. 10.—VIEW DOWN A THREE-BAY WORKSHOP

FIG. 11.—THREE-BAY WORKSHOP: THE FRAME IN COURSE OF CONSTRUCTION

Fig. 13.—The capping cable method as used in France

Fig. 16.—Completed multi-storey building

The construction was rapid and straightforward, and except for the careful positioning of the cables in each beam there was no complicated steel reinforcement to fix, and it was easy to place high-quality concrete. The average concrete strengths obtained were 6,850 lb/sq. in. at 14 days (when stressing was carried out) and 7,500 lb/sq. in. at 28 days. The mix was 4½:1 with a water/cement ratio of 0·4.

The Building Research Station undertook measurements on this building during construction, during testing, and subsequently, and these have been reported in detail.[3] The completed building is shown in Fig. 16.

Other buildings have been erected abroad to which the Authors would like to refer before describing possible future developments.

The first of these is in Munich, Germany, where a similar building to the one described has been completed. This is of nine storeys and again employs Magnel–Blaton cables. It is significantly different in one detail, which is that instead of divorcing the floor from the framework, it was made an integral part, and reinforcement was placed in the floor to ensure that the stress applied through the columns was distributed to the whole width of the floor. In this method a lighter form of construction is obtained, as will be observed from a brief comparison of the beam and cable sizes in Table 3.

TABLE 3

	Span: ft in.	Design load: lb/sq. ft	Beam size: in.	Beam cables (wires)
M.o.W. office	46 9	96	28 × 15	56 @0·276 in.
German building . . .	52 6	112	25 × 14½	80 @0·200 in.

The second of these is the construction of additional floors inside the Galeries Lafayette, Paris. This again is a single-bay multi-storey construction six storeys in height in which full continuity is obtained by using storey height column cables with anchorages on both sides of the knee. It would be difficult to obtain a satisfactory appearance if the knees were exposed. This is shown in Fig. 17. One of the merits of this particular scheme was that precast segments could be used and could be manhandled into the store through existing doorways with the minimum of interference to the normal working of the shop. These segments were stressed together, the whole frame being in dry construction except for the packing of the joints.

Two other multi-storey buildings, at least, have been built, one at Le Havre and the other in America. They have one thing in common: they are stressed longitudinally. This appears to have been achieved without the attendant difficulties of longitudinal shortening by stressing the whole floor as opposed to "isolated" beams; in this way the stresses are low so that the shortening due to stressing and subsequent creep is small. The transverse beams are effectively simply supported. These two buildings are referred to since they draw attention to a problem which appears to be a real one to the Authors, and to a solution of it which, however, does not seem to have a universal application.

There is no doubt that the simple grid of columns and beams is the most useful one for general office or similar use, being in general also the cheapest method on a cubic-

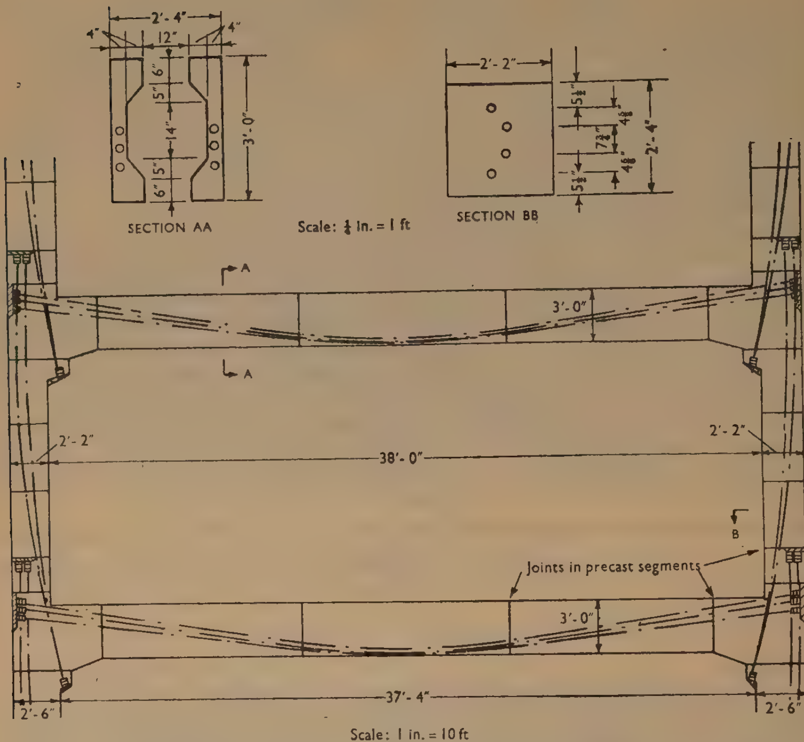

FIG. 17.—SINGLE-STOREY MULTI-BAY CONSTRUCTION FOR GALERIES LAFAYETTE, PARIS

foot basis. It is to the construction of such a frame that the Authors feel some attention should be paid.

The single-bay and double-bay multi-storey structures appear to be readily soluble in the design office and easily constructed in the field, provided that the columns are of reinforced concrete and the floors span from one prestressed beam to another so that the longitudinal beams are merely tie-beams.

More than two bays would give trouble to the design office, and in the field, from frictional effects if the cables were of the normal sinusoidal shape. Similarly, if the longitudinal beams were load-carrying and not tie-beams, problems would arise in the longitudinal direction because of the stiffness of the columns and the subsequent building-in of longitudinal walls would permit the beams to shrink and creep relatively to the end anchorage. One solution would be to use simply supported prestressed beams longitudinally with nominal mild-steel reinforcement at each end. Another solution would be to use a method similar to that described for the multi-bay workshop, with cables protruding from the end which can be stressed in the adjacent span. A variant on this would be to use the prestressing steel to pre-tension the beam using temporary anchorages, the final anchorages being in the adjacent beam. This would be possible with either the 0·200-in.-dia. wire or the new indented Macalloy bar. There must be many variations of this sort; what is required is something really practical that requires little or no finesse on the building site.

One of the difficulties of building frames as compared with say, bridges, is the small size of the members. This means that if sinusoidal cable shapes—which are the most economic—are used, they must be placed carefully if the design is to be faithfully interpreted in the field. For many frames, therefore, the Authors are convinced that uniform stressing is probably the answer for multi-bay construction. It is realized that it is "uneconomic" in the sense that it uses a little more concrete and steel, but its advantages both in the design office and the field must surely outweigh this factor. To illustrate the point, two part sections through a telephone exchange building are shown in Fig. 18, Plate 2. It will be seen that four cables, each of twelve 0·276-in.-dia. wires, replace five 1-in.-dia., two ⅞-in.-dia., and four ⅝-in.-dia. bars, plus stirruping; the section is also reduced. There is little design or detailing to do since the sections are determined directly from the moment and allowable stress, and there is no difficulty in placing four cables and nominal stirruping. Its ultimate resistance to failure can be readily checked using Guyon's method.[4] In the field, too, the placing of the cables is simpler.

Conclusions

This Paper has attempted to describe the application of prestressing to a variety of building frames. They are, so to speak, isolated solutions and as more experience is gained, better solutions will be forthcoming. The work has been done in general as part of the normal work of the Ministry of Works and represents only a tiny fraction of the total output of framed buildings built in conventional methods using steel and reinforced concrete. At this stage it would be unwise to draw any conclusions as to its relative advantages compared with either or both of the normal methods. Whilst reinforced concrete is detailed in the engineer's design office and not in the contractor's design office as opposed to steelwork, prestressed concrete should appeal to the engineer since far less detailing is required, and in most cases the design work is also simplified. There is, of course, far more work in the engineer's office than is required for steelwork.

The chief problem which still requires more attention is the longitudinal stressing of long buildings with continuous tendons. It is felt that in general it should work satisfactorily, but theoretically it is difficult to justify. It is true that on the ultimate basis, provided the steel is there and in the right place, the structure would be as safe as any other structure, yet permanent cracks in a prestressed structure would be undesirable.

The Authors are aware that there must be many other framed buildings embodying prestressed concrete and it is hoped that this Paper will focus and draw together experience that has been gained in the past decade on this aspect of prestressed concrete.

Acknowledgement

This Paper is published by permission of the Ministry of Works, but the opinions expressed in it are the Authors'.

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APPENDIX I

CALCULATIONS FOR A FULLY PRESTRESSED SINGLE PORTAL FRAME WITH PINNED FEET

Dimensions

Span: 43 ft
Height: 24 ft
Frames at 15-ft ctrs

Loads

Assuming that the frame is stressed when the roof units are in place the load on beam, including self-weight = 433 lb/ft run
Added loads due to lightweight screed, felt and 15 lb/sq. ft super load = 450 lb/ft run
Unit wind load, p = 7 lb/sq. ft

Working Stresses

Stress limits on concrete:—
(i) in compression = 2,000 lb/sq. in.
(ii) in tension = 200 lb/sq. in.

Sections

(1) *Columns*

Architectural treatment, with exposed columns, requires a maximum width of 10 in. and a minimum depth of 24 in., without taper. Such a section is deeper than required. Due to extensive ducting in the floor it is impractical to provide fixed feet.

(2) *Beams*

Preliminary investigation indicates a section 24 in. deep and 5 in. wide with flanges at top and bottom of 10 in. × 3 in. deep, the bottom flange "tapering in" from the knees to give a tee section over the central 3/5 of span.

From tables, this beam has the following frame properties:—

$$\text{Fixed-end moment for U.D. load} = 1{\cdot}05 \times \frac{wl^2}{12}$$

$$\text{Stiffness} = 0{\cdot}84 \times \frac{I_{\text{knee}}}{l}$$

$$\text{Carry-over factor} = 0{\cdot}53$$

The properties of beam and column sections are as in Table 4.

TABLE 4

	Area, A: sq. in.	Distance of extreme fibres from centroid: in. y_1	y_2	I_{xx}: in.^{4}	Section moduli: in.^{3} Z_1	Z_2	Core point distances from centroid: in. C_1	C_2	k^2: sq. in.
Beam centre .	157·5	10·02	13·98	7,850	784	562	4·98	3·57	49·9
Beam knee . .	181·5	11·60	12·40	10,840	935	874	5·15	4·81	59·7
Column . . .	240	12·00	12·00	11,520	960	960	4·00	4·00	48·0

k denotes radius of gyration

Moments

Analysis gives moments (in thousands of lb.-in.) as in Table 5.

TABLE 5

	Beam centre	Knee
Minimum gravity load	+ 541	− 663
Maximum gravity load	+ 1,100	− 1,350
Wind	− 8	{ − 150 + 134
Minimum total load	+ 533*	− 529*
Maximum total load	+ 1,100	− 1,500

* When clad.

Prestress

Adopting the Lee-McCall system using H.T.S. rods with an initial stress of 94,000 lb/sq. in. and assuming initially 15% losses from creep and shrinkage, it seems that two 1⅛-in.-dia. rods in the beams (final prestress force $F = 159{,}000$ lb.), and two 1-in.-dia. rods ($F = 126{,}000$ lb.) in the columns, are suitable.

If no tension is assumed under maximum load, the required eccentricities become:

(1) *Beam at the knee*

$$e_0 \geqslant \frac{M}{F_B} - C_1 \geqslant \frac{1{,}500{,}000}{159{,}000} - 5\cdot15$$
$$\geqslant 4\cdot3 \text{ in.}$$

(2) *Beam at midspan*

$$e_B \geqslant \frac{M}{F_B} - C_2 \geqslant \frac{1{,}100{,}000}{159{,}000} - 3\cdot57$$
$$\geqslant 3\cdot35 \text{ in.}$$

(3) *Column at knee*

$$e_C \geqslant \frac{1{,}420{,}000}{126{,}000} - 4 \geqslant 7\cdot3 \text{ in.}$$

To satisfy the condition of equal rotation of beam and column at the knee under prestress

$$\frac{2}{3} F_C \times \frac{e_C}{\rho} + \frac{F_b}{1} \int_0^l y\,dx = 0$$

$$\text{where } \rho = \frac{\text{column stiffness}}{\text{beam stiffness}}$$

and y = eccentricity of beam tendon from the neutral axis at any point distant x from knee.

Whence

$$\int_0^l y\,dx = -\frac{2}{3} \times 43 \times 12 \times \frac{126{,}000}{159{,}000} \times \frac{7\cdot3}{1\cdot7} = -1{,}170 \text{ sq. in.}$$

For a parabolic cable :

$$\int_0^l y\,dx = \tfrac{2}{3}[e_B + e_0]l - e_0 l = -1{,}170 \text{ sq. in.}$$

if $e_0 = 4\cdot3$ in. this gives $e_B = 5\cdot5$ in.

Stresses

The above eccentricity gives stresses at midspan due to prestress as follows, in lb/sq. in:—

	Top fibre	Bottom fibre
(i) Initial stresses	− 124	+ 3,010
(ii) Final stress	− 105	+ 2,565

Load stresses are:—

(iii) Minimum load	+ 690	− 960
(iv) Maximum load	+ 1,400	− 1,955

Giving combined stresses:—

(i) + (iii) Minimum load	+ 565	+ 2,050
(ii) + (iv) Maximum load	+ 1,295	+ 610

Similarly combined stresses at the beam ends are:—

Minimum load	+ 1,180	+ 867
Maximum load	0	+ 1,808

and combined column stresses at knee are:—

	Outer fibre	Inner fibre
Minimum load	+ 1,090	+ 220
Maximum load	0	+ 1,200

(Note: Portal thrust under maximum load produces a mean compression of about 35 lb/sq. in. in the beam. The elastic shortening of the beam under prestress produces a beam moment of about + 8,000 lb.-in. Both these effects may therefore be neglected in this case.)

Linear transformation

On examination it is possible, with the above tendon positions, to effect a linear transformation.

The knee detail and anchorage dimensions make a reduction of about 4·8 in. in the column eccentricity desirable.

This entails a vertical transformation in the beam tendon of:—

$$4{\cdot}8 \times \frac{126{,}000}{159{,}000} = 3{\cdot}8 \text{ in.}$$

This gives actual eccentricities of tendons as:—

Column knee $= 7{\cdot}3 - 4{\cdot}8 = 2{\cdot}5$ in.
Beam mid-span $= 5{\cdot}5 + 3{\cdot}8 = 9{\cdot}3$ in.
Beam knee $= 4{\cdot}3 - 3{\cdot}8 = 0{\cdot}5$ in.

Mid-span tendon eccentricity provides adequate cover and the above transformation will be adopted.

Effect of stressing column cables before beam cables

Analysis gives moments in the beam of 72,000 lb-in. at mid-span, and 243,000 lb-in. at the knee, or a maximum tensile stress of − 210 lb/sq. in. This is acceptable.

Load factors

Taking a load factor of 1·5 on dead loads, the load factors on super loads are:—

(i) for column 2·5
(ii) for beam (at knee) 2·8
(iii) for beam (mid-span) 5·8

Shear

Maximum load = 883 lb/ft run

Uniform upward load on beam due to tendon curvature (parabolic)

$$= q = 8F_B \frac{(e_0 + e_B)}{l^2} = 588 \text{ lb/ft. run}$$

Hence net maximum shear (neglecting wind)

$$= \frac{(883 - 588) \times 43}{2} = 6{,}350 \text{ lb.}$$

Hence at beam knee, where section approximates to symmetrical, the maximum shear stress at the neutral axis

$$< \frac{3}{2} \times \frac{6{,}350 \text{ lb}}{5 \times 24} = 79 \text{ lb/sq. in.}$$

Compressive prestress at neutral axis = 1,230 lb/sq. in.
Hence maximum principal tensile stress at beam knee

$$= \frac{1{,}230}{2} - \sqrt{\left\{\frac{1{,}230}{2}\right\}^2 + 79^2}$$

$$\approx 0$$

Column shear is small.

APPENDIX II

NOTES ON THE DESIGN OF AN OFFICE BUILDING AT KILBURN, LONDON

For the analysis of this frame the moment distribution method was used, sway being treated by Naylor's method.[5] The frames were stressed as each floor was completed and analyses had therefore to be made at each stage of construction, in addition to the analysis for the complete frame. Typical stress and bending-moment diagrams are shown in Fig. 19.

The tendon traces were arrived at using Guyon's *fuseau* envelope. To obtain the required condition of no rotation at the beam ends due to prestress, a method devised by the late H. A. Whitaker was used in conjunction with this and is illustrated in Fig. 20. It is a useful device, particularly where an asymmetric cable is required, as in the fourth-floor beams.

Fig. 20a represents a tendon consisting of a parabolic central section with tangential straights for the outer sections.

It can be shown that area ABEFD $= lM\left[\frac{12ab - c^2}{12ab}\right]$

and $\bar{x}$ = distance of centroid of ABF from A $= \frac{4a^2b + 4abl - ac^2}{12ab - c^2}$

Fig. 20b represents a symmetrical tendon with centroid on the centre line with base AB horizontal, and chosen so that area ABB′A′ = area enclosed by tendon

$$= lM\left[\frac{12ab - c^2}{24ab}\right].$$

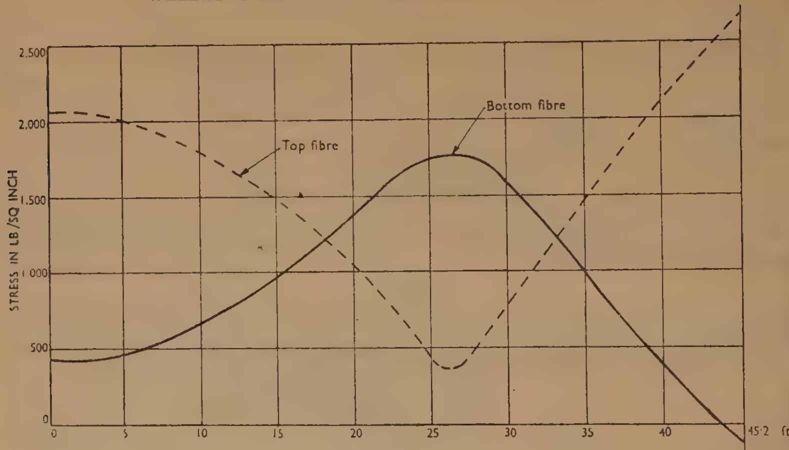
In Fig. 20c base AB has been rotated about J to position A″B″. Area A″B″B′A′ = area ABB′A′, but the position of the centroid will have been changed. If no rotation is to be produced at A and B, the centroid of A″B″B′A′ must lie on the same vertical as the centroid of AFB.

The distance through which the centroid is moved

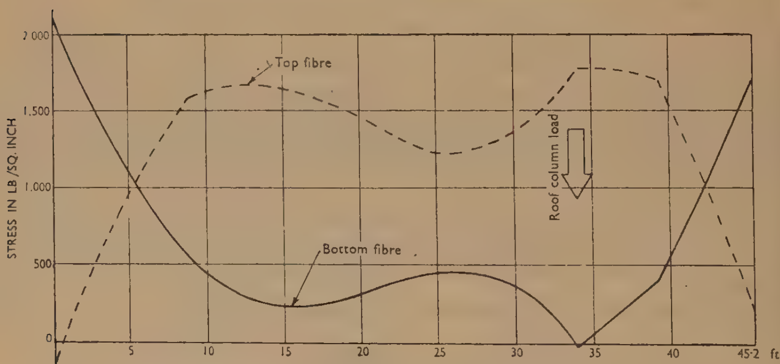
$$= \frac{8a^2b - 4abl - c^2(2a - 1)}{24ab - 2c^2}$$

and this therefore $= \frac{dl^2}{6} \frac{24ab}{lM(12ab - c^2)}.$

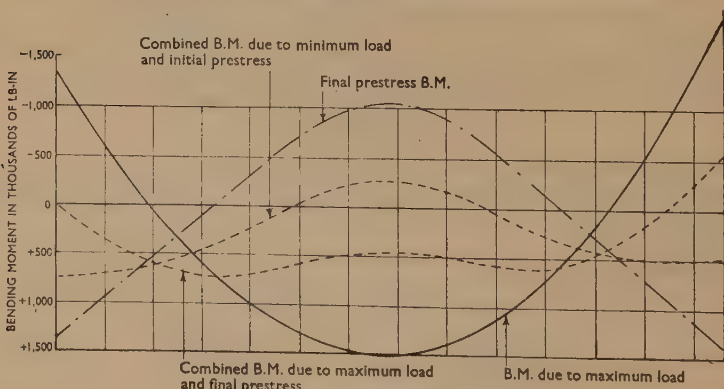
or
$$d = \frac{8a^2b - 4abl - c^2(2a - 1)}{8abl} M$$



(a)—Fourth-floor beam prestress stresses



(b)—Fourth-floor beam: combined stresses under maximum load



(c)—Roof-beam: bending moments

Fig. 19.—Typical stress and bending-moment diagrams

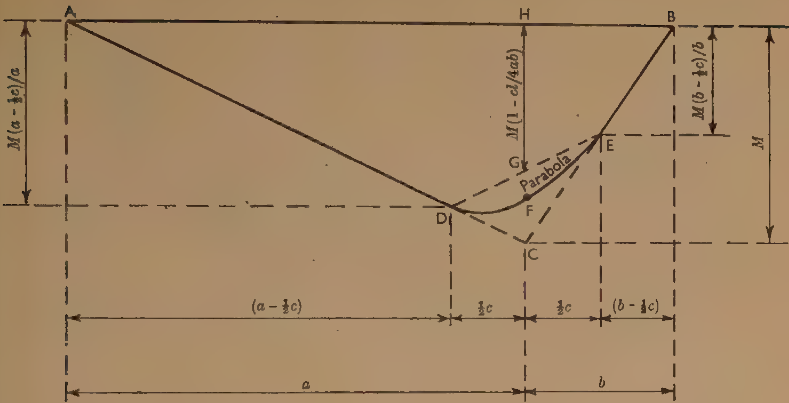


FIG. 20a

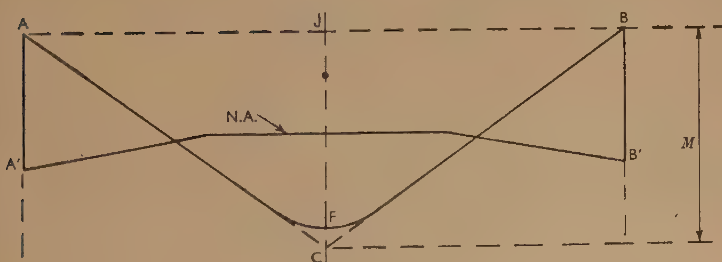


FIG. 20b

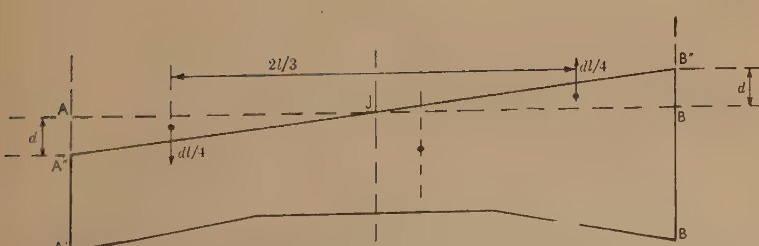


FIG. 20c

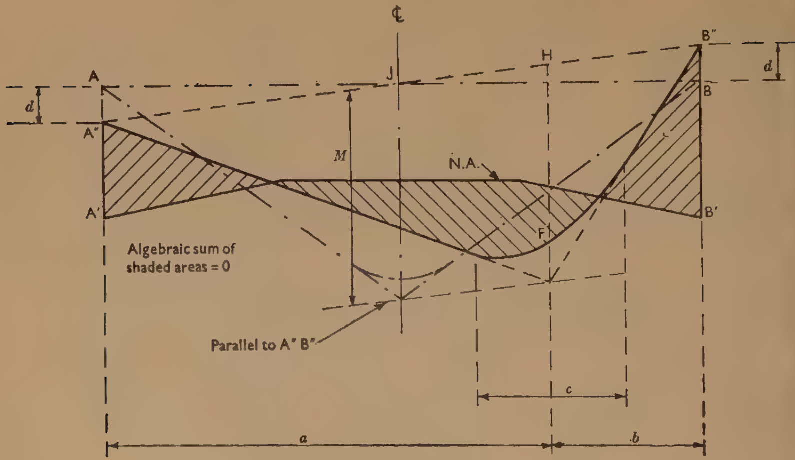


FIG. 20d

Fig. 20d shows the transformation.
When the beam is of uniform section, the neutral axis is straight and

$$\mathrm{A'A} = \mathrm{B'B} = M\left[\frac{12ab - c^2}{24ab}\right]$$

$$\mathrm{HF} = M\left[1 - \frac{cl}{8ab}\right]$$

For a symmetrical cable $a = b = \frac{l}{2}$, $\bar{x} = \frac{l}{2}$ and $d = 0$

and
$$\mathrm{A'A''} = \mathrm{B'B''} = M\left[\frac{1}{2} - \frac{c^2}{6l^2}\right]$$

$$\mathrm{HF} = M\left[1 - \frac{c}{2l}\right]$$

$$\therefore \frac{\mathrm{A'A''}}{\mathrm{HF}} = \frac{\left[\frac{1}{2} - \frac{c^2}{6l^2}\right]}{\left[1 - \frac{c}{2l}\right]}$$

When $c = 1$, i.e., the whole cable is parabolic,

$\frac{\mathrm{A'A''}}{\mathrm{HF}} = \frac{2}{3}$, or the end eccentricity is twice that at midspan.

This is often impracticable from stress considerations.
When $c = 0$, i.e., the cable consists of two straights only,

$\frac{\mathrm{A'A''}}{\mathrm{HF}} = \frac{1}{2}$, or the end and midspan eccentricities are equal.

For partial parabolic tendons the relative eccentricities lie between these values.
In practice one or more of the unknowns a, c, M, and d are defined fairly closely by the *fuseau* envelope and other considerations and a suitable tendon path can quickly be found

Fig. 21

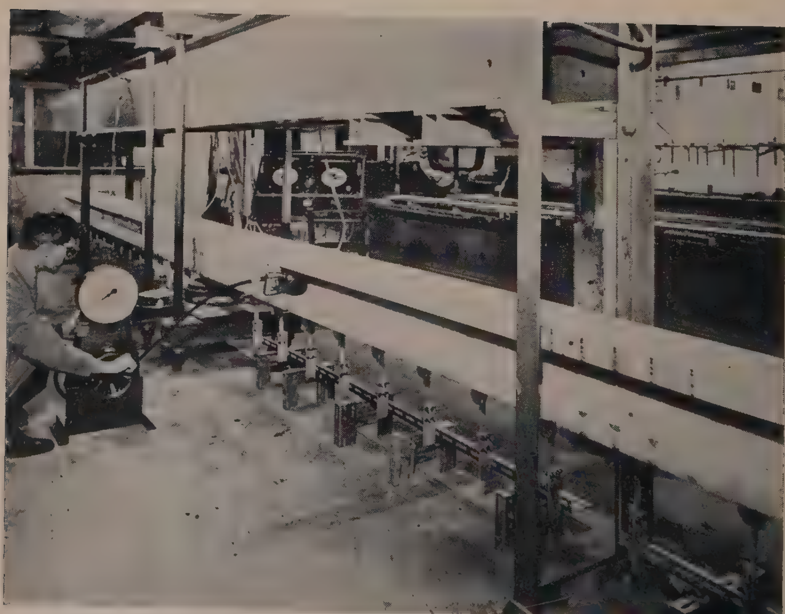


Fig. 22

The Paper, which was received on 17 August, 1955, is accompanied by eight photo-raphs and seventeen sheets of diagrams and drawings, from which the half-tone page lates, the Figures in the text, and the folding Plates 1 and 2 have been prepared, and y two Appendices.

Discussion

The Authors introduced the Paper with the aid of a series of lantern slides.

Professor A. L. L. Baker (Professor of Concrete Technology, Imperial College of cience and Technology) referred to three significant points in the Paper. First, the pecification of the concrete by minimum cube strength at 6,000 lb/sq. in. and the chievement of the contractor of a variation of cube strengths from 7,000–10,000 lb/sq. in., ndicated the quality of work possible. That demonstrated the importance of following he recommendations of the Institution's report on "Quality Control" by specifying ninimum strength. He hoped that the New Code of Practice would encourage similar tandards of quality control in the case of ordinary reinforced concrete, where the higher nit cost of a high-strength mix could be justified.

In Table 2 it was indicated that the cost of a frame for a 51-ft-span workshop building as the same for prestressed concrete as for structural steel. The cost of maintenance ainting was not mentioned. Presumably, therefore, it could be concluded that the pre-tressed concrete design was the most economical. That information was significant.

Professor Baker then showed some slides of recent tests carried out at Imperial College y one of his students, Mr H. C. Visvesvaraya, who was to speak later.

Fig. 21 showed a special jack which had been designed to facilitate the joining together, n full continuity, of precast framework members. The jack was, so far as he knew, the mallest and lightest yet made for its capacity; it could easily be fitted into a small pocket o that two independent precast beams could be joined by a prestressed tie or tendon hich followed a fairly flat arc coming close to the top of the beam at the joint and then own again to the anchorage at the jack. That enabled the precast units to be joined ogether and prestressed continuously in a very simple manner.

It was intended to continue the research and to try to achieve the same advantages of ll effective depth around the corners in the end joints of a building frame and a portal rame. There was full effective depth at the support and at mid span in the case shown n Fig. 22. At less than 80% working load there had been no cracks in the test and there ad been good agreement with the calculations. Under working load, assumed to be half he ultimate load, there had been a few very fine cracks, the kind of cracks which in normal einforced concrete construction were not deemed to be serious.

It therefore seemed that there was scope for a type of construction in which the main mphasis was on the ultimate strength, with the full sections of the beam used at the ipport and at mid-span in order to carry the maximum ultimate load. Such a design, owever, might have the slight disadvantage of being slightly highly stressed in regard o tension in the concrete under working load. In many cases that did not seem a serious atter, particularly if cracking would not lead to any serious results if it occurred and articularly if such beams were indoors and under cover.

Before failure occurred in the beam in Fig. 22 a plastic hinge had begun to develop at e support and the bending moment at the support had been practically equal to the ending moment at mid-span. He hoped that later they would be able to report the sults of similar tests going around the corner of a portal frame with full effective depth eing used. There might be an objection to the possibility of very fine hair cracks occur-ng in prestressed concrete construction but such cracks had been tolerated for years in dinary reinforced concrete and it seemed that under certain circumstances there was stification for continuing that practice, particularly since the ultimate strength could be onsiderably increased and the method of construction facilitated.

Mr Donovan H. Lee (Consulting Engineer) said he agreed with all the principa opinions expressed by the Authors. He suggested that there was not yet a method of framing and prestressing of prestressed buildings which was generally agreed as satisfactory. The difficulties arising from movements in long structures continuously stressed had been rightly emphasized by the Authors, but he agreed there were now a number of successfully prestressed buildings where shortening due to stressing had been avoided. Some designers, he thought, were inclined to forget that to keep down the cost of stressing, the number of stressing operations should be kept to the minimum. Contractors' rates for stressing seen as rates in Bills of Quantities, varied over a wide range and indicated that there was still uncertainty in contractors' experience, or lack of it, as to the cost of stressing.

Mr A. Goldstein (Partner, Messrs Travers Morgan and Partners, Consulting Engineers, London), asked for the Authors' comments on some problems generic to the question of continuous prestressed portals. Referring specifically to a 110-ft-span two-pin portal, he said that if the portal were suitably shaped then, in addition to easing the cable curvature, the moments returned to the corners, which meant that the critical position was frequently at the corners. If the cables were taken right through instead of stopping along the length of the beam, there was some eccentricity to spare at the middle, which meant that the transformation which the Authors had mentioned could be obtained very comfortably.

Further research was still required on the question of transformation. Suppose, for instance, when designing the cables for a portal, it was found that the anchorages at the corners came outside or near the top of the section and transformation was carried out. Then, assuming point anchorages, there must be a certain amount of stress diffusion from the anchorage. What happened in that corner if the stress diffusion left some areas of the corner not stressed to the required degree? Little data on that point was available and the Authors' opinion would be useful.

The Authors had stated that in one of their designs the ultimate load had been reduced by transformation. But some tests carried out by the Cement and Concrete Association, showing transformation of continuous beam cables, did not indicate an effect on the ultimate load. Some tests had also been carried out by the same organization on ultimate load of portals and it would be useful to have those results and the Authors' comments on them. If the tests were representative it seemed that transformation would not affect ultimate load.

Finally, he agreed with the Authors entirely about through cables for small sections—putting a uniform prestress in the section. Working on a design stress of 2,000 lb/sq. in., the section was stressed to a uniform stress of 1,000 lb/sq. in. The moments at mid-span would produce a negative stress of 1,000 lb/sq. in., and the moments over the support would occur in the same way, and the section would be only half used.

What was the Authors' opinion of the idea of forming hinges over the supports and putting the cable right through, thereby getting not continuity but something which might be more economical The cables could be placed at the bottom of the core or middle third without causing tension or, if certain tension were accepted at the ends, they could be placed even lower to allow for permanent finishes. The effective moment which could be used if that were done was that due to a stress of 2,000 lb/sq. in. in the example he had chosen, for by placing the cable at the bottom of the core a positive stress of 2,000 lb/sq. in. could be applied at mid-span. There were then the two conditions: in the first case, a prestress of 1,000 lb/sq. in. with bending moment $WL/12$; and in the second case a prestress of 2,000 lb/sq. in. with bending moment $WL/8$ at mid span.

The second method seemed to give certain economies but it had its snags; the rigidity of the continuous beams was lost. Nevertheless, the method had possibilities and Mr Goldstein knew that on one occasion its use had proved on investigation to be more economical than uniformly prestressed multi-span continuous beams.

Mr F. J. Samuely (Consulting Engineer) said that the fact that rigid frames were possible should not be an inducement to use rigid prestressing in all circumstances. For multi-storey buildings, for instance, whenever possible it was better to have a pinned deck

-prestressed concrete which was not rigidly fixed, taking the wind thrust by special rrangements. He had recently undertaken an investigation concerning the wind support or a large one-storey building. It had been found to be cheaper to have a roof deck with olumns which were simply leaning against the solid roof. The roof was held up by he gable walls. That was cheaper than a rigid frame.

There were a number of instances where rigid frames had to be used, however, and it hould not be forgotten that the designer could alter the bending-moment diagram onsiderably to suit himself. The Author had shown examples taking the dead load on a eam on two supports and designing the corners accordingly.

One important factor in prestressed concrete was that the cost depended on the difference etween maximum and minimum bending moments; it was practically the same whether was $\pm$ 3,000 lb/sq. in. or plus 6,000 lb/sq. in. and zero. Therefore, by superimposing ending moment on to a corner, without making the corner more expensive, other bending oments could be reduced. In a rigid frame of conventional shape a hinge would not ormally be introduced, but Mr Samuely was convinced that if it were prestressed it as worthwhile introducing that hinge frequently, to make certain of a negative oment at the corner. The cost of the construction could be reduced by reducing the cost f the top.

It should not be forgotten that it was always possible to introduce a bending moment rtificially by prestressing. Assuming the presence of a prestressing bar, it was possible have any amount of positive bending moment superimposed on the whole frame or, lternatively, looking at the picture from the other side, it was possible to superimpose ny amount of negative moment. Once it was decided to prestress it was possible to alter e bending-moment diagram at will.

In a Paper[6] to the Institution of Structural Engineers a few years ago, Hendry had re- erred to rigid corners in steel and pointed out that a sharp corner gave very much heavier resses in compression than those calculated. That could be shown theoretically. Mr oldstein had been worried about the outer corners of a rigid frame, but it seemed that the ame sort of problem arose at the inner corners. If the section were an I-section, the roblem could be overcome to a certain extent by completely filling in at the corner to give reater resistance. It was a problem which ought to be investigated and it would be useful Professor Baker could give some time to it.

Mr J. A. Derrington (Engineer, Sir Robert McAlpine & Sons Ltd) said the Authors ad stated that they had found it unwise to draw conclusions about the relative merits f normally reinforced and prestressed concrete, but, in the Paper, he thought, they ould have brought out the influence which the relation between the live load and the ead load of a structure had on the choice of the medium of construction. That was made ear, particularly on p. 71 by reference to the 40-ft-span frame, which was cheaper than e normal concrete alternative, even though additional columns might have been troduced into the latter. From his own experience he could recall a three-storey uilding of roughly the same character, in which the difference in cost between prestressed oncrete and normal reinforced concrete was very small, but undoubtedly the introduction f two additional framing columns to the normal reinforced concrete alternative would ave reduced that cost by about 30%. The only reason was that whereas one of the ructures was a light roof structure, the other was an office building with much heavier ve loads.

One could only agree with the Authors' remarks that prestressed columns generally did ot pay for themselves, even though by using the magic word "prestressing" the allowable resses shot up from 1,140 to 2,000 lb/sq. in. for concrete which was common to both ructures.

He was intrigued by the novel approach to the design of portals whereby one made a

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[6] A. W. Hendry, "An investigation of the stress distribution in steel portal frame nees." *Structural Engineer*, vol. 25, p. 101 (Mar.–Apr. 1947).

full analysis, taking into account the moment of inertia of the sections, and then, by process of linear transformation, decided what the moment would be at the knee. Con sidering the total moment as for a simply supported beam and deciding what resistanc could be provided at the joint by prestressing forces or by mild-steel or high-tensile-ste reinforcement, would give the same answer a little quicker.

Mr Derrington then showed a number of slides illustrating a building in which pre stressing and normal reinforced concrete had been combined economically to give a attractive final appearance.

The Authors had shown most of their columns as hinged and it would be interesting t know the reason. The hinge cost money to produce and it raised complications in erectio and construction. It was also rather difficult to guarantee in practice.

With reference to the continuous prestressing of a framed building, the stress which wa applied when spread over the entire floor and beam construction generally worked ou fairly low and the resultant movement was much less than that expected from temperatur expansion and contraction.

Mr Derrington found prestressing had to justify with multi-storey office blocks in norma circumstances. He had worked out some figures and had found that the cost of providin 100,000 lb. prestress force for a 40-ft span was between 9*s* and 12*s* per foot run. Tha force could be resisted by high-tensile steel without prestressing for between 7*s* and 9 per foot run, depending on the type of high-tensile steel used.

An attractive way of blending the two materials was to use the long-line fully bonde prestressed unit with cast-in-situ slab, and the cost of providing that force then droppe to between 4*s* 6*d* and 5*s* per foot run. Continuity could be provided if necessary o desirable by the use of mild-steel or high-tensile-steel bars and the cost was much le than when introducing short prestressing cables, which raised those figures to between 16 and 20*s* per foot run.

Mr E. W. H. Gifford (Consulting Engineer) referred to Guyon's statement of 2 or years ago that prestressed concrete would not become a complete framework materi until it could achieve continuity. The way in which that had been done in Great Britai had been due in no small part to the work of Mr Walley and his colleagues.

The assumption of composite action between an in-situ floor and a simple bea presented no particular problem but obviously, with continuity, that floor coming int tension at reversal of moment changed the value of I for the section at that point. Th exact point where the change occurred was rather difficult to establish. Although was not necessary to split hairs over it, it would be interesting to know whether, in th tests which had been carried out, the Authors had discovered whether that point wa significant or not.

It had been pleasant to see from the Paper that two-stage prestressing had been use to a larger extent. In the light of later experience would the Authors have considere that in the Kilburn project? The use of two stages seemed very attractive for two reason First, in the average prestressed frame, because of the virtues of prestressing of being ab to reduce the effective dead-load moment going round the corner, they were not concerne to the same extent with reducing the mid-span moment by having very stiff columns. stressing were done in two stages, the first cable, which had no effect on the corner, too up a large part of the shortening of the beam. The second system of cables was availab for putting in the heavy live-load moment in the span and taking care of the live-loa moment going round the corner of the frame. The dead load had been virtually stresse out by the primary cable.

The Authors had mentioned the difficulty of longitudinal stressing particularly in lon buildings and they had mentioned a certain figure of elastic shortening in long building If the edge beams of a building were to be precast and some form of joint made between th beams and the columns they had to take into account not only the elastic shortening of th beam but also the effect of shrinkage of the mortar joint. Shrinkage could be offset b using very dry caulking, but 3 years ago he had had an interesting experience with te 33-ft crane beams where a 330-ft cable had been used as a secondary, with mortar pressu

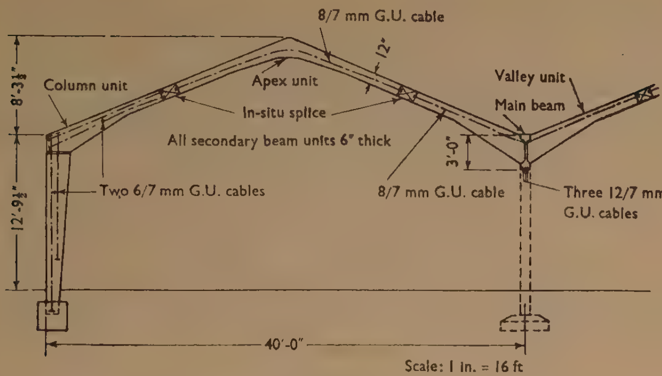
8/7 mm G.U. cable
12"
Column unit
Apex unit
Valley unit
Main beam
In-situ splice
All secondary beam units 6" thick
8'-3½"
3'-0"
12'-9½"
Two 6/7 mm G.U. cables
8/7 mm G.U. cable
Three 12/7 mm G.U. cables
40'-0"
Scale: 1 in. = 16 ft

FIG. 23

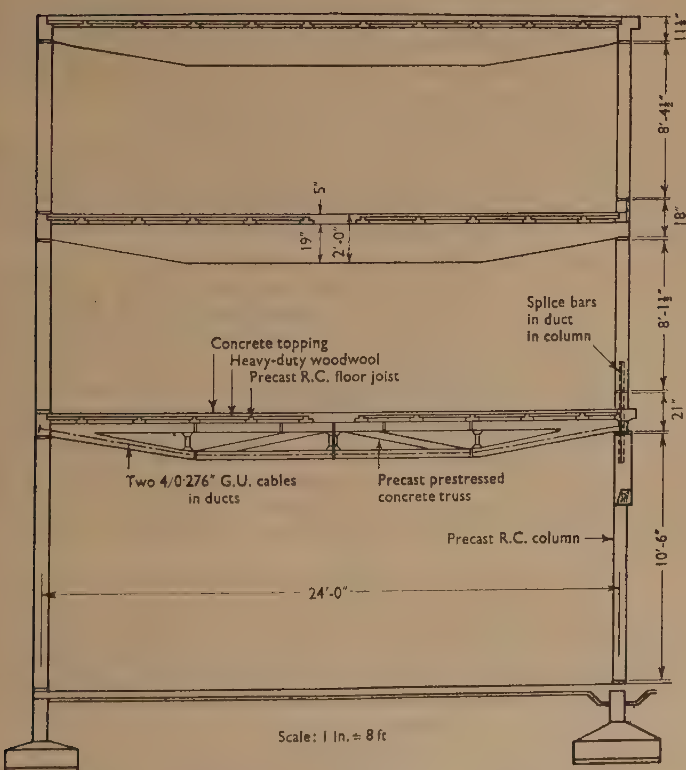
11½"
8'-4½"
5"
18"
19"
2'-0"
Splice bars in duct in column
8'-1½"
Concrete topping
Heavy-duty woodwool
Precast R.C. floor joist
21"
Two 4/0·276" G.U. cables in ducts
Precast prestressed concrete truss
Precast R.C. column
10'-6"
24'-0"
Scale: 1 in. = 8 ft

FIG. 24

pads. The whole was stressed in one operation. He had been suspicious of what would happen and had left the end columns free. There had been nearly ½-in. shortening or push-in on each end column, which was very nearly twice the elastic shortening.

Fig. 23 showed part of a five-span, five-bay multi-pitch roof. The secondary beams were at 15-ft centres supported by 60-ft-span main beams. There were two spans, each of 60 ft, for each bay. The secondary beam valley unit formed a diaphragm to the main beam itself. In some ways it was a little crude from the technical point of view but it had the advantage of economy on the site and in the drawing office.

The apex units had been prestressed in the factory, as had been the valley units and the external corner units. Reinforced concrete splices had been used at the points of minimum bending moment. The main beams had been cast in 15-ft sections with the valley unit stressed in between, using temporary false work. The main beam had been stressed with two cables as primaries and there was one secondary cable passing continuously through the building on the same system as used at Sighthill. The column had a precast cap and a mortar pressure pad for 3,500 lb/sq. in. was placed on each side. The cable transferred the prestress and at the same time performed the useful function of holding the beam on to the column.

There would be no point in carrying out single-stage prestressing on the 60-ft-span beam in that case because in such a two-span system there was no economy in making full continuity. There would be full shortening of the beams by all cables, which would be embarrassing to the outside columns.

Fig. 24 showed a frame building, although not in the sense used in the Paper. There was a prestressed truss of precast sections joined together with two tendons passing through the bottom boom. They were divided over the column so that splice bars passed through a hole in the end of the truss and down into the 6-in. × 6-in. precast reinforced concrete columns. Obviously the frame did not carry wind, which was carried in the way which Mr Samuely had described—by the floor acting as horizontal beam.

It was a school building and for its particular size and function it was a remarkably cheap job. The floor acted continuously with the truss and had been placed on it after the truss had been erected and before grouting. Only the live load was carried round the corner of the frame and because of the form of the truss the live-load moment at that point was very small.

Fig. 25 showed another small building which worked out cheaply. The main beam was a prestressed composite but there was a soffit unit similar to the type described by Mr Samuely[7] some years ago. The inverted U was stressed first with the primary wires along the bottom, and those were anchored. The work was done on a scaffold and the precast reinforced concrete column was placed against the end of the soffit system. The secondary cables followed the path indicated to the anchorage in the outer column itself. Wind was not carried on those columns. By so proportioning the beam and the column the moment passing round was extremely small and quite a small prestressing force was required in the second cable to offset it. It had been so arranged that that prestressing force was sufficient to balance the dead-load bending moment at mid-span, so that the live-load moment in the column was the only concern and the shortening effect of the relatively small force was almost negligible on that span. The roof was a normal three-pin frame.

Mr Gifford then turned to the application of temporary hinges to multi-span beams and particularly to systems of unequal span. The Authors might agree that on multi-span buildings equal spans in prestressed concrete tended to be a little uneconomic, and it seemed that in many factories and similar buildings a system of spans of the type more familiar in bridge work would be useful. He hoped shortly to have the opportunity to work out the details. He sketched a three-bay frame in which the outside spans carried

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[7] F. J. Samuely, "Some Recent Experience in Composite Precast and In-Situ Concrete Construction, with Particular Reference to Prestressing." Proc. Instn Civ. Engrs, Part III, vol. 1, p. 222 (Aug. 1952).

the central span as a suspended span for dead-load condition only. The floor beams, which would be of precast construction, later to be covered with the floor slab, were placed so that the outer-span beams, with their overhang into the centre bay carried nearly 80% of the dead load of the system. Pressure pads were situated at points of negative moment. A cable could be introduced running through the pressure pads into the required positions at the back and over the top section. The cable could be stressed and the joint would be filled at the bottom. That placed the cables exactly in the position where they were needed and avoided any unfortunate results from secondary moments. It also seemed to give a simple method of erection. It was obviously designed for a job

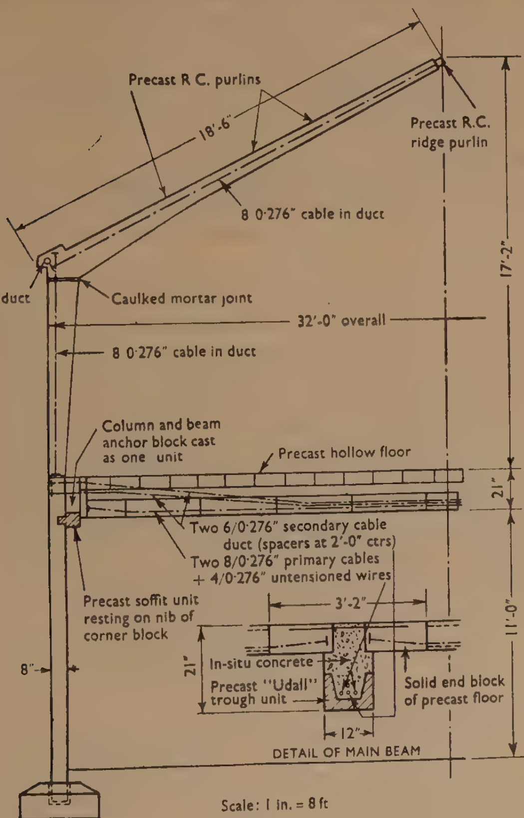

FIG. 25

where framing would be used in any case and the whole work could go through with no interruption through the secondary stressing, which could follow with the erection.

After the meeting, Mr Gifford wrote that he had examined that proposal in detail on a scheme for beams of 44–63–44-ft span at 20-ft centres. The very heavy live load of 300 lb/sq. ft produced such large reversals of moment in the end spans that continuity was obviously not economical under those conditions. But the use of the suspended span for live and dead loads appeared to be satisfactory. He expected that the factory would be built with that type of beam.

Mr J. N. Lowe (Pre-stressed Concrete Co. Ltd) said that because of the existing economic circumstances in Great Britain generally, engineers were spending a great deal of time and energy on the design of structures for buildings. It was important to remember that in building, more than in civil engineering, the design was closely associated with architecture, quantity surveying, and probably R.I.B.A. general conditions.

The work of the engineer might be associated with only 25–40% of the total cost of the work. Nevertheless it should be realized that the engineer's work could greatly affect the remaining cost and it was not always the cheapest structure that produced the cheapest total job. In particular, the effects of installation of services should be borne in mind.

First, the engineer and the architect would choose the medium in which they would work and then select the method of using that medium. The Authors had presented a useful summary for helping the architect and engineer to make that choice.

Mr Lowe's firm had followed much the same path of development as used by the Authors, starting with a frame which was basically beams on load-bearing walls and following through with work not unlike Sighthill. The next step was to continuous frames, leading to the method of supporting floors on pin-jointed columns, staying the building against horizontal deflexions by anchoring the floors (which then acted as horizontal beams) to suitable and convenient vertical cantilevers such as stairwells, lift wells, and internal or external walls. The company had used that method satisfactorily for millions of square feet. They were now, however, once again examining the usefulness of beams on load-bearing walls.

On p. 74 there was a very pointed reference to how much one part of a structure acted with another part. Tests had indicated that it was difficult to prevent interaction between all parts of a structure, and the engineer had to examine that question very carefully.

On pp. 75 and 76, the Authors had mentioned the importance of carefully planned erection procedure. Before embarking upon a complicated erection plan the engineer should satisfy himself that unforeseen delay in one item of the erection procedure did not involve accumulative delay to the whole job. The engineer should avoid the possibility of having to recalculate an erection procedure during the work to reduce such accumulative delay.

Well-planned erection procedure could possibly save materials, but it should be borne in mind that continuous design did not always do so—particularly if the design was not according to ultimate loading—for it might increase labour costs and might also tie the hands of the contractor more than the engineer or architect for the job would wish.

On p. 79 the Authors had suggested that the prestressed bending-moment diagram was always worthwhile drawing. That was a vital point. It was absolutely essential for an engineer to remember that prestressing was the application of an external force. Without that realization he could never work satisfactorily in the medium.

Turning to p. 81 Mr Lowe was not sure that the simple grid of columns and beams was the cheapest method on a cubic-foot basis—always bearing in mind that the structure was but part of the cost of a building.

On the other hand he agreed strongly with the comment on p. 83 that there were advantages in uniformly stressing a member. The engineer should never be goaded into liking the pretty technical solution in building. His answer affected the cost of the work of architects and consulting engineers.

Summarizing, Mr Lowe said that the decision to be reached was whether to frame or not to frame. Any choice made now between load-bearing walls, fixed building frames, and pinned building frames, might not be a final one, for the ratio of cost of labour to materials was changing rapidly and what was right today might be wrong tomorrow.

Mr H. C. Visvesvaraya (Civil Engineering Department, Imperial College of Science and Technology) said that Professor A. L. L. Baker, at the beginning of the discussion, had already indicated their general approach to the problem. The continuous beams tested were 30 ft 8 in. long on two spans each of 15 ft with third-point loads. The continuity was established by short internal ties at the support, anchored within the depth of the beams. On pp. 77 and 78 the Authors had indicated some of the disadvantages of the capping cable system as used in France. There were of course many more difficulties with them,

all of which had been solved by the suggested new method of construction. Further, certain additional advantages had been gained. The details would be published shortly. The beams tested so far had been pretensioned prestressed units joined together, as indicated in Fig. 26a. The resultant line of pressure was indicated by the dotted line. There was an unavoidable discontinuity or sudden jump in the line of pressure at the anchorages of the internal tie but that should not cause any worry because in general those anchorage points could be located in the zone of contraflexure where the "cable zone" was widest and most suited to deal with such discontinuities. The condition of the line of pressure could be improved when curved cables were used in the span such as in post-tensioned units, cast-in-situ or precast, as indicated in Fig. 26b. It might be relevant to

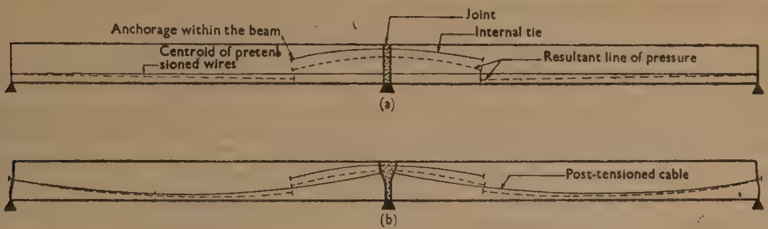

Fig. 26

point out that the invariance of ultimate load on prestressed continuous beams with linearly transformed cables shown by Guyon and Dr Morice had serious limitations and those limitations and the consequent drawbacks in any new way of approach to ultimate load design should be borne in mind.

Very little consideration seemed to have been given to joints although they had been used on many occasions. Joints should receive greater attention in prestressed concrete construction than in reinforced concrete construction because realization of high working load meant prevention of cracking at those loads, which in turn meant obtaining the maximum resistance against tension. In the case of normal reinforced concrete the quality of the concrete used was not generally high and hence the modulus of rupture of a well-made joint would not be very different from that of the adjoining concrete; whereas in the case of prestressed concrete the concrete adjoining the joints might have a modulus of rupture of the order of 600 to 1,000 lb/sq. in. or even more; all that would be wasted unless the joint itself could take a high tensile stress.

In the case of tests carried out at Imperial College, Professor Baker had mentioned the minute cracks which had occurred at about 80% of the working load (taken as half the ultimate load). If cracking was considered to be the criterion for working load the load factor would be about 2·3. The cracking occurred at a place where it could be tolerated in a normal structure, such as the buildings under discussion, because from the point of view of aesthetics it was at the support and did not matter; from the point of view of corrosion of steel there would normally be the column and hence the cracks were unlikely to form at that load.

The Authors had indicated on p. 86 in Appendix I that the load factor for the frame was 5·8. That gave an indication how uneconomical a structure might be from an ultimate-load point of view when the designs were based only on elastic theories.

He did not agree with the Authors that uniform prestressing in building frames was a solution of the problems which were being faced today.

He entirely agreed, however, with their suggestion that the prestress-moment diagrams and secondary-moment diagrams should always be plotted. He found the influence-coefficient method of analysis to be the easiest though the Authors preferred the theorem of three moments. The advantage of the influence-coefficient method was, he thought,

that it looked upon the structure as a whole and also that very little extra labour was required to check by both elastic and plastic methods of design.

The Authors had mentioned on p. 71 that 2,700 lb/sq. in. compression and 600 lb/sq. in. tension had been allowed at working loads without any mild-steel reinforcement and that no trouble had been experienced. That, he thought, should serve to stimulate confidence that so long as the structure had the required factor of safety in regard to the ultimate load, those high stresses could be allowed without any harm at working loads. However, the cube strength of the concrete in that case had not been mentioned; what was it?

On p. 83 the Authors had stated "it is true that on the ultimate basis . . ., yet permanent cracks in a prestressed structure would be undesirable." Mr Visvesvaraya wished to qualify that statement by saying that "as long as these permanent cracks are not wider than those permitted in corresponding ordinary reinforced concrete structures."

Mr G. O. Kee (Assistant Engineer, Donovan H. Lee, Consulting Engineer) said that ultimately the ideal design would be for fully prestressed frames, but that stage had not yet been reached. Each material had to be used to its best individual advantage with the present knowledge Cost was generally the deciding factor and a prestressed member with in-situ concrete, mild-steel reinforced, seemed at present to be the most economical solution in most cases. He showed a slide illustrating a large structure of composite construction in the United States, covering 6 acres, where the beams were prestressed and designed for continuity by the use of mild steel across the supports. The in-situ deck was designed as a car park. The precast members had been cast and erected within 75 days.

Dr P. B. Morice (Head of the Structures Department of the Research and Development Division of the Cement and Concrete Association) said the Paper represented the first considered discussion of the application of continuity to prestressed concrete building structures in the United Kingdom. So far as he could see, continuity in beams became advantageous in the design-load range only when the dead weight was too great to allow a direct design method to be applied. In some cases the moment variations in continuous beams could be as much as 15% greater than those in the corresponding simple beam system. Probably a similar effect occurred in some forms of building frame.

He could not understand why the Authors had restricted the ends of their beams in simple pin-jointed frames to a state of no rotation, particularly where they used leg prestressing, since they could use, with some advantage, other concordant profiles which bent legs or allowed them to rotate. Also, he could not see why the small eccentricity variations required to eliminate elastic, creep, and shrinkage shortening were not applied.

Mr Goldstein had mentioned the effects of linear transformation on ultimate load. With full moment redistribution linear transformations theoretically had a very small effect, if any, on the ultimate strength of the structure as a whole. Dr Morice had carried out tests on both continuous beams and portal frames in which transformations had varied the strength ratios at critical sections from 1 : 1 to as much as 20 : 1, and the ultimate strength had remained constant. However, in mentioning those results he pointed out that at the moment they referred to structures in which centre-span point loading had been applied, but full moment redistribution might not occur, for example, in the case of sway in frames.

Mr R. W. Pearson (Senior Structural Engineer, Chief Structural Engineer's Branch, Ministry of Works) said he had been directly concerned with the Sighthill Stationery Office store at the erection end. The shortening of beams when stressed across the full width of the building, especially with a multi-storey building, was, he was convinced, of great importance, particularly where the columns were very stiff. In the case of Sighthill the secondary beam had been stressed across the full width of the building—120 ft—and the shortening which had taken place during that secondary prestressing was about $\frac{3}{8}$ in. in 120 ft. There was a very stiff column and the effect on the column would have been

serious if they had not permitted the beams to move relatively to the column. The main beams were simply supported and rested on a bracket. While the secondary beams had been stressed, the main beams had slid on the bracket a distance of about half the total shortening. The main beams were jacked off the column, thus permitting the column to spring back and allowing the rest of the movement to take place.

That movement had taken place with only a second cable, which in fact had fewer wires than the primary prestressing cable in those beams. Subsequently further movement would take place due to creep and shrinkage. It had been interesting to hear mention of shrinkage in the joints and the design of joints for connecting precast members would be an interesting subject for a Paper.

The form of the joint was of great importance because of the cost. The question was whether such joints could be filled with mortar or concrete, and whether the cable which passed across them could be sheathed, at a reasonable cost.

*** **Mr J. S. Shipway** (Assistant Civil Engineer, Messrs Maunsell, Posford, and Pavry, Consulting Engineers), referring to the Stationery Office Building, asked if the stresses of 2,700 lb/sq. in. compression and 600 lb/sq. in. tension were the initial stresses in the beams before anticipated losses of prestress due to creep and shrinkage occurred, or the final working-load stresses. If the former, what were the stresses under working load and what percentage loss in prestress had been allowed for?

Longitudinal prestressing had been mentioned in the Paper and in the discussion, but Mr Shipway was not clear as to its precise purpose in the case when the longitudinal beams were pure tie-beams. Was it designed merely to hold the whole structure firmly together and if so, was the amount of prestress required determined by calculation or by engineering judgement? Was the longitudinal prestressing of the whole floor, as in the examples quoted at Le Havre and in America intended to have a function similar to the transverse prestressing of bridge decks?

There was no mention in the Paper of any difficulties arising in the design of end blocks, and it would seem that they no longer presented any problems. Was it becoming quite usual now to design the reinforcement for end blocks and such points as portal knees more by judgement and experience than by actual detailed calculation? Where the tendons passed over the supports for continuity and were anchored in the respective beams, was there any special reinforcement required to take care of the high local stresses at those points, and if so was the amount required calculated for, or otherwise? Could the Authors supply a detail for the reinforcement, if any, at one of those points?

Mr Walley, replying on behalf of Mr Adams and himself, referred first to Professor Baker's remarks on concrete strength and the correlation between design cube strength asked for and that obtained. That was still largely in the hands of the man who put the concrete into the job. The Ministry had a section for advising contractors on the possible best mix to use on a job, but in the end they were left in the hands of the man doing it. The relatively high concrete strengths quoted had been obtained without elaborate or expensive precautions.

He agreed with Professor Baker that probably the ultimate strength was more important than the stresses at normal design load, provided that under normal working conditions the deflexion and general behaviour of the structure were satisfactory. Figs 21 and 22 showing the method used in the Imperial College experiments were interesting. He was not sure that he could agree entirely to cracking over supports if it occurred under the real load which the building was called upon to carry. His only worry was with corrosion. He was not sure that enough was yet known about corrosion in prestressing wire and rods but he had no qualms about the cracking from the theoretical or any other point of view.

*** This contribution was submitted in writing after the closure of the oral discussion. —Sec.

It was true, as Mr Lee had said, that prestressing was not the only answer. The fact that prestressing was a successful medium for a given structure did not mean that it would necessarily be successful or cheap even in a similar problem. Each case had to be considered on its merits and an unbiased view given to the architect or whoever required the information. He hoped that he had not given the impression that there was no other method but prestressing; in fact, he had been at pains to say that it was in reality a very small part of any large building programme although it was a desirable method in certain well-defined cases.

Mr Goldstein had asked what happened at the knee: everyone would like to know the answer to that question. The stresses at that point were extremely high. With normal distribution on the anchorage there was a zone of high compression spreading out from the distribution plates with possibly idle concrete at the corner. In the slide shown by Mr Adams a mild-steel cage had been put at that point; the size of the rods and their disposition being determined by experience. That was the method adopted on a great many engineering problems, except that the experience was hidden in a Code of Practice. On one of the portals there had been trouble on the corner with cracking and crushing of the concrete. It had been found that the contractor had omitted some of the rods in the cage at that corner. A portion had had to be cut away for the steel to be inserted, and after that there had been no trouble.

He agreed with Mr Goldstein and Dr Morice on the fact that linear transformation did not necessarily affect ultimate load conditions. Both Mr Guyon's and Dr Morice's experiments confirmed that in certain cases that was so. Nevertheless, it would be reassuring to have a theoretical basis for it.

Mr Goldstein had discussed whether uniform stressing was better than putting the cable at the lower core point. The second method had been used at Sighthill but it had the disadvantage of lack of continuity. In his Introduction he had apologized for having mentioned ancient buildings—Sighthill was now in that category, though not yet an ancient monument! His apology had been unnecessary since many of the speakers had described the same devices as those used at Sighthill in connexion with other structures.

Mr Samuely had raised an important point which was not appreciated by people who were not very familiar with prestressing. It was that the bending-moment diagram could be varied in an infinite number of ways and the prestressing forces could be used to influence the bending-moment diagram into the desired shape for the structure. By placing cables relative to some hinge that alteration could be made to give the portal structure the requisite shape. The placing of pins in a portal structure, as Mr Samuely had described, was also an important device in framing.

Mr Derrington had complained that nothing had been said about the influence of the relation between the live load and the dead load. Had the Authors tried to weigh all the pros and cons the Paper would have been extremely lengthy. Prestressing certainly became a possibility worth considering whenever the dead loads formed the major portion of the loads to be carried. In the various roof structures shown by the Authors the major load was the self-load and the external loads were essentially very small by comparison.

He did not agree with Mr Derrington on the detailed calculations in connexion with the design of portals; he did not think the calculation was exactly the same whether the moment was put on the knee with prestressing tendons or reinforcement. Redundant reactions were being introduced which were not being introduced when taking the continuity moment with mild-steel rods. There were two ways of dealing with the problem: one was to avoid rotation and secondary moments; in the other the rotation and the moment induced in the legs were calculated and allowances made. The Authors could not agree with Mr Derrington that a linear transformation altered the knee moment. They had assumed that it had no effect at all on the moments. It did not matter very much which way was used. The slides showing composite action between prestressed members and in situ concrete were extremely interesting and confirmed the Authors' feeling that it was a very promising form of multi-storey construction.

Mr Gifford had asked whether they thought two-stage prestressing always an advantage.

Where dead load and live load were very different there were significant theoretical advantages in using two-stage prestressing but a question to be considered was that asked by Mr Lee—whether the equipment was brought back to the job or left on the job in idleness. All such factors had to be taken into account in deciding whether to use one-stage or two-stage prestressing.

Mr Gifford had also raised the question of elastic shortening and the subsequent movement of long beams. It was not clear where the movement came from when it was equal to twice the elastic shortening when stressed. That would be expected to happen later as a result of creep and shrinkage but not immediately. In the Paper the Authors had said that more work should be done on the problem so that they might feel more confident in carrying out long sequences of stressing and so eliminating intermediate anchorages. It was a convenient form of construction because the anchorage devices were external and accessible, which was not always the case with internal jacking arrangements.

His drawings illustrating the system of supports at points of contraflexure had been used many times, for instance many years ago on crane gantries in Orleans. In that case continuity had been established over the supports.

Mr Lowe had outlined the three choices of building frames. The third was becoming more and more popular. It was a useful way of looking at the problem because in so many cases the moment from wind and sway was carried twice over—in the frame and the diaphragm walls, which made it impossible for the frame to act if it wanted to. There was no reason that the wind moments should not be carried, thus pleasing the architect by reducing all the column sizes.

Mr Lowe's remarks on erection procedure were all-important. When introducing prestressing forces it was possible to juggle in many ways but the result could be to make it very difficult for the site. It was necessary to go back to something simple, and the Authors had appealed for that in the Paper.

Mr Visvesvaraya had given some interesting comments on continuity and on the effective centre of the prestressing forces. That could be quite remote from the position of the prestressing tendons. He had drawn attention to the dead load factor found by calculation on one of the jobs in the Paper, and that sort of thing arose from time to time when adhering rigidly to the working-load conditions. On the other hand, with a load factor of 2 only might it not behave very well at working-load conditions. The load factor of 5·8 related to the centre of the beam after linear transformation. The true load factor was about 3 in both beams and columns. In that case the working load and not ultimate strength was used for design. The average 28-day cube strength at Sighthill was just over 7,000 lb/sq. in.

The Authors could not agree that cracks as wide as those acceptable in reinforced concrete should be allowed.

Replying to Mr Lee, Mr Walley said he was sure that composite construction was a powerful tool when designing continuous buildings. He agreed with Dr Morice that continuous beams were not always advantageous; Sighthill was a case in point.

He thanked Mr Pearson for his comments on Sighthill; the views on the additional movement which could take place were important. That had not been so important at Sighthill because it was a completely glazed building, but it would become important where diaphragm walls, which were solid, were introduced, since they tried to stop the building from moving. Provided it could move he was not worried but once they tried to stop it from moving different considerations arose.

In answer to Mr Shipway, the stresses quoted for Sighthill were erection stresses. The stresses under working-load conditions were 2,000 lb/sq. in. (compression) and for zero (tension). The percentage loss of prestress assured was 15. The Authors agreed that where longitudinal beams were merely tie-beams there was no need to prestress. That was the case on the Kilburn building. Where, however, intermediate columns were omitted it became a necessity. The longitudinal stressing of the floors in Le Havre and America was, as far as the Authors knew, the primary stressing. The detailing of the end blocks was often done by eye provided there was nothing unusual in the disposition of the

anchorages, but the Authors would hesitate to recommend that method to anyone who had not done a detailed calculation of such a block. It would not perhaps be wise to supply a detail of a particular end block since it would not necessarily apply to another job.

The closing date for Correspondence on the foregoing Paper was 15 March, 1956. No contribution received later than that date will be printed in the Proceedings. —Sec.

STRUCTURAL AND BUILDING ENGINEERING DIVISION MEETING

29 November, 1955

Mr F. M. Fuller, Member, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Author.

Structural Paper No. 46

THE STANCHION PROBLEM IN FRAME STRUCTURES DESIGNED ACCORDING TO ULTIMATE CARRYING CAPACITY

by

* Michael Rex Horne, M.A., Ph.D., A.M.I.C.E.

SYNOPSIS

The general problem of stanchion behaviour in structures proportioned by collapse as opposed to safe-stress methods of design is discussed. The loading and restraint onditions operating on a stanchion length contained in a structure which is on the point of collapse are classified. Each class of behaviour provides a potential basis for design, provided it is well enough understood and leads to results which can be expressed to a ufficient accuracy in simple terms.

Earlier investigations concerned with stanchions in plastic structures are reviewed nd classified according to the adopted scheme, and the merits and demerits of design nethods based on the various classes of behaviour are assessed. It is concluded that here is a possibility of using a design method in which the stanchions, designed elastially, give maximum support to the beams which are designed according to the simple lastic theory with, in general, hogging-moment hinges at both ends. Since this will esult in large bending moments being induced in the stanchions refined methods must e used in the design of the latter, the basis of design being the attainment of the yield tress in the most highly stressed fibres. Whilst existing design methods for stanchions night be adapted for this purpose by operating upon them with a suitable load factor, t is shown that they are, in this context, not entirely satisfactory, and the remainder of he Paper is concerned with the elaboration of a suitable method.

The design method evolved allows for the possibility of flexural-torsional buckling of -section members bent about the major axis. Account may be taken of bending moments pplied simultaneously about the major and the minor axes, and in any ratio as between he two ends. The permissible loads given by this method are compared with those erived from B.S. 449 and from the "Recommendations for Design" of the Steel Strucures Research Committee, proper allowance being made for the implied load factors.

Results in close agreement with those obtained from the "Recommendations for esign" are obtained except where, in the proposed design method, some of the simplifyng assumptions made in the Recommendations in order to achieve a manageable proedure are avoided.

INTRODUCTION

HE design of single-storey rigid-jointed structures by the plastic theory is now an ccepted alternative to the use of orthodox design methods based on so-called safe

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stresses. According to the former procedure, the structure is so proportioned tha collapse would not occur at loads less than the working loads multiplied by an agreec load factor. Methods of calculating the collapse loads of a given structure and o choosing suitable sections to enable a structure to support given loads are avail able.[1, 2, 3] In these methods collapse loads are derived without reference to defor mation and deflexions, apart from the simple requirement that sufficient plasti hinges must occur to transform either the whole or part of the structure into a mechanism. Whilst it is not always possible to calculate the bending moment throughout the structure at collapse without reference to a complicated elastic plastic analysis, all the bending moments needed to confirm the accuracy of a collapse load may be obtained by considering the requirements of equilibrium only This fact, together with the attainment of considerable economy in the use of steel makes the simple plastic theory an attractive basis for the design of rigid-jointec structures possessing, in the elastic range, many degrees of redundancy.

It has, of course, to be recognized that the collapse load of a structure, as calcu lated by the simple plastic theory, may not be the only criterion on which desigr has to be based. As when using orthodox safe-stress methods, conditions of fatigue produced by repetitive loading, or the incidence of excessive deflexions, may be the over-riding factor and must naturally be allowed for where necessary. There are however, two limitations to the application of the plastic theory in its simple forr which are of more potential importance than either fatigue loading or excessive deflexions. First, compressive loads in a member may endanger its internal stability before the collapse load of the structure, as calculated by the simple plastic theory is reached. Secondly, the deflexions of the structure just prior to collapse may sc change the conditions of equilibrium for the structure as a whole, that there is ar appreciable alteration in the collapse load. These effects may be referred to briefly as "member instability" and "frame instability" respectively, the latter term being used since the alteration in the collapse load of the structure is virtually always a decrease, thus implying a form of instability. These two factors have caused the application of the plastic theory to be limited mainly to single-storey construction.

In multi-storey buildings the total vertical loads may be sufficient to cause frame instability of the structure in general, or member instability in the stanchions, to a sufficient extent to render the simple plastic theory inapplicable. Accordingly, ir presenting the plastic theory, warnings have frequently been given that the method are not appropriate for structures containing members with high mean axial stresses Unfortunately, because of the great complexity of the subject and despite a sus tained effort directed towards its solution, it has been impossible to state limiting values of axial stress and slenderness below which the simple methods are applicable This complexity exists not only in the plastic range but also in elastic structures the difficulty of the subject is in fact partly responsible for the illogical nature o existing orthodox methods of designing stanchions in multi-storey frames.

The present Paper is concerned primarily with member stability in stanchions and is divided into two parts. Part 1 sets out to describe, in general terms, the nature of the stanchion problem in structures which are to be designed on the basi of their ultimate carrying capacity. A review is then given of the experimenta and theoretical progress which has been made in studying various aspects of stanchior behaviour, and an assessment is made of possible design methods for multi-storey multi-bay frames. The conclusion is reached that, in the immediate future, th

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The notation is given on p. 143.

ost promising approach is to design the beams on the basis of the plastic theory, e stanchions being designed so that they just remain elastic under such conditions. art 1 closes with a study of the possibility of using, for this purpose, existing anchion design rules, suitably modified to allow for their introduction into a load-ctor method for the frame as a whole. It is shown that neither the regulations ontained in B.S. 449 (1948),[4] nor those suggested by the Steel Structures Research ommittee [5, 12] are entirely satisfactory.

The second part of the Paper is concerned with the derivation of an elastic design ethod for I-section stanchions subjected to any combination of terminal moments bout the two principal axes at either end. The resulting curves may be used to esign the stanchions in structures in which the beams are designed by the plastic eory, as suggested in the first part of the Paper. The present Paper does not resent a complete design method for multi-storey multi-bay frames. Various roblems remain to be solved before this objective can be achieved, and Part 2 oncludes with a discussion of these remaining difficulties.

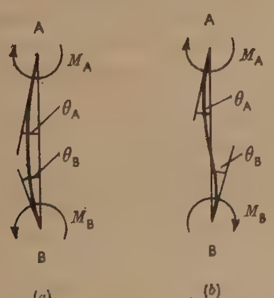
Part 1

The general stanchion problem

) The stanchion problem in elastic structures

The general stanchion problem will be approached by considering the behaviour f a stanchion in a multi-storey multi-bay frame in which all members meet at ght-angles, and in which the joints are rigid. This course is adopted as the best eans of arriving at a general classification of all the many cases which may arise. is not, of course, completely general, but it will be found that the system of assification includes most of the important loading conditions, including those countered in pitched-roof portal frames.

The main factors influencing the general behaviour of stanchions in the elastic nge are well known. A quantity of great significance is the Euler critical load lenoted hereafter by P_E) for buckling of the member as a pin-ended strut about e principal axis under consideration. When the axial load P is less than this uantity, an increase in either of the terminal bending moments will lead to an crease in the corresponding end rotation of the member relative to a straight line inting the two ends, provided the rotation is measured in the same direction as e bending moment. When P is greater than P_E at least one end of the stanchion



IG. 1.—Elastic behaviour of stanchions when the axial load is less than the Euler crippling value for a pin-ended strut

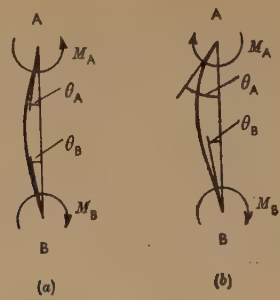


FIG. 2.—ELASTIC BEHAVIOUR OF STANCHIONS WHEN THE AXIAL LOAD IS GREATER THA THE EULER CRIPPLING VALUE FOR A PIN-ENDED STRUT

shows reversed behaviour, i.e., increase of moment is only accompanied by a increase in the rotation provided this is measured in a direction opposite to tha of the applied bending moment. These facts are illustrated in Figs 1 and 2, th rotations θ_A and θ_B being shown in each case as positive in those senses in whic an increase in any rotation is accompanied by a numerical increase in the corre sponding bending moment. The stanchion lengths AB in Figs 1a and 1b carr axial loads below the Euler value, and in each case the rotation is measured positiv in the same direction as the corresponding terminal moment. In Figs 2a and 2 the stanchions carry axial loads in excess of the Euler value. Both end moment are measured opposite to the rotations in Fig. 2a, whilst in Fig. 2b the lower bendin moment only is measured opposite to its rotation.

The apparent change-over of behaviour of a stanchion when the axial load passe the Euler value is not, of course, as sudden as the above simplified account indicate A quantitative definition of this behaviour may be obtained by reference to th ratio of increase in terminal moment in a stanchion per unit increase of end rotatio measured in the same direction, the remote end of the stanchion being assumed t be subjected to a constant bending moment. This quantity may be termed th stiffness and decreases as the axial load is increased, becoming zero at the Eule load and negative at higher loads. The decrease of stiffness with increase of axi load is usually ignored when analysing continuous structures, since it leads to con siderable complications, but analytical methods are available, usually based on th use of Berry functions.[6] An ingenious solution based on the use of nomogran has been given by Wood.[7]

In design methods based on elastic behaviour the effect of axial loads on stanchio stiffness is almost invariably neglected. In the research leading up to the Recon mendations for Design of the Steel Structures Research Committee, Baker ar Holder[8, 11] came to the conclusion that in practical structures the working loa in the stanchions would be too far below the Euler values for the decrease in stanchio stiffness to be important. The same procedure was adopted by Wood[9] in a lat design method which, although more versatile and in general more accurate, w unfortunately more difficult in application than the Recommendations for Desig Orthodox design methods, such as B.S. 449,[4] make allowance for the type of stanchio behaviour indicated in Fig. 2 by taking the equivalent length of the stanchion

e less than its actual length. It has, however, been shown [10] that this treatment as no logical basis when the criterion of safety is the attainment of a specified stress -as it must be when an elastic design method is being employed.

) *The stanchion problem in structures stressed beyond the elastic limit*

The history of elastic design methods for multi-storey multi-bay frames might ad one to suppose that it is unnecessary as well as impracticable to take account f the reduction of stanchion rigidity due to axial load. There are, however, several easons why such a course should not be adopted, at any rate without further ivestigation, when considering the design of structures on the basis of ultimate rength. The mean axial stresses due to the ultimate loads are much greater ian those due to the working loads, thus causing a significant decrease in stanchion gidity. This is true even when the stanchions behave entirely elastically; if the ads are increased beyond the values at which the yield stress is reached in a stanion, behaviour of the type shown in Fig. 2 may occur at an axial load considerably elow the Euler value. Finally, there is a vicious circle involving, on the one hand, ie behaviour of the stanchions and, on the other, the selection of a design method ading to the choice of the stanchion sections themselves. This latter point calls or elucidation.

If it is assumed that the stanchions behave as shown in Fig. 1, the surrounding eams are evidently being helped to support the loads which they carry. To the xtent that a design method is rational, it will take advantage of this fact in deterining the beam sections, and will at the same time proportion the stanchion to ustain bending moments in the directions indicated. This procedure will of itself isure that axial loads are considerably below the Euler values. Conversely, if anchion behaviour of the type shown in Fig. 2 is assumed, the beams will have be made of sufficient strength and rigidity to withstand the bending moments hich the stanchions exert upon them. It will then be found that the most econoical stanchion is so strengthened by the presence of the beams that it will indeed e capable of carrying an ultimate load greater than the Euler value. The effects f the design method itself on stanchion behaviour and *vice versa* are in fact so itimate that a design method cannot be derived by any logical process without rst making certain assumptions regarding the interactions between the beams and ie stanchions.

The above arguments show the necessity of considering all the types of stanchion ehaviour shown in Figs 1 and 2, not forgetting the complications introduced by lastic or partially plastic behaviour. To solve such a problem in its entirety is of ourse impracticable, and one can only hope to take certain typical cases. As a arting point, one may discuss the two conditions represented by Figs 1a and 2a.

In Fig. 1a, the stanchion AB gives active support to at least some of the surounding beams. In order to achieve economy in design, it will be desired to lect the cross-sections of the members such that, at the collapse load, at least one iember can deform indefinitely without increase of load. If the beams attached end A deform, leaving stanchion AB unaffected, then plastic hinges must form etween the beams and the stanchion, and the bending moment M_A will remain onstant. It would, however, be impossible for the stanchion to deform without ie beams collapsing, since in this case θ_A would increase with decrease of the moment M_A thus violating the assumed condition of the stanchion.

Turning now to Fig. 2a it is readily seen that if the beams were on the point of ollapse with plastic hinges at their ends, the stanchion could not remain stable.

Hence, in this case, if only one type of member fails it must be the stanchions, th beams being still predominantly elastic, so that the bending moments M_A and M may increase with increasing values of θ_A and θ_B.

The above discussion shows that if failure occurs either in the beams or in th stanchions, but not in both, then the loading condition in Fig. 1a is produced b beams which are on the point of collapse and exert constant bending moments o the stanchion (giving "plastic loading") whilst that in Fig. 2a corresponds to beam still in the elastic condition (giving "elastic loading"). These two types of loadin may be represented diagrammatically as in Figs 3b and 3a respectively. Plasti loading is represented by a loaded cantilever since the plastic hinge at the end of beam on the point of collapse will apply a bending moment which remains constan if the joint rotates.

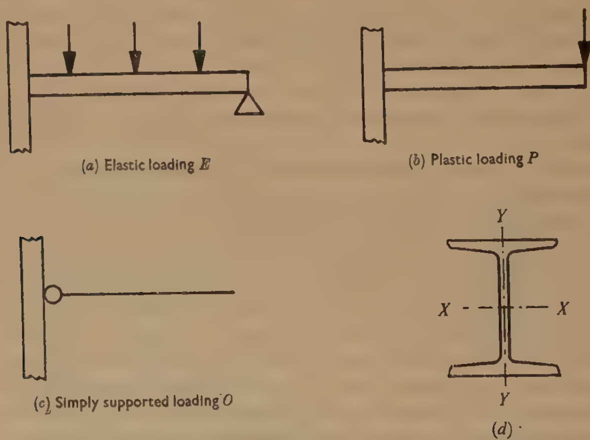


FIG. 3.—DIAGRAMMATIC REPRESENTATION OF STANCHION LOADING CONDITIONS

Between the two extreme cases in which the stanchions give maximum suppo to the beams and *vice versa*, there lie a whole range of differing conditions. I general, the intermediate cases may be expected to be more difficult to solve tha the extreme cases and for this reason unlikely to lead to practicable design method There is, however, one particular condition of great importance, namely, that which the interaction moment between the beam and the stanchion is zero, a remains zero, as the structure collapses under the ultimate load. The structure then behaving as though it had non-rigid joints, since the beams and stanchio can only deform if they are independently in a state of collapse. Hence, even wh the joints are rigid, this condition may be represented diagrammatically by a pi joint as in Fig. 3c and referred to as "simply supported loading."

One of the three conditions "plastic," "elastic," or "simply supported" loadi may be taken as a first approximation to the type of interaction actually occurri between one end of a stanchion and a beam when either or both are on the poi of collapse. Whilst this classification is discontinuous and only very approxima it has the advantage of possessing simple physical interpretations. It is an ov simplification in that reference is made only to a single stanchion length in co junction with a single beam, whereas both the stanchion and the beam may

ontinuous through the joint. With respect to the beams, no great difficulty hould be encountered in choosing the correct classification for a particular case, ince the behaviour of continuous beams in the plastic range is well understood. n general, if either of the beams on the two sides of a joint shows a change of end aoment when the joint is rotated, the stanchion loading is "elastic." The loading s otherwise "plastic" except when the net moment applied by the two beams is ero, when it becomes "simply supported." Stanchion continuity presents a more lifficult problem. In the "elastic loading" case, the stanchion is on the point f collapse while the beam remains elastic and is thus largely insulated from the nfluence of stanchion lengths above and below. In the "plastic loading" case, owever, there must be a certain distribution of the net beam moment between the ipper and lower stanchion lengths, whilst even in the simply supported case there s interaction between a stanchion length and the lengths above and below. The roblem of the continuous stanchion is thus of considerable importance when dealing vith the "plastic" and the "simply supported" loading conditions.

3) *The classification of stanchion behaviour in a structure at its ultimate load*

The remarks made in the previous section lead to a general method of classifying he loading conditions in a stanchion length when a structure is on the point of ollapse. The stanchion will be assumed, in the general case, to be connected at its nds to beams which could apply bending moments about either or both of the rincipal axes, XX denoting the major axis and YY the minor (see Fig. 3d). When stanchion is subjected about its major axis to plastic loading, this will be denoted y the letters P_x, the beams responsible for this loading being termed the major-axis eams. If, for the same stanchion, the minor-axis beams were simply supported, he total loading condition would be described as $P_x . O_y$. When either set of beams re non-existent, this fact will also be denoted by the letter O, whilst the letter E vill be used for elastic loading. Hence a stanchion subjected to elastic bending bout the minor axis but with no major-axis beams would have the loading classiication $O_x . E_y$.

The categories into which the stanchion loading conditions have been divided do ot allow for any displacement in a horizontal direction of one end of a stanchion ength relative to the other. Such displacements occur when the mechanism of ailure according to the simple plastic theory involves sidesway, as shown by the xamples in Figs 4a and 4b. Such mechanisms may be referred to as "overall" nechanisms in contrast to "local" mechanisms, which involve only a few members nd introduce no sidesway. When considering the behaviour in bending of an -section stanchion about its minor axis, it has to be assumed when using the present ystem of classification that sway deflexions are prevented. Overall collapse nechanisms may, however, be allowed in the plane of the web of I-sections, provided he axial loads are not excessive. The criterion to be adopted is that the additional ending moments induced by sway deflexion when the structure is just on the oint of collapse shall be small compared with the existing moments about the najor axis. This condition is usually satisfied in single-storey frames and someimes in other cases. The placing of types of loading into the elastic and plastic ategories may then be achieved by noting that plastic loading only occurs if the noment applied to the end of the stanchion remains constant during the deformation f the structure. Thus in Fig. 4a, stanchion BF is under elastic loading at each nd, whilst stanchion DH is at each end subjected to plastic loading. Stanchion G is under elastic loading at C and plastic loading at G. In Fig. 4b the stanchion

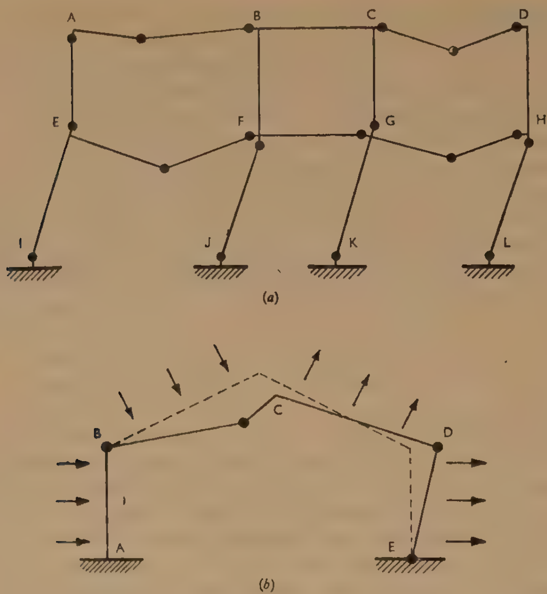


FIG. 4.—OVERALL PLASTIC COLLAPSE MECHANISMS

DE is under plastic loading at each end, as also is end B of stanchion AB. At A, the bending moment would not vary even if the stanchion AB underwent deformation, since the bending moments throughout the structure are statically determinate owing to the presence of four plastic hinges. Hence the conditions at the lower end of stanchion AB are also those associated with plastic loading.

The three possible loading conditions about both the major and the minor axis of a stanchion length give a total of nine possible combinations for consideration. These are shown diagrammatically in Fig. 5. It should be noted that no significance attaches to the relative sizes of the beams and stanchions as shown. Considering both ends of a particular stanchion length, it is evident that each of the nine possible loading descriptions for one end could be associated with nine possible loading classifications at the other. If it is acknowledged that the conditions at the upper and lower ends could be reversed without affecting the axial load at collapse, it is found that there are forty-five classes into which the loading conditions for a stanchion length could be placed.

It is quite clear that the complete exploration of the stanchion problem in the above terms is a practical impossibility. Fortunately, the aim of the engineer is not to discover all the possible conditions which might conceivably arise, but to deal with the worst cases and to establish solutions which will lead to manageable design methods. The designer has a large measure of control over the bending moments to which a stanchion is to be subjected and it will usually be possible to ensure that the classification of loading conditions according to Fig. 5 is the same at either end. Such a limitation is indeed essential if design methods are to be established, and so nine types only of loading condition will be considered. Each

case will be discussed, and a brief review given of the investigations carried out and progress made towards an understanding of the corresponding stanchion behaviour. In this discussion attention is directed primarily to doubly symmetrical I- or H-section members, although some of the conclusions refer to any stanchion with a section having two axes of symmetry.

Stanchion simply supported about both axes. (*Case* $O_x \,.\, O_y$)

This is the case of a pin-ended strut. Orthodox design methods have, for the most part, been derived by considering this case only, the effect of beam moments and beam rigidity being either ignored or allowed for by methods which bear no

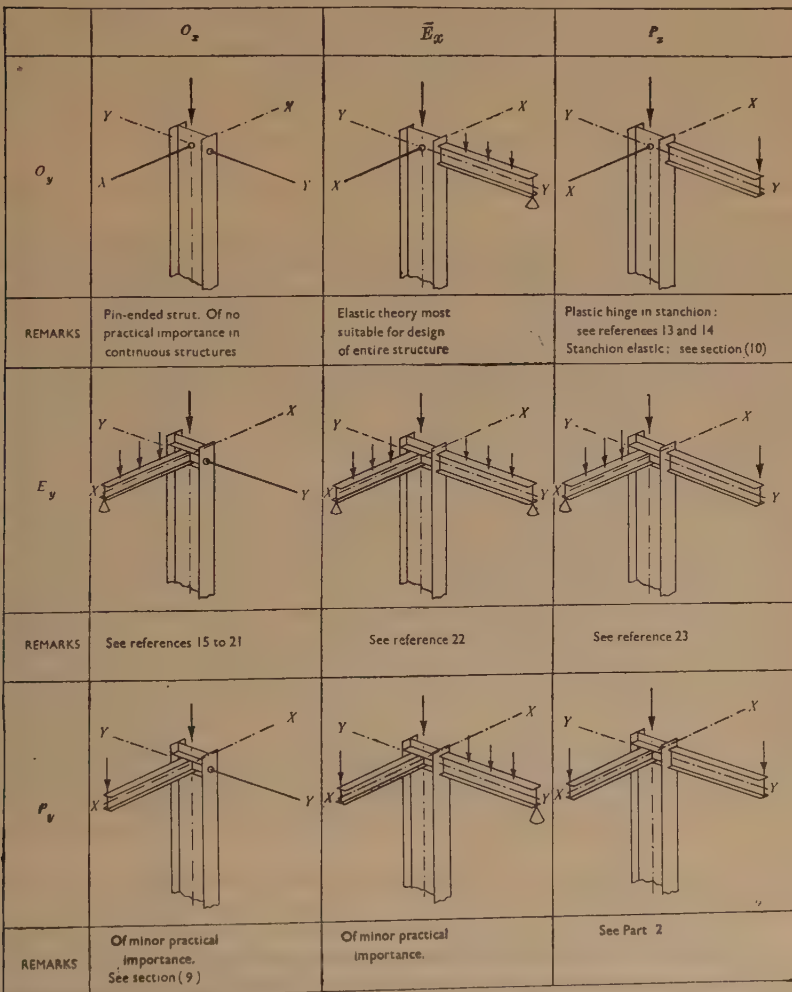


FIG. 5.—SUMMARY OF THE STANCHION PROBLEM

relation to real behaviour. The inadequacy of such an approach was fully shown by the work on stanchions of the Steel Structures Research Committee, and has recently been discussed again by Baker.[10]

Stanchion with elastic loading about the major axis, simply supported about the minor axis. (*Case* $E_x . O_y$)

This type of stanchion loading is not one in which plastic behaviour is likely to feature, either in the design of the beams or in that of the stanchions. The minor-axis beams receive no moment restraint from the stanchions and therefore no advantage is to be gained by designing them plastically. Since the stanchion is not restrained at the ends against rotation about the minor axis, complete collapse will occur, except in very stocky members, at a load very little greater than that at which the yield stress is first reached in the extreme fibres. Hence, a design method based on this type of stanchion loading should be derived from elastic analysis, as for example in the methods evolved by the Steel Structures Research Committee.[5, 12]

Stanchion with plastic loading about the major axis, simply supported about the minor axis. (*Case* $P_x . O_y$)

In this case, as in the last, the stanchion has little reserve of strength after the yield stress has been reached. Plastic design methods are, however, necessarily involved for the major-axis beams and the stanchion may therefore be required to participate, in the plane of its web, in an overall collapse mechanism. When this involves a plastic hinge (with flexure about the major axis) at either or both ends of the stanchion, a well-defined loading condition is obtained, of particular importance in the design of single-storey buildings such as that shown in Fig. 4b. Hitherto, very little guidance has been available on the limitation of the slenderness ratios in such structures and this has been left to the judgement of the designer. Considerable progress has now been made towards a solution of this problem and it is hoped that the results may soon be published. It appears from this work [13, 14] that stanchions with a plastic hinge at one end may be allowed up to certain slenderness ratios, provided the terminal major-axis bending moments do not bend the stanchion in nearly symmetrical single curvature as in Fig. 1a. Plastic hinges at both ends can be allowed provided the stanchion is bent in double curvature, that is, with the two moments acting in the same direction as in Fig. 1b.

It will be evident that, if the stanchion is required to develop one or two plastic hinges before complete collapse occurs, there must be a moderately severe restriction on the mean axial stress. When no plastic hinge is required in the stanchion it is more profitable, with this loading condition, to design the stanchion on the basis of elastic theory. The loading condition $P_x . O_y$ is then best considered as the special case of the condition $P_x . P_y$ in which the moment about the minor axis is zero, and a suitable design method is given in Part 2, under section (10).

Stanchion simply supported about the major axis, with elastic loading about the minor axis. (*Case* $O_x . E_y$)

This loading condition has been the subject of a long series of experimental and theoretical investigations carried out since 1944.[15–21] Tests have been carried out on mild-steel stanchions rigidly connected to high-tensile-steel beams as shown in Figs 6a and 6b. The beams were simply supported at their outer ends, and the frame set up in a special test rig [15] in which loads could be applied

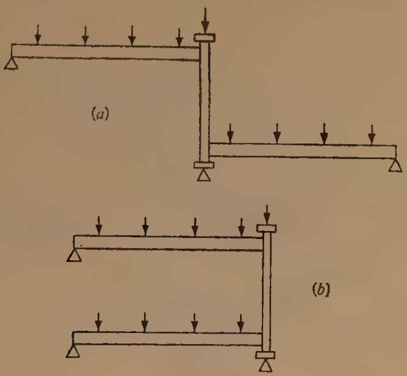


FIG. 6.—FRAMES USED IN TESTING STANCHION LENGTHS BENT ABOUT THE MINOR AXIS IN (*a*) SINGLE CURVATURE AND (*b*) DOUBLE CURVATURE

by means of dead weights acting through levers. These levers were arranged to apply vertical loads distributed at four sections along each beam, and direct axial load to the stanchions. The stanchions of the type shown in Fig. 6a were thus bent by the beams into a single C-curve (single curvature bending), whilst those in Fig. 6b were bent in an S-curve (double curvature bending). Beam loads to some predetermined value were first applied and direct axial load was then added until collapse of the stanchion occurred by buckling. A theoretical explanation of the test results has been obtained [20, 21] and the results of a typical analysis are shown in Fig. 7. This shows the growth of the plastic zones with increase of axial load up to the theoretical collapse condition represented by Fig. 7e. The further plastic deformation depicted in Fig. 7f is only reached theoretically after a decrease in the applied load, and therefore represents a state reached at some stage when the member is failing catastrophically owing to its inability to sustain the peak value. Such analyses, based on a complete elastic-plastic treatment of the plane stanchion problem, have been shown to be in agreement with test results. Recently, the problem has been further explored by the use of an electronic digital computor to derive the behaviour of stanchions of rectangular and I-section under a wide range of conditions. A tentative design method based on these results is now being studied, and gives maximum stanchion economy at the expense of the beams, which may be heavier than in orthodox design; it remains to be seen whether an overall economy is possible. In general, it may be said that the design method is not particularly promising, being mainly of interest as an extreme case.

Stanchion with elastic loading about both axes. (*Case* $E_x . E_y$)

When attention was first turned from the plane stanchion problem represented by case $O_x . E_y$ to the problem of bending about both axes, this was the first loading condition to be investigated. A number of tests have been performed [22] on stanchions connected to beams arranged to lie along the principal axes at each end, as shown in Fig. 8. It had been hoped that the theory of plane stanchions might have been extended to deal with this problem, but it has unfortunately proved so difficult to deal with analytically that little progress has been

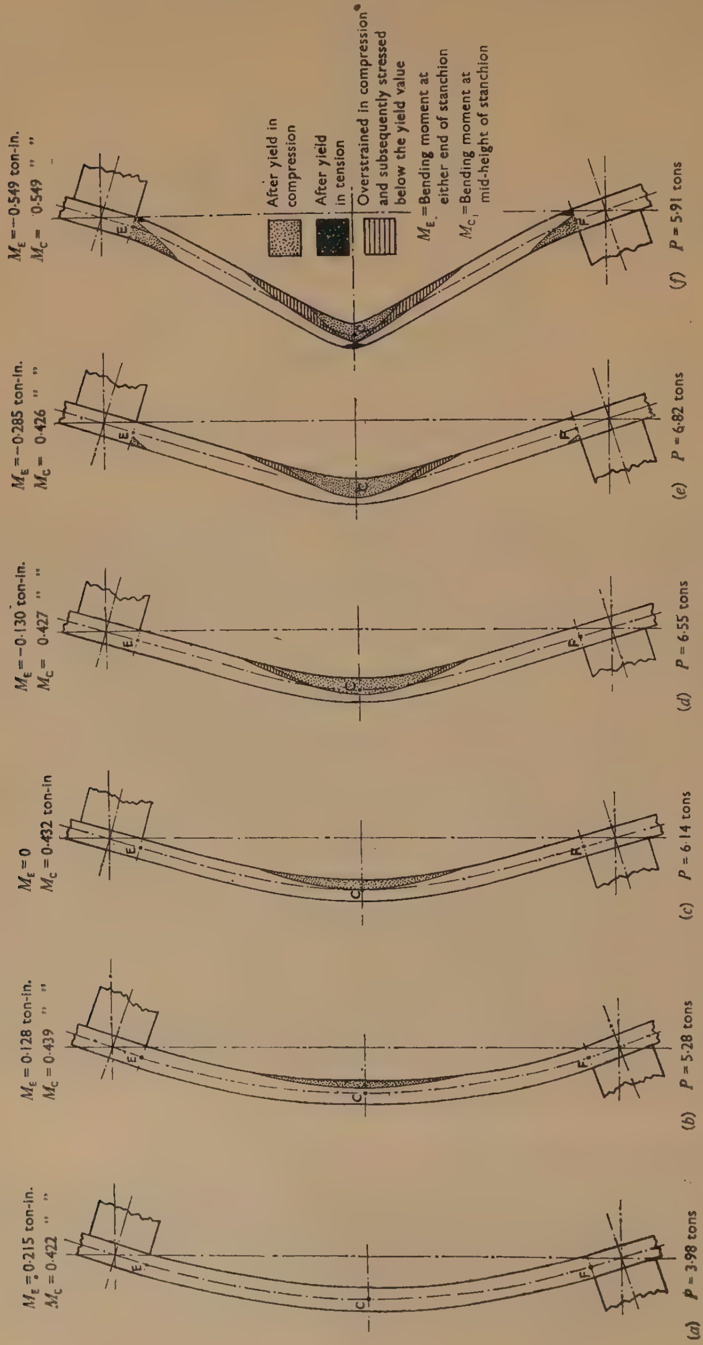
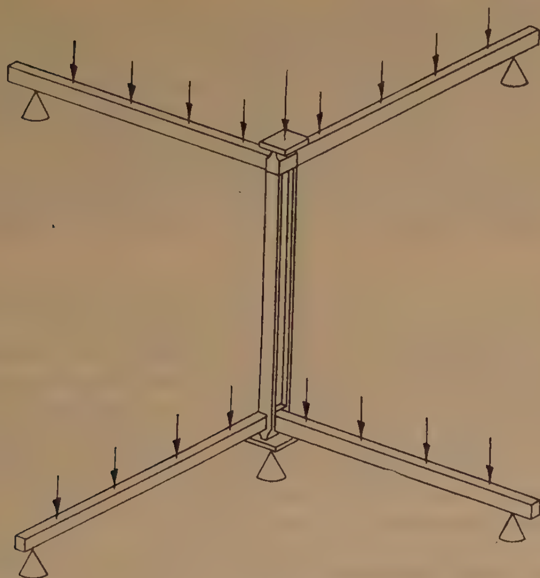


FIG. 7.—ELASTIC-PLASTIC ANALYSIS OF A STANCHION BENT ABOUT THE MINOR AXIS BY ELASTIC BEAMS



.8.—FRAME USED IN TESTING STANCHION LENGTHS BENT ABOUT BOTH AXES IN SINGLE CURVATURE

made. The tests have indicated that bending moments introduced by elastic major-axis beams have comparatively little effect on the axial load in the stanchion at collapse. It might thus be possible to make allowance for them by empirical methods, but there seems to be little promise that this approach is likely to lead to a tractable design method.

Stanchion with plastic loading about the major axis and elastic loading about the minor axis. (*Case* $P_x \,.\, E_y$)

This loading condition is of great potential importance in the evolution of a design method taking full account of the benefits of joint rigidity. The principal load-carrying beams in the structure should be those which frame into the flanges of the stanchions (the major-axis beams), whilst the minor-axis beams, which remain elastic, stabilize the stanchions by providing restraint against the rotation of the ends about the minor axis. The stanchions are in this way enabled to sustain high mean axial stresses, despite the high terminal moments applied by the major-axis beams. Unfortunately, this case is intractable analytically and although some tests have been performed [23] a satisfactory theoretical treatment is lacking.

Under certain circumstances, the behaviour of a stanchion in the class $P_x \,.\, E_y$ at collapse may be expected to differ but little from that of a similar stanchion in the category $P_x \,.\, O_y$. This occurs when the stanchion, because of its participation in an overall collapse mechanism, is required to develop plastic hinges about the major axis at both ends, with bending in double curvature. For the structure as a whole to develop its full load-carrying capacity, some degree of

rotation in the stanchion about its major axis may have to take place at these hinges. The resulting plastic deformation at the ends of the stanchion must compromise the restraint offered by the elastic minor-axis beams, and the only safe course in design is then to assume no directional restraint, so that the loading becomes equivalent to case $P_x . O_y$. When a plastic hinge occurs at one end only, then the stanchion loading condition is $P_x . O_y$ at one end and $P_x . E_y$ at the other. Since it appears at present to be impracticable to evolve design methods for mixed classes of end conditions, such a stanchion should be designed on the safe assumption that the more severe condition, $P_x . O_y$, applies at both ends.

Stanchion simply supported about the major axis, with plastic loading about the minor axis. (*Case* $O_x . P_y$)

Under this loading condition, the stanchion would provide full resistance to a bending moment applied by a beam about its minor axis, whilst no use would be made of its superior resistance to bending about its major axis. Whilst this is not apparently an economic arrangement, it may be necessary in part of a structure because of the intervention of other factors which dictate the disposition of the members. The stanchion has little reserve of strength beyond the load which just produces yield in the extreme fibres, and it should therefore be designed by elastic theory. A method which takes account of the actual bending moments applied at both ends of the stanchion has been obtained as a special case of the design method evolved for the more important loading condition $P_x . P_y$ and this is given in section (9).

Stanchion with elastic loading about the major axis and plastic loading about the minor axis. (*Case* $E_x . P_y$)

As with case $O_x . P_y$, this loading condition will tend to be uneconomic. Elastic design methods for the stanchions and major-axis beams, similar to those suited to case $E_x . O_y$, could be applied, provided the necessary modifications were made to allow for the effect of the known bending moments applied to the stanchions by the plastic minor-axis beams. If desired, a design procedure using the Recommendations of the Steel Structures Research Committee [5, 12] could be applied without difficulty.

Stanchions with plastic loading about both axes. (*Case* $P_x . P_y$)

Except in stanchions of very low slenderness ratio the attainment of the yield stress in the most highly stressed fibres must necessarily be the criterion of stanchion design for this type of loading. The design will evidently sacrifice stanchion economy in the interest of obtaining minimum beam sizes, a course which will be particularly desirable when large clear spans are required. The extent to which the stanchions so designed provide end restraint for both the major- and the minor-axis beams renders most important the use of an economic design method for the stanchions, otherwise the economy achieved in the beams may be lost in the stanchions. The derivation of such a design method is the subject of the remainder of this Paper.

(4) *Existing design methods for stanchions*

In the three loading classifications $P_x . O_y$, $O_x . P_y$, and $P_x . P_y$ discussed in the previous section, the stanchion is subjected by plastic beams to given terminal

moments about either or both of the principal axes. It has been maintained that in the cases $O_x \cdot P_y$ and $P_x \cdot P_y$, the stanchions are most suitably designed by elastic theory, whilst elastic design is also suitable for case $P_x \cdot O_y$ except when the stanchion participates in an overall collapse mechanism and contains a plastic hinge at one or both ends. Before proceeding to the derivation of a suitable elastic design method for these cases, a critical examination will be made of two existing sets of regulations, namely those given in B.S. 449 [4] (clauses 18, 19, 22, 35, 36, and 37) and those in the "Recommendations for Design" of the Steel Structures Research Committee [5] (clauses 11 to 19).

B.S. 449 (1948)

The "safe" stanchion stresses given both in this British Standard and in its predecessor (B.S.S. 449, 1937) are based on the Perry–Robertson strut formula.[24] This gives the mean axial stress at which the yield point is just reached in a pin-ended strut with an initial imperfection. The imperfection is assumed to take the form of a sine wave, $u = u_0 \sin \frac{\pi z}{l}$, in a plane perpendicular to the minor axis, where z denotes distance along the longitudinal axis OZ, l is the length of the strut, and u_0 is the initial central deflexion (see Fig. 9a).* Under an axial load P the central deflexion increases, as a result of elastic flexure, to $u_1 = u_0 \frac{P_E}{P_E - P}$ where P_E is the Euler load for buckling about the minor axis (see Fig. 9b). The central bending moment is then $u_0 \frac{P_E P}{P_E - P}$. Robertson suggested that u_0 should have a value $\epsilon \frac{l r_y}{a_y}$ where ϵ is a constant, a_y is the distance from the extreme compression fibres

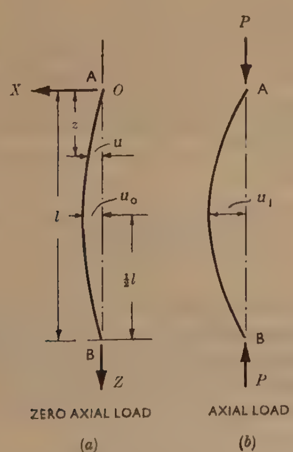


FIG. 9.—DEFLEXIONS OF A STRUT AS ASSUMED IN THE PERRY–ROBERTSON TREATMENT

* The notation used here differs from that of Robertson.

of a cross-section to the minor principal axis, and r_y is the least radius of gyration. It follows that if the yield stress (denoted by f_L) is just reached in compression when the mean axial stress is p, then:

$$f_L = p\left\{1 + \frac{\epsilon \dfrac{l}{r_y}}{1 - \left(\dfrac{l}{r_y}\right)^2 \dfrac{p}{E\pi^2}}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where E is the modulus of elasticity. Solving this quadratic equation for p, the well known Perry–Robertson formula is obtained.

In B.S. 449 the working stresses for axially loaded struts having l/r_y equal to or greater than 80 are computed from the Perry–Robertson formula. It is assumed that $f_L = 15{\cdot}25$ tons/sq. in., $E = 13{,}000$ tons/sq. in., and $\epsilon = 0{\cdot}003$. The safe stress quoted is $0{\cdot}5p$, thus introducing a load factor of 2·0. (See Table 7 of B.S. 449 and Appendix C of the Specification.) When l/r_y is less than 80, the safe stress is obtained by linear interpolation with respect to l/r_y from 9·00 tons/sq. in. when $l/r_y = 0$ to 5·12 tons/sq. in. (the value obtained from the Perry–Robertson formula) when $l/r_y = 80$.

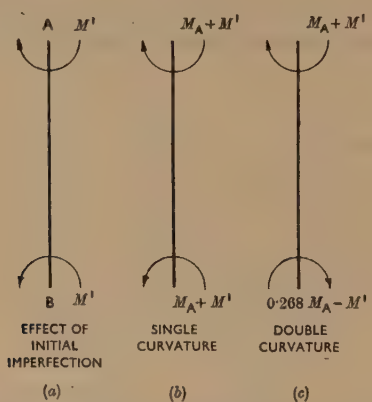


FIG. 10.—STANCHION TERMINAL MOMENTS ASSUMED BY THE STEEL STRUCTURES RESEARCH COMMITTEE

The use of the Perry–Robertson formula in B.S. 449 for the calculation of safe working stresses for axially loaded struts is entirely reasonable, but the regulations for members under axial load combined with bending moments do not possess any obviously rational basis (clause 22). If f_a denotes the mean axial stress in a member and f_{bc} the maximum total compressive stress due to bending about both principal axes, then the specification imposes the restriction:

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} \leqslant 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Here, F_a is the maximum permissible compressive stress in axially loaded struts (as already quoted) and F_{bc} is the permissible compressive bending stress in the

absence of axial load (clause 19). This formula is a marked departure from the corresponding regulation in the previous edition of this British Standard (B.S.S. 449, 1937), which (using the same symbols) stipulated that:

$$f_a + f_{bc} < F_a$$

i.e.,

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_a} < 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Since $F_a < F_{bc}$ the new regulation (2) is considerably less stringent than the old.

No differentiation is made in B.S. 449 between bending stresses caused by major- and minor-axis bending moments. The symbol f_{bc} denotes the sum of the bending stresses about both axes.

It appears from the above discussion that the maximum combined stanchion stresses specified in B.S. 449 are not based on any consistently rational treatment. As part of the whole specification the regulations have apparently led to the design of safe structures. The success of the whole specification in producing safe structures cannot, however, guarantee the safety of abstracting that part dealing with loads on stanchions and using it in an entirely new design method. The regulations contained in B.S. 449 cannot therefore be used reliably in conjunction with a plastic method of beam design with rigid joints unless their use for this purpose can be justified as both safe and economic by comparison with results obtained in a fuller treatment of the problem.

Steel Structures Research Committee

In 1936 a rational treatment of the stanchion problem was made for the Steel Structures Research Committee (S.S.R.C.) by Baker.[10, 25] For his General Method of Design families of curves were produced showing the bending stress allowable in the presence of any ratio of end moments and a given mean axial stress for the full practical range of slenderness ratios (length divided by least radius of gyration). These curves give those combinations of axial load and bending moment which, if multiplied by a load factor of 2·0, would be just sufficient to produce a maximum extreme fibre stress in the stanchion of 18 tons/sq. in., the value of the elastic modulus being 13,000 tons/sq. in. In calculating these curves allowance for initial imperfections in the stanchions was made, not directly by assuming an eccentricity of applied load, but by introducing an additional uniform bending moment M' applied to the stanchion (Fig. 10a). The value of this bending moment was taken as $0{\cdot}012\, EI_y\, (r_y/la_y)$ where I_y denotes the moment of inertia about the minor axis. This moment is equivalent to an initial central deflexion of $0{\cdot}0015\, lr_y/a_y$ compared with an initial deflexion of $0{\cdot}003\, lr_y/a_y$ assumed in B.S. 449.

In his Final Simplified Method of Design which was adopted by the Committee and published as its Recommendations for Design,[5, 10] Baker reduced these curves to one set which covered stanchion behaviour for the most critical load distributions in continuous elastic structures producing single-curvature bending (Fig. 10b) or the worst case of double-curvature bending with $M_B/M_A = -0{\cdot}268$ * (Fig. 10c).

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* The ratio $M_B/M_A = -0{\cdot}268$ in the double-curvature case was chosen as the result of an analysis of certain idealized continuous frames, and was such that the highest stress induced in the stanchion was a maximum for given maximum beam loads.[25] It should be noted that this particular ratio only has meaning when considering stanchions as components of an elastic structure, and has no significance when dealing with structures in which the beams become plastic.

Whilst no simpler but still rational method than this is likely to be produced it would not, owing to the simplifications involved, achieve the economies which should be aimed at in a plastic method of design.

There are two other respects in which the S.S.R.C. Recommendations for Design may be unsatisfactory for the present purpose. The first is that all bending was assumed to be about the minor axis, that is to say the stresses due to major-axis bending were simply added to those due to minor-axis moments. This has the great merit of simplicity and was shown by Baker and Holder [8] to be a safe procedure; they pointed out its extravagance, however, and derived a method for treating the bending moments about the major and minor axes separately, but it applied only to pure single curvature bending and was rather cumbersome. The second limitation is that the S.S.R.C. Recommendations were drafted to apply to multi-storey buildings in which torsional failure is prevented by the cladding of the steelwork. The main application of the plastic theory has hitherto been to industrial buildings where the steelwork is uncased so that it is important to consider the possibility of combined torsional and bending failure.

The conclusion has therefore been reached that, whilst Baker's curves for his General Method of Design may be useful when it comes to designing multi-storey frames which are to have normal fire-resisting casing, it might be unwise to rely on them for all cases of plastic design. For this reason the method described in detail in Part 2 of the Paper has been evolved.

Part 2

THE DESIGN OF I-SECTION STANCHIONS SUBJECTED TO TERMINAL BENDING MOMENTS ABOUT BOTH AXES

(5) *General basis of design*

The basis of the design method given below is the limitation of the maximun extreme fibre stress in a stanchion to the yield value. The stanchion is assumec to have an initial imperfection in the form of a sinusoidal curvature about the mino axis, this being in accord with the Perry–Robertson assumption for the plane bendin problem. In the general case, the stanchion is subjected at each end to independen bending moments about both axes in the presence of an axial load P, as shown i Fig. 11. The terminal moments about the major axis are denoted by M_x' an M_x'', whilst those about the minor axis are denoted by M_y' and M_y''. The shea forces R_x and R_y are required to maintain equilibrium. The derivation of th maximum extreme fibre stress for this condition is difficult and the problem is deal with in stages. In section (6) is discussed the elastic instability of an I-sectio member subjected to axial load and arbitrary terminal moments about the majc axis. A design method for stanchions subjected to uniform bending momen about both principal axes is derived in section (7), and its extension to stanchior with unequal terminal moments is discussed in section (8). This represents a desig method appropriate to the general loading condition $P_x \, . \, P_y$. The design of stan chions subjected to terminal moments applied about the minor axis only (cas $O_x \, . \, P_y$) is discussed in section (9), whilst section (10) deals with stanchions ben about the major axis only (case $P_x \, . \, O_y$). A comparison between the propose design methods and the results obtained from a modified application of the appr priate clauses in B.S. 449 and the Recommendations for Design of the Steel Stru tures Research Committee is contained in section (11).

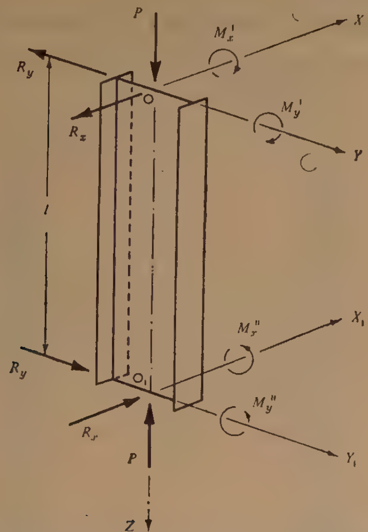


FIG. 11.—GENERAL LOADING CONDITIONS FOR AN I-SECTION STANCHION LENGTH

3) *The elastic instability of I-section stanchions*

The loads which produce elastic instability in stanchions loaded as shown in ig. 11, but with zero minor-axis moments ($M_y' = M_y'' = 0$), have been btained [26, 27] for all ratios of the major-axis terminal moments M_x' and M_x''. enoting the ratio M_x''/M_x' by β, bending in uniform single curvature corresponds $\beta = 1$, whilst bending in symmetrical double curvature is represented by $\beta = -1$. he critical load combinations depend not only on the slenderness of the stanchion bout its minor axis l/r_y, but also upon the torsional rigidity of the stanchion and s resistance to warping. Failure occurs by twisting of the stanchion combined ith flexure about the minor axis. The exact results cannot be expressed in a mple analytical form, but it has been shown [26] that safe combinations of the ial load P and the larger terminal bending moment M_x' are given by the equation:

$$\frac{1}{F}\left(\frac{M_x'}{M_E}\right)^2 + \frac{P}{P_E} = 1 \qquad \text{(4)}$$

here F is a function of β only. The relation between β and $1/\sqrt{F}$ is shown aphically in Fig. 12 and in tabular form in Table 1. The symbol P_E denotes the uler critical load for the stanchion, treated as a pin-ended strut. Hence if I_y is e moment of inertia of a cross-section about the minor axis and l is the length $_E = \frac{EI_y\pi^2}{l^2}$. The symbol M_E represents the uniform moment which, if applied out the major axis of the stanchion in the absence of any axial load, would just use lateral instability, the resistance of the member to warping being neglected. G denotes the elastic modulus of rigidity and K the torsional modulus for the tion, then: $M_E^2 = \frac{\pi^2}{l^2}(EI_y)(GK)$.

When the terminal bending moments are equal, bending the stanchion in single rvature, the value of F in equation (4) becomes unity. Hence, so far as elastic

TABLE 1.—VALUES OF $1\sqrt{F}$ (EQUATION (4))

Ratio of end moments: β	$\dfrac{1}{\sqrt{F}}$
1·0	1·000
0·9	0·950
0·8	0·902
0·7	0·854
0·6	0·807
0·5	0·762
0·4	0·714
0·3	0·677
0·2	0·637
0·1	0·600
0	0·565
—0·1	0·532
—0·2	0·502
—0·3	0·475
—0·4	0·451
—0·5	0·429
—0·6	0·410
—0·7	0·394
—0·8	0·384
—0·9	0·381
—1·0	0·391

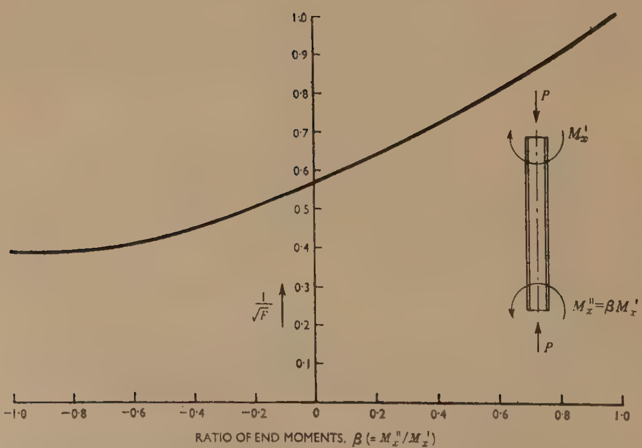


FIG. 12.—DEPENDENCE OF $1/\sqrt{F}$ ON THE RATIO OF END MOMENTS β

nstability is concerned, terminal moments of M_x' and $\beta M_x'$ are equivalent to a niform moment of M_x where $M_x = \frac{1}{\sqrt{F}} M_x'$. The condition for instability then ecomes:

$$\left(\frac{M_x}{M_E}\right)^2 + \frac{P}{P_E} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

7) *Design method for bending by uniform moments about both principal axes.* (*Case $P_x . P_y$*)

When a stanchion is subjected to major- and minor-axis terminal moments of $M_x' = M_x'' = M_x$ and $M_y' = M_y'' = M_y$ respectively (Fig. 11), the maximum stress lways occurs at the mid-height of the stanchion. Following Robertson [24] the imerfections in the stanchion may be allowed for by assuming the longitudinal axis o have an initial curvature in a plane perpendicular to the web (plane OZX in 'ig. 11), the displacement in the direction XO being given by $u_o \sin \frac{\pi z}{l}$. In this xpression, $u_o = \epsilon\left(\frac{l r_y}{a_y}\right)$ where r_y is the radius of gyration about the minor axis and $_y$ the distance of the extreme fibres from the minor axis.

It is convenient to represent the applied uniform moment M_y by the equivalent 'ourier half-range series $\frac{4}{\pi} M_y \sum_{n=0}^{n=\infty} \frac{1}{(2n+1)} \sin (2n+1)\frac{\pi z}{l}$. The deflexions produced y the minor-axis bending moment in the OZX plane, in the absence of the axial load P and the major-axis bending moment M_x, are $\frac{4}{\pi^3}\frac{M_y l^2}{EI_y}\sum_{n=0}^{n=\infty}\frac{1}{(2n+1)^3}\sin(2n+1)\frac{\pi z}{l}$.

'aking the first term only of the series, the deflexions become $\frac{4}{\pi^3}\frac{M_y l^2}{EI_y}\sin\frac{\pi z}{l}$, hat at mid-height $\left(z = \frac{l}{2}\right)$ being $0{\cdot}129 \frac{M_y l^2}{EI_y}$. The accurate value of the central eflexion is $0{\cdot}125 \frac{M_y l^2}{EI_y}$, so that by taking the first term only a safe result is obtained.

The total deflexions in the OZX plane in the absence of the axial load P and the ajor-axis bending moment M_x are thus given by:

$$u = \left\{u_o + \frac{4}{\pi^3}\frac{M_y l^2}{EI_y}\right\}\sin\frac{\pi z}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Vhen P and M_x are applied, it is readily shown that the deflexions increase to u' here, if

$$\left(\frac{M_x}{M_E}\right)^2 + \frac{P}{P_E} = \gamma_y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

hen

$$u' = \frac{u}{1-\gamma_y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Vhen $M_x = 0$, equation (8) reduces to the familiar expression $u' = u\frac{P_E}{P_E - P}$ for he deflexion of a pin-ended strut with initial imperfection.

It is now possible to calculate the minor-axis bending moment at mid-height. Before P and M_x are applied, this bending moment is M_y. The application of P and M_x increases this bending moment by an amount equal to the value of $\left\{-EI_y \frac{d^2(u'-u)}{dz^2}\right\}$ when $z = \frac{l}{2}$. Hence it may be shown that the central bending moment is $[M_y]_{\frac{l}{2}}$ where:

$$[M_y]_{\frac{l}{2}} = \frac{EI_y\pi^2}{l^2}\left\{\frac{\gamma_y}{1-\gamma_y}\right\}u_o + \left\{1 + \frac{4}{\pi}\frac{\gamma_y}{1-\gamma_y}\right\}M_y \quad . \quad . \quad . \quad (9)$$

The central bending moment about the major axis $[M_x]_{\frac{l}{2}}$ may be obtained in a similar manner. The deflexions v in the plane OYZ in the absence of the axial load P are given approximately by:

$$v = \frac{4}{\pi^3}\frac{M_x l^2}{EI_x}\sin\frac{\pi z}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where I_x is the moment of inertia about the major axis. On the application of the axial load P this deflexion increases to v' where, if

$$\frac{Pl^2}{\pi^2 EI_x} = \gamma_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

then

$$v' = \frac{v}{1-\gamma_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The symbol γ_x denotes the ratio of the axial load P to the Euler critical load for the stanchion treated as a pin-ended strut buckling about the major axis. The central bending moment about the major axis in the presence of the axial load is then the value of $\left\{M_x - EI_x\frac{d^2(v'-v)}{dz^2}\right\}$ when $z = \frac{l}{2}$. Hence

$$[M_x]_{\frac{l}{2}} = \left\{1 + \frac{4}{\pi}\frac{\gamma_x}{1-\gamma_x}\right\}M_x \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The value of the maximum stress at the mid-height of the stanchion may now be calculated. Equating this stress to the yield stress, denoted by f_L, the following equation is obtained:

$$\frac{P}{A} + \frac{a_x}{I_x}[M_x]_{\frac{l}{2}} + \frac{a_y}{I_y}[M_y]_{\frac{l}{2}} = f_L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

A denotes the area of cross-section and a_x the distance of the extreme fibres from the major principal axis.

Let the mean stress $\frac{P}{A}$ be denoted by p. Let $f_x = \frac{a_x}{I_x}M_x$ and $f_y = \frac{a_y}{I_y}M_y$ so that f_x and f_y are the externally applied bending stresses about the two axes. Then it follows from equations (9), (13), and (14) that

$$p + N_x f_x + N_y f_y = f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where

$$N_x = 1 + \frac{4}{\pi}\frac{\gamma_x}{1-\gamma_x} \quad \ldots \quad (16)$$

$$N_y = 1 + \frac{4}{\pi}\frac{\gamma_y}{1-\gamma_y} \quad \ldots \quad (17)$$

$$f = f_L - E\pi^2\epsilon\left\{\frac{r_y}{l}\right\}\left\{\frac{\gamma_y}{1-\gamma_y}\right\} \quad \ldots \quad (18)$$

Equation (11) may be expressed in the form

$$\gamma_x = \frac{1}{E\pi^2}\left(\frac{l}{r_x}\right)^2 p \quad \ldots \quad (19)$$

and equation (7) in the similar form

$$\gamma_y = \frac{1}{E\pi^2}\left(\frac{l}{r_y}\right)^2 p' \quad \ldots \quad (20)$$

where

$$p' = p + \frac{f_x^2}{T} \quad \ldots \quad (21)$$

and

$$T = \frac{AGKa_x^2}{I_x^2} \quad \ldots \quad (22)$$

Hence N_y is the same function of l/r_y and p' as is N_x of l/r_x and p. The value of f given by equation (18) is also a function of l/r_y and p'. If a fixed value is adopted for G, the elastic modulus of rigidity, then T is a constant for a member of any given cross-section.

Charts from which the values of f, N_x, and N_y may be obtained are given in Figs 13 and 14. In calculating these charts the following values have been taken: $f_L = 15{\cdot}25$ tons/sq. in., $E = 13{,}000$ tons/sq. in., $\epsilon = 0{\cdot}0015$. In Fig. 13 the heavy curve corresponds to the condition $f_x = f_y = 0$, and represents the allowable mean axial stress in a stanchion with zero applied bending moments. The value of N_x may be read on the vertical scale of Fig. 14 by entering the chart with the axial stress p and the major-axis slenderness ratio l/r_x, whilst N_y is obtained similarly by entering the chart with the stress p' (equation (21)) and the minor-axis slenderness ratio l/r_y. It may be noted that the figures in parenthesis give an alternate pair of scales for p, p' and l/r_x, l/r_y. The values of the constant T for B.S. rolled-steel joists [28] are given in Table 2, using the torsional moduli K as given by Cassie and Dobie [29] and a value for the modulus of rigidity G of 5,000 tons/sq. in. The value of T for the built-up welded section in Fig. 15 is $5{,}000\dfrac{AK}{Z_x^2}$ where A is the cross-sectional area, K the torsional rigidity, and Z_x the elastic modulus for bending about the axis XX. The torsional rigidity is given with sufficient accuracy by

$$K = \tfrac{2}{3}Bt_1^3 + \tfrac{1}{3}(D - 2t_1)\,t_2^3 \quad \ldots \quad (23)$$

In checking the ability of a stanchion to carry a given axial load together with uniform applied moments about the two axes, the corresponding stresses p, f_x, and f_y are first calculated. Taking $p' = p + \dfrac{f_x^2}{T}$, f is obtained from Fig. 13 and N_x and N_y from Fig. 14 as already described. The stanchion will then support the given loads provided

$$p + N_x f_x + N_y f_y < f \quad \ldots \quad (24)$$

TABLE 2.—VALUES OF T FOR B.S. ROLLED-STEEL JOISTS

Section of R.S.J.: in. × in. × lb.	T: tons/sq. in.	Section of R.S.J.: in. × in. × lb.	T: tons/sq. in.
3 × 1½ × 4	143·7	12 × 5 × 32	34·8
3 × 3 × 8½	254·5	12 × 6 × 44	56·0
4 × 1¾ × 5	65·8	12 × 6 × 54	85·8
4 × 3 × 10	155·3	12 × 8 × 65	87·5
4¾ × 1¾ × 6½	83·5	13 × 5 × 35	33·9
5 × 3 × 11	98·1	14 × 6 × 46	39·1
5 × 4½ × 20	194·2	14 × 6 × 57	60·8
6 × 3 × 12	68·4	14 × 8 × 70	64·9
6 × 4½ × 20	120·3	15 × 5 × 42	32·1
6 × 5 × 25	162·6	15 × 6 × 45	30·8
7 × 4 × 16	55·8	16 × 6 × 50	31·2
8 × 4 × 18	48·0	16 × 6 × 62	47·8
8 × 5 × 28	89·0	16 × 8 × 75	51·1
8 × 6 × 35	111·8	18 × 6 × 55	27·1
9 × 4 × 21	46·8	18 × 7 × 75	43·1
9 × 7 × 50	135·2	18 × 8 × 80	41·0
10 × 4½ × 25	41·7	20 × 6½ × 65	25·8
10 × 5 × 30	53·9	20 × 7½ × 89	40·5
10 × 6 × 40	77·3	22 × 7 × 75	23·0
10 × 8 × 55	100·8	24 × 7½ × 95	27·0

Example 1.—A stanchion of length 14 ft is required to withstand the following factored loads: axial load 8 tons, major-axis bending moment 120 tons-in., minor-axis bending moment 11 tons-in. Would a 10-in. × 4½-in. × 25-lb. R.S.J. be satisfactory?

It is found that:

$T = 41{\cdot}7$ (Table 2) $\qquad l/r_x = 41 \qquad l/r_y = 179$

$f_x = 4{\cdot}90$ tons/sq. in. $\qquad f_y = 3{\cdot}82$ tons/sq. in.

$$p = 1{\cdot}09 \text{ tons/sq. in.} \qquad p' = \left(1{\cdot}09 + \frac{4{\cdot}90^2}{41{\cdot}7}\right) = 1{\cdot}67 \text{ tons/sq. in.}$$

From Fig. 14: $N_y = 1{\cdot}91$ (using $l/r_y = 179$, $p' = 1{\cdot}67$ tons/sq. in.)

$N_x = 1{\cdot}02$ (using $l/r_x = 41$, $p = 1{\cdot}09$ tons/sq. in.)

Hence $p + N_x f_x + N_y f_y = 13{\cdot}4$ tons/sq. in.

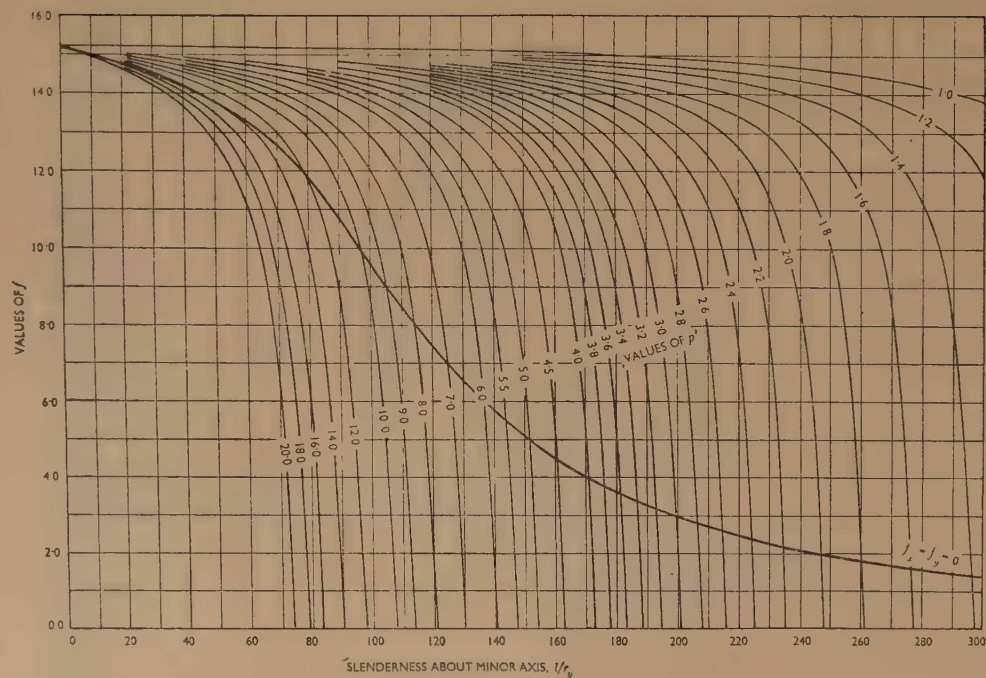
From Fig. 13, $f = 14{\cdot}5$ tons/sq. in. (using $l/r_y = 179$, $p' = 1{\cdot}67$ tons/sq. in.)

The stanchion section is therefore satisfactory.

(8) *Design method for arbitrary ratios of end moments about both principal axes. (Case $P_x . P_y$)*

When the applied bending moments vary along the stanchion it is difficult to determine the maximum extreme fibre stress, which may occur at any section. This considerably increases the difficulty of deriving a general design method. It is evident that an exact solution would be excessively complicated and that resort must be had to safe approximate methods.

In discussing the elastic instability of a stanchion under unequal terminal moments M_x' and M_x'' applied about the major axis (see section (6)), it was found that these



For safety, $p + N_x f_x + N_y f_y \leqslant f$
where:
f_x = stress due to equivalent uniform moment about major axis (tons/sq. in.)
f_y = stress due to equivalent uniform moment about minor axis (tons/sq. in.)
p = mean axial stress
$p' = p + f_x^2/T$ (value of T from Table 2 or calculated from equations (22) and (23))
N_x, N_y obtained from Fig. 14

FIG. 13.—STANCHION DESIGN CHART—VALUES OF f

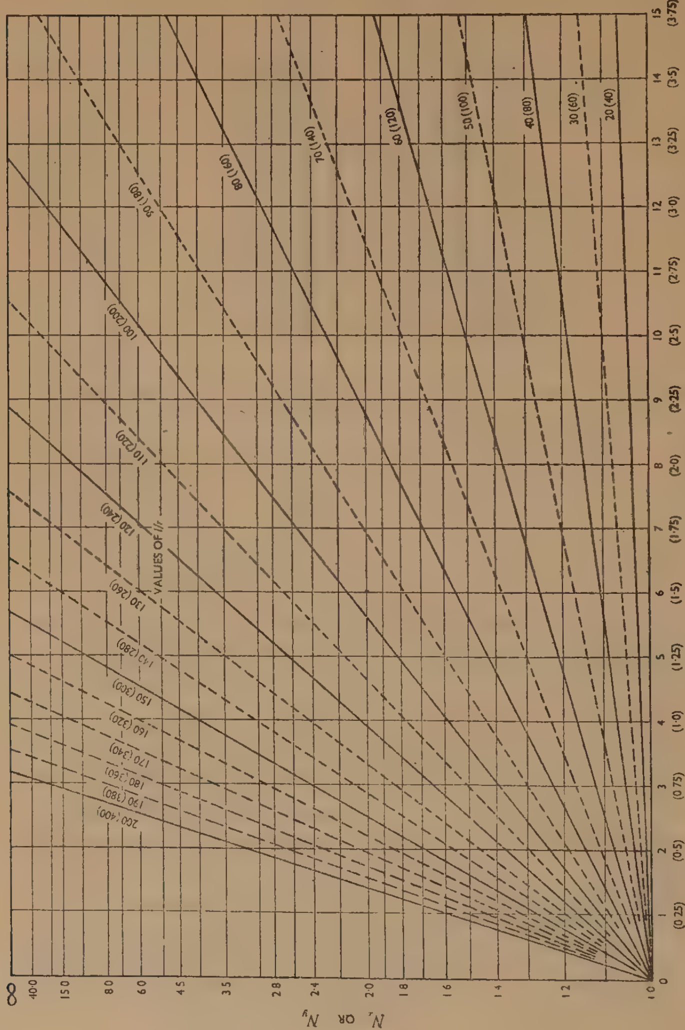


FIG. 14.—STANCHION DESIGN CHART—VALUES OF N_x AND N_y

were very nearly equivalent to equal terminal moments $\frac{1}{\sqrt{F}} M_x'$, bending the stanchion in single curvature, where the value of F depends on the ratio $\beta = \frac{M_x''}{M_x'}$ (see Table 1). It may be shown that, when imperfections are present, the substitution of similarly calculated uniform moments for both the major- and the minor-axis applied moments leads to a close estimate of the loads just sufficient to produce yield, provided this does not occur at the end of the stanchion. If the yield stress

is first reached at either end, the combinations of axial load and bending moments at which this occurs may be calculated by elementary methods.

Consider the plane problem of a stanchion length AB (Fig. 16a) with an initial imperfection $u = u_0 \sin \frac{\pi z}{l}$ where, as before, $u_0 = 0{\cdot}0015 \frac{l r_y}{a_y}$. The stanchion is subjected to a load P (Fig. 16b) which is eccentric at the ends by amounts e and βe in the plane of the major principal axes. The stanchion is thus subjected to terminal moments about the minor axis of eP and βeP. If the eccentricities e and βe and the slenderness l/r_y of the stanchion are defined, then at some definite value of P, corresponding to a mean axial stress p_y, the yield stress will just be reached in the most

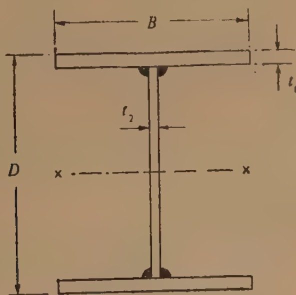


FIG. 15.—DIMENSIONS OF WELDED I-SECTION USED IN THE CALCULATION OF THE PROPERTY T

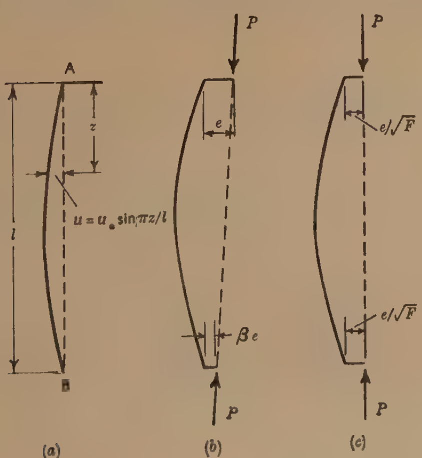


FIG. 16.—STANCHION LENGTH (a) UNDER NO LOAD, (b) WITH AXIAL LOAD AND UNEQUAL ECCENTRICITIES, (c) WITH AXIAL LOAD AND "EQUIVALENT" EQUAL ECCENTRICITIES

highly stressed fibres. Suppose that the loading in Fig. 16b is replaced by that in Fig. 16c, in which the terminal moments are equal and of value $(1/\sqrt{F})\,eP$. The values of $1/\sqrt{F}$ depend on β according to Table 1. Let p_y' denote the mean axial stress at which this substituted loading just produces yield in the stanchion. Then provided p_y' is very nearly equal to p_y the loading in Fig. 16c may, for the purpose of designing the stanchion, be regarded as equivalent to that in Fig. 16b. Expressions for maximum stresses in stanchions with initial imperfections subjected to unequal terminal moments have been derived by Baker and Holder.[8] These expressions have been used to derive values of p_y and p_y' for stanchions with initial imperfections equivalent to the deflected form $u = 0{\cdot}0015\left(\frac{lr_y}{a_y}\right)\sin\frac{\pi z}{l}$, and the results of this analysis are contained in Fig. 17. The curves show, for given values of l/r_y and β, the percentage by which p_y' exceeds p_y for values of p_y varying from zero (corresponding to $e = \infty$) to the axial stress at which the permissible terminal moments are zero ($e = 0$). The greatest discrepancies between p_y and p_y' are seen to occur when $\beta = -1$ and l/r_y is large, with p_y not far below its maximum value. In these cases p_y' is less than p_y, and so to treat the loading in Fig. 16c as equivalent to that in Fig. 16b would lead to safe estimates of the loads required to produce yield. The greatest amount by which p_y' exceeds p_y is only 5% and the general level of agreement is surprisingly good.

The curves in Fig. 17 cover the whole possible range of values of the eccentricity e, together with widely ranged values of β and l/r_y. It may therefore be concluded that, for the plane problem of an eccentrically loaded stanchion, terminal moments M and βM may always be regarded in design as equivalent to equal moments of value $\frac{1}{\sqrt{F}}M$, whatever the axial force and slenderness ratio, except when yield occurs first at one end. The much more complicated problem of the incidence of yield in stanchions with initial imperfections subjected to unequal terminal moments about the major axis has not been investigated. Since, however, the replacement of unequal terminal moments by equal moments of value $\frac{1}{\sqrt{F}}M$ reproduces the condition for lateral instability, it may be concluded that the same substitution will lead to sufficiently accurate predictions of the major-axis moments required to produce yield, again provided this does not occur at either end.

Suppose now that an I-section stanchion is subjected to major-axis terminal moments M_x' and M_x'' and minor-axis terminal moments M_y' and M_y'' as shown in Fig. 11. An axial load P, applied through the centroids of the end sections, is also present. Then the major-axis moments may be replaced by a uniform applied moment of value $M_x = \frac{1}{\sqrt{F}}M_x'$ where $\frac{1}{\sqrt{F}}$ corresponds to $\beta = \frac{M_x''}{M_x'}$, it being assumed that M_x' is numerically greater than M_x''. If M_y' is likewise greater than M_y'', the minor-axis moments may similarly be replaced by a uniform applied moment $M_y = \frac{1}{\sqrt{F}}M_y'$ where $\frac{1}{\sqrt{F}}$ corresponds to $\beta = \frac{M_y''}{M_y'}$, whilst if M_y' is less than M_y'', then the equivalent moment is $M_y = \frac{1}{\sqrt{F}}M_y''$ for $\beta = \frac{M_y'}{M_y''}$. Equations (16) to (24) may be applied to ensure that the maximum stress does not exceed the

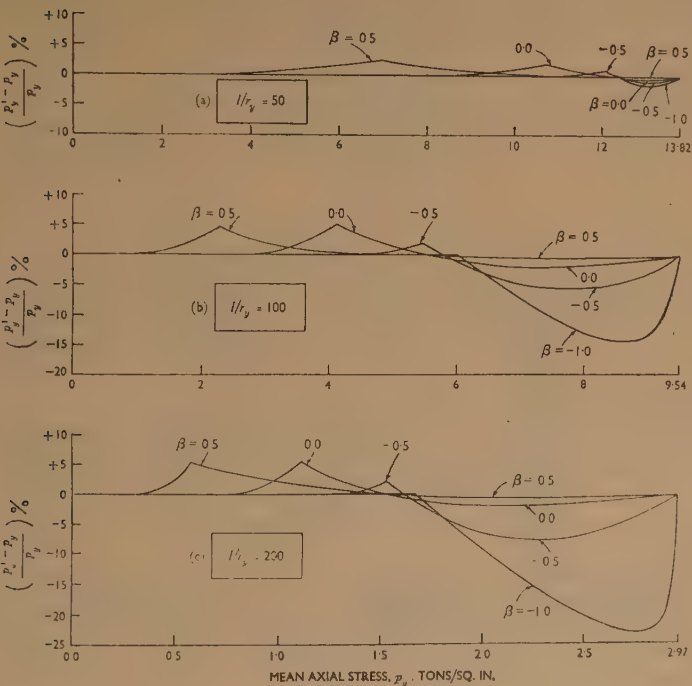


FIG. 17.—AXIAL STRESSES AT YIELD p_y FOR UNEQUAL ECCENTRICITIES COMPARED WITH ESTIMATES OF $p_y(p_y')$ OBTAINED BY SUBSTITUTING "EQUIVALENT" EQUAL ECCENTRICITIES

yield value provided this maximum does not occur at either end. To ensure that the yield stress shall not be exceeded anywhere in the stanchion, a total of three conditions must be satisfied, namely:

$$\left.\begin{array}{l} p + f_x' \quad + f_y' \quad \leqslant 15{\cdot}25 \\ p + f_x'' \quad + f_y'' \quad \leqslant 15{\cdot}25 \\ p + N_x f_x + N_y f_y \leqslant f \end{array}\right\} \quad \ldots \ldots \ldots \quad (25)$$

where f_x', f_x'', and f_x are the extreme fibre stresses corresponding to major-axis bending moments M_x', M_x'', and M_x respectively, whilst f_y', f_y'', and f_y similarly correspond to minor-axis bending moments M_y', M_y'', and M_y. The values of N_x, N_y, and f are derived as before from the values of p, $p'\left(= p + \frac{f_x^2}{T}\right)$, $\frac{l}{r_x}$, and $\frac{l}{r_y}$ by using Figs 13 and 14. The bending moments M_x and M_y may be referred to as the "equivalent uniform moments" about the major and minor axes respectively, and are obtained by multiplying the larger corresponding terminal moment by the appropriate coefficient derived from Table 1.

Example 2.—A 9-in. × 7-in. × 50-lb. R.S.J. of length 14 ft carries an axial load

of 60 tons. The upper and lower terminal bending moments are respectively 20 and 10 tons-ft. (double curvature) about the major axis and 2 and 6 tons-ft (single curvature) about the minor axis. Check the ability of the stanchion to support these loads.

The loads and section properties are given in Fig. 18.

$T = 135{\cdot}2$	$l/r_x = 45$	$l/r_y = 102$
$f_x' = 5{\cdot}19$ tons/sq. in.	$f_x'' = 2{\cdot}60$ tons/sq. in.	$f_x = 2{\cdot}23$ tons/sq. in.
$f_y' = 2{\cdot}09$,,	$f_y'' = 6{\cdot}27$,,	$f_y = 4{\cdot}32$,,
$p = 4{\cdot}08$,,	$p' = 4{\cdot}12$,,	

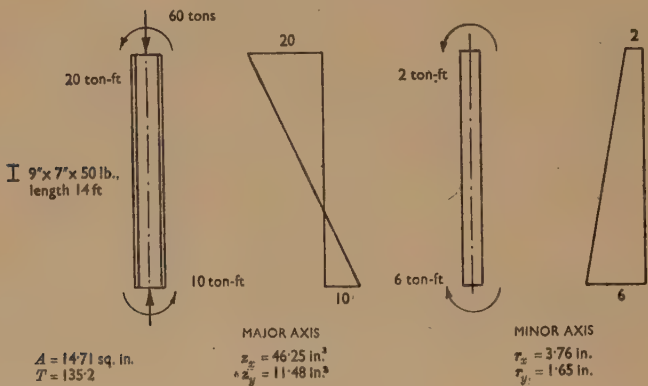


FIG. 18.—LOADING CONDITIONS IN EXAMPLE 2

Maximum stress at top

$$p + f_x' + f_y' = 11{\cdot}36 \text{ tons/sq. in. (less than } 15{\cdot}25 \text{ tons/sq. in.)}$$

Maximum stress at bottom

$$p + f_x'' + f_y'' = 12{\cdot}95 \text{ tons/sq. in. (less than } 15{\cdot}25 \text{ tons/sq. in.)}$$

Instability

From Fig. 13, $f = 14{\cdot}2$ tons/sq. in. $(l/r_y = 102,\ p' = 4{\cdot}12)$

From Fig. 14, $N_x = 1{\cdot}09$ $(l/r_x = 45,\ p = 4{\cdot}08)$

$N_y = 1{\cdot}64$ $(l/r_y = 102,\ p' = 4{\cdot}12)$

$$p + N_x f_x + N_y f_y = 13{\cdot}59 \text{ tons/sq. in. (less than } f).$$

The stanchion section is therefore satisfactory.

Approximate expression for N_x

The value of the coefficient N_x in the last inequality in (25) is found in practice to differ not greatly from unity, its value usually lying between 1·0 and 1·2. It is therefore worth enquiring whether a simple approximation can be found in order to avoid reference to the chart in Fig. 14.

It follows from equations (19) and (20) that

$$\gamma_x = \frac{p}{p'}\left(\frac{r_y}{r_x}\right)^2 \gamma_y$$

Now γ_y is always less than unity (equation (18)), whilst p is less than p'. Hence $\gamma_x < \left(\frac{r_y}{r_x}\right)^2$ and therefore it follows from equations (16) and (19) that:

$$N_x < 1 + \frac{4}{E\pi^3}\frac{p\left(\frac{l}{r_x}\right)^2}{1 - \left(\frac{r_y}{r_x}\right)^2}$$

Hence the equation

$$N_x = 1 + \frac{4}{E\pi^3}\frac{p\left(\frac{l}{r_x}\right)^2}{1 - c^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

will give a safe (high) estimate for N_x provided $c > r_y/r_x$. The maximum value of r_y/r_x for B.S. joists is 0·57 (for a 3 in. $\times$ 3 in. at 8·5 lb.). The substitution of $c = 0{\cdot}57$ and $E = 13{,}000$ tons/sq. in. in equation (26) gives

$$N_x = 1 + p\left\{\frac{1}{261}\frac{l}{r_x}\right\}^2$$

This represents a safe expression for N_x for all B.S. joists. In its derivation a number of approximations (all on the safe side) have been made. In view of this, the following expression may with confidence be adopted:

$$N_x = 1 + p\left\{\frac{1}{300}\frac{l}{r_x}\right\}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

When dealing with sections other than B.S. joists the above approximation is satisfactory provided that the ratio of major to minor radius of gyration is greater than about 1·75, or the ratio of major to minor moment of inertia greater than about 3·0.

In many cases $1{\cdot}00 < N_x < 1{\cdot}02$, and it is then sufficiently accurate to take $N_x = 1{\cdot}00$. It follows from equation (27) that, provided $\frac{l}{r_x} < \frac{40}{\sqrt{p}}$, the last inequality in equations (25) may be modified to

$$p + f_x + N_y f_y < f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

9) *The design of stanchions subjected to bending moments about the minor principal axis only.* (*Case* $O_x \, . \, P_y$)

When bending moments are applied about the minor axis only, the methods eveloped in the previous section are directly applicable. The design method may, owever, be simplified considerably, since there is no possibility of torsional failure. utting $f_x = 0$ in equations (15) to (21) it is found, upon the elimination of the mbols f and N_y, that the maximum permissible minor axis bending stress f_y is ven by:

$$f_y = \frac{\left\{f_L - p\right\}\left\{E\pi^2\left(\frac{r_y}{l}\right)^2 - p\right\} - E\epsilon\pi^2 p\left(\frac{r_y}{l}\right)}{E\pi^2\left(\frac{r_y}{l}\right)^2 + \left(\frac{4}{\pi} - 1\right)p} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

hen the terminal moments are unequal, the stress f_y is that which corresponds the equivalent uniform moment M_y.

A chart giving the relation between f_y, p, and l/r_y is given in Fig. 19. As in the rivation of the chart in Fig. 13, it has been assumed that $f_L = 15{\cdot}25$ tons/sq. in.,

$E = 13{,}000$ tons/sq. in., and $\epsilon = 0{\cdot}0015$. From this chart may be derived directly the greatest permissible equivalent bending stress f_y for any combination of mean axial stress p and slenderness ratio l/r_y. The additional restriction that the combined stress at either end must not exceed the yield value must also be observed.

Example 3.—A 10-in. × 8-in. × 55-lb. R.S.J. of length 20 ft sustains minor-axis bending moments of 12 tons-ft at the top and zero at the bottom. Estimate the maximum axial thrust which the stanchion will support.

$A = 16{\cdot}18$ sq. in. $\qquad l/r_y = 130 \qquad Z_y = 13{\cdot}69$ in.^{3}

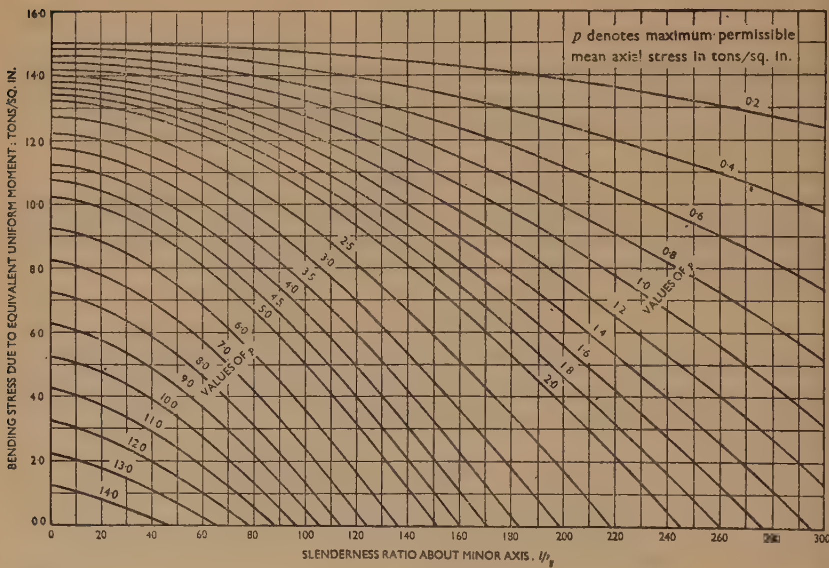


Fig. 19.—Stanchion design chart—bending about the minor axis only

Maximum stress at ends

At top, $f_y' = 10{\cdot}52$ tons/sq. in., giving a limiting axial stress of $15{\cdot}25 - 10{\cdot}52 = 4{\cdot}73$ tons/sq. in.

Instability

$$\frac{M_y''}{M_y'} = 0, \text{ hence } \beta = 0 \text{ and } \frac{1}{\sqrt{F}} = 0{\cdot}565$$

$$f_y = 0{\cdot}565 \times 10{\cdot}52 = 5{\cdot}95 \text{ tons/sq. in.}$$

Hence from Fig. 19 maximum axial stress is 3·1 tons/sq. in.

The stanchion will thus sustain a mean axial stress of 3·1 tons/sq. in. giving an axial load of 50·1 tons.

(10) *The design of stanchions subjected to bending moments about the major principal axis only.* (*Case* $P_x \cdot O_y$)

The methods described in section (8) may be used, but with considerable simplifi-

cation since $f_y' = f_y'' = f_y = 0$. Using the approximate expression for N_x represented by equation (27) the conditions that have to be satisfied are:

$$\left.\begin{aligned} p + f_x' &< 15{\cdot}25 \\ p + f_x'' &< 15{\cdot}25 \\ p + \left\{1 + p\left(\frac{1}{300}\frac{l}{r_x}\right)^2\right\} f_x &< f \end{aligned}\right\} \quad (30)$$

where f is obtained by entering the chart in Fig. 13 with l/r_y and $p' = p + \frac{f_x^2}{T}$. When $\frac{l}{r_x} < \frac{40}{\sqrt{p}}$, the last condition in (30) may be modified to:

$$p + f_x < f \quad (31)$$

Example 4.—An 18-in. × 6-in. × 55-lb. R.S.J. is 18 ft long and supports an axial load of 25 tons. Check the ability of the stanchion to carry terminal bending moments about the major axis of 100 tons-ft at the top and 10 tons-ft at the foot, tending to bend the member in double curvature.

It is found that:

$T = 27{\cdot}1$ $\quad l/r_x = 30$ $\quad l/r_y = 179$

$f_x' = 12{\cdot}83$ tons/sq. in. $\quad f_x'' = 1{\cdot}28$ tons/sq. in. $\quad \beta = -0{\cdot}1$

$\frac{1}{\sqrt{F}} = 0{\cdot}532$ $\quad f_x = 12{\cdot}83 \times 0{\cdot}532 = 6{\cdot}82$ tons/sq. in.

$p = 1{\cdot}55$ tons/sq. in. $\quad p' = 1{\cdot}55 + \frac{6{\cdot}82^2}{27{\cdot}1} = 3{\cdot}27$ tons/sq. in.

Maximum stress at top

$p + f_x' = 14{\cdot}38$ tons/sq. in. (less than $15{\cdot}25$ tons/sq. in.)

Instability

From Fig. 13, $f = 10{\cdot}4$ tons/sq. in. (using $l/r_y = 179$, $p' = 3{\cdot}27$).

$$\frac{40}{\sqrt{p}} = 32\text{; hence } N_x = 1\text{, since } \frac{40}{\sqrt{p}} > \frac{l}{r_x}$$

$$p + N_x f_x = 8{\cdot}37 \text{ tons/sq. in. (less than } f)$$

The stanchion section is therefore satisfactory.

.1) *Numerical comparison between suggested design method and previous methods*

It is desirable to make some comparison between the results given by the sug-esuggested design method and previous design specifications. A difficulty arises in iat, whilst the suggested design method is for use with factored loads, previous gulations have been derived as part of "safe stress" methods of design. It is erefore necessary to multiply the loads given by previous specifications by appro-iate factors.

The load factor appropriate to B.S. 449: 1948[4] may be calculated by considering e load factor implied in the specification for beam stresses. The working stress bending is taken as 10 tons/sq. in., whilst the yield stress is assumed to be 15·25 ns/sq. in. Taking an average beam-shape factor of 1·15, this gives a load factor collapse for simply supported beams of $(15{\cdot}25 \times 1{\cdot}15) \div 10 = 1{\cdot}75$. This load tor differs considerably from the value of 2·0 assumed in B.S. 449 when deriving e stanchion stresses for slenderness ratios greater than 80. It is, however, possible to correlate the design method given by B.S. 449 with any consistent

load factor and 1·75 may be taken as being most appropriate for the specification as a whole.

The load factor assumed in calculating the stanchion curves given by the Steel Structures Research Committee was 2·0. The assumed yield stress was, however, 18 tons/sq. in., compared with the 15·25 tons/sq. in. now usually accepted in the design of mild-steel structures. If the approximation is made that the loads at the commencement of yield in a stanchion are proportional to the yield stress, the effective load factor for the S.S.R.C. curves becomes $2{\cdot}0 \times 15{\cdot}25/18 = 1{\cdot}69$. Alternatively, the curves could be recalculated using the new value for the yield stress. The former procedure gives stresses which are slightly lower than those which would result from a complete recalculation. The difference is, however, slight and for low slenderness ratios it becomes completely negligible. The course will therefore be adopted of using the original curves (for single and double curvature separately, see Figs 13.2 and 13.3 of reference 10) with a load factor of 1·69.

The axial stresses given by the three methods in the absence of applied bending moments are compared in Fig. 20, the curve shown for the new method being the same as the heavy curve in Fig. 13. The new formula is quite close to the stresses obtained from the design charts of the Steel Structures Research Committee, and the two curves would become practically identical were the S.S.R.C. results recalculated without a load factor but with a yield stress of 15·25 tons/sq. in. The B.S. 449 curve is on the whole lower than the other curves, owing partly to the higher assumed value of initial imperfections ($\epsilon = 0{\cdot}003$ in place of $\epsilon = 0{\cdot}0015$).

When bending moments are applied about the minor axis only, the S.S.R.C. recommendations may be expected to agree closely with the new method for equal end moments in single curvature and for end-moment ratios of 0·268 in double curvature ($\beta = 1$ and $-0{\cdot}268$ respectively). The allowable terminal bending stresses when the mean axial stress is 8 tons/sq. in. are shown for these two ratios of end moments in Figs 21 and 22. Since no allowance is made for differing ratios of end moment in B.S. 449, the corresponding curves are the same in each Figure. The stresses given by the S.S.R.C. recommendations are seen to be in quite close agreement with those given by the new method.

Turning now to comparisons involving bending about the major axis only, it is necessary to consider actual sections, since the new method introduces the property T. Flexural-torsional instability is most important when T is small and the rolled section with the lowest value is a 22-in. × 7-in. B.S. joist, for which $T = 23{\cdot}0$ (see Table 2). The allowable major-axis bending stresses for bending in uniform single curvature for this section when the mean axial stress is 2 tons/sq. in. are shown in Fig. 23. The new method is seen to be in good agreement with the S.S.R.C. curve, this latter being obtained by treating the applied bending stresses as though they were due to moments acting about the minor axis. The S.S.R.C. improved curve has been obtained from the analysis [8] due to Baker and Holder to which reference has already been made, which contains a method for dealing with stanchions subjected to uniform bending moments about both axes. This analysis does not allow for the incidence of flexural-torsional instability and this explains the excessively high values given by this method in the present example.

When dealing with a section for which flexural-torsional buckling is not critical, the S.S.R.C. improved method is in good agreement with the treatment derived in this Paper. This may be seen from Fig. 24, which refers to an 8-in. × 6-in. B.S. joist ($T = 111{\cdot}8$) under a mean axial stress of 8 tons/sq. in. Thus neither of the S.S.R.C. methods are in consistently good agreement with a procedure which

takes account of the danger of flexural-torsional buckling. The Final Report does not give an improved analysis for bending about the major axis in double curvature, and the simple formula for $\beta = -0{\cdot}268$ gives results intermediate between the new method and the values given by B.S. 449. This is illustrated in Fig. 25, which refers to a 22-in. × 7-in. B.S. joist subjected to a mean axial stress of 8 tons/sq. in.

Summarizing, it may be said that the new method is in good agreement with the S.S.R.C. curves when bending moments are applied about the minor axis only for the two ratios $\beta = 1$ and $\beta = -0{\cdot}268$. Comparisons could, of course, be made between the new method and Baker's curves for the "General Method of Design" for any ratio of end moments, but the good agreement obtained for the two particular ratios makes such further comparisons scarcely necessary and provides evidence that the suggested design method may be expected to be generally reliable. The lack of consistent agreement when bending is about the major axis shows that a design method, to be fully satisfactory, cannot be based on the simple substitution of equal bending stresses about the minor axis, such a substitution being particularly unsatisfactory when bending is in single curvature. Whilst no comparisons between the various methods have been made for combined bending about both axes, it may reasonably be assumed that the levels of agreement would be intermediate between the levels obtained when considering bending about the two axes independently.

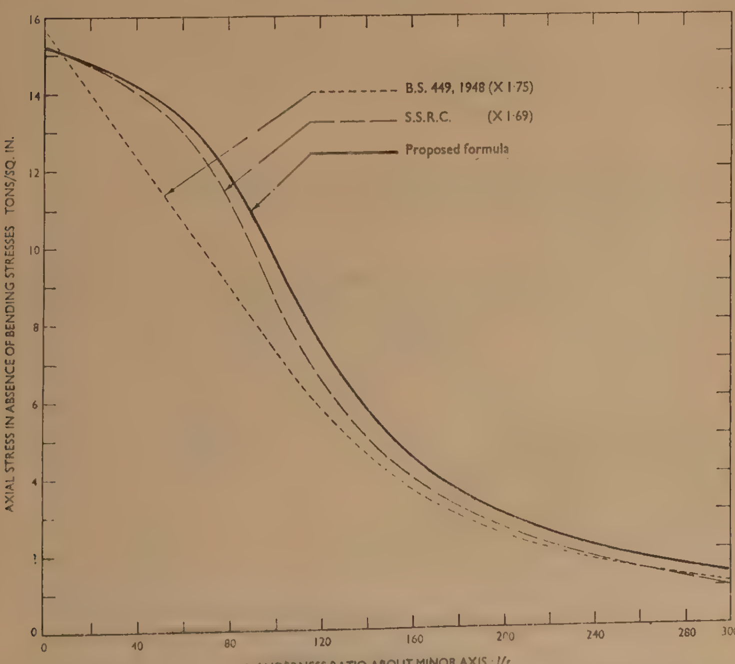


FIG. 20.—PERMISSIBLE AXIAL STRESSES IN THE ABSENCE OF APPLIED BENDING MOMENTS

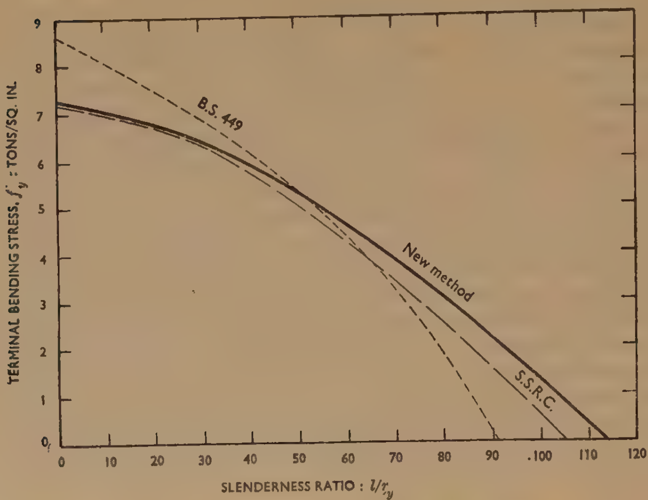


FIG. 21.—BENDING ABOUT THE MINOR AXIS IN SINGLE CURVATURE ($\beta = 1$) AT A MEAN AXIAL STRESS OF 8 TONS/SQ. IN.

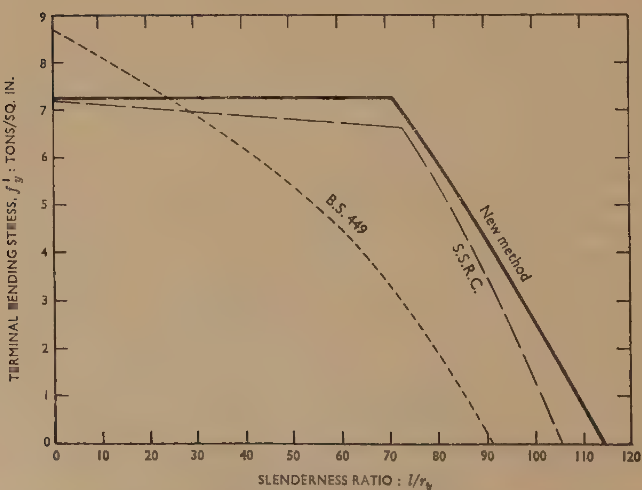


FIG. 22.—BENDING ABOUT THE MINOR AXIS IN DOUBLE CURVATURE ($\beta = -0{\cdot}268$) AT A MEAN AXIAL STRESS OF 8 TONS/SQ. IN.

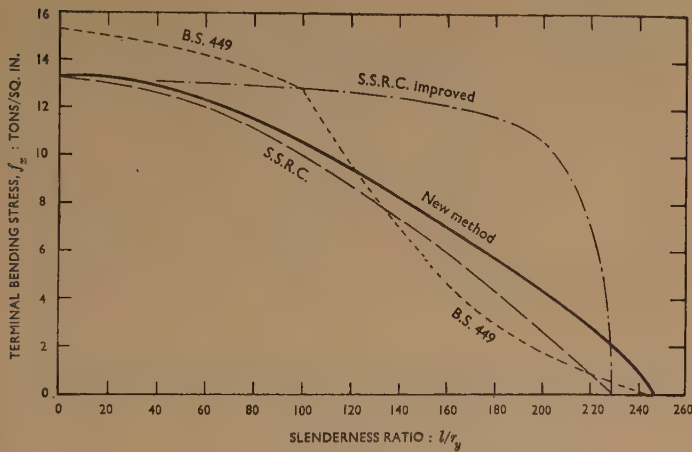


FIG. 23.—BENDING ABOUT THE MAJOR AXIS OF A 22-IN. × 7-IN. B.S. JOIST IN SINGLE CURVATURE ($\beta = 1$). MEAN AXIAL STRESS, 2 TONS/SQ. IN.

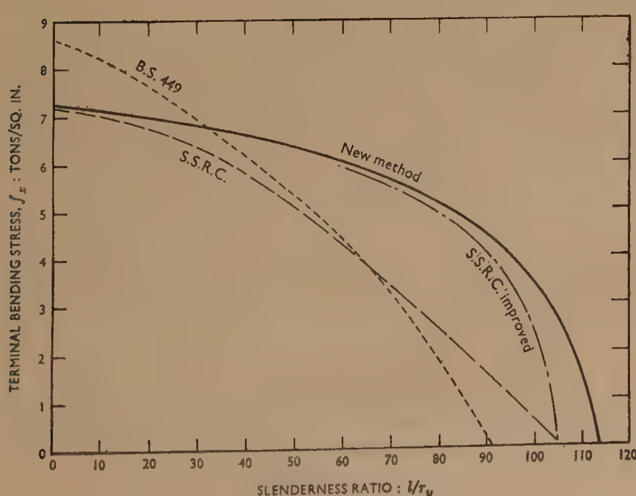


FIG. 24.—BENDING ABOUT THE MAJOR AXIS OF AN 8-IN. × 6-IN. B.S. JOIST IN SINGLE CURVATURE ($\beta = 1$). MEAN AXIAL STRESS, 8 TONS/SQ. IN.

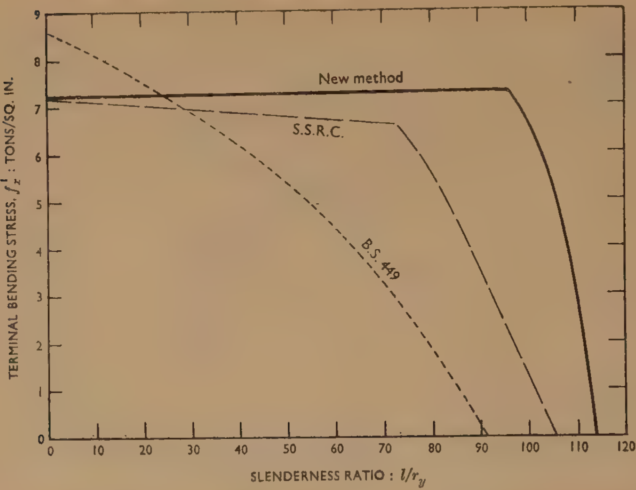


FIG. 25.—BENDING ABOUT THE MAJOR AXIS OF A 22-IN. × 7-IN. B.S. JOIST IN DOUBLE CURVATURE ($\beta = -0{\cdot}268$). MEAN AXIAL STRESS, 8 TONS/SQ. IN.

(12) *The determination of stanchion terminal moments in continuous structures*

The design method which has been presented for stanchions has been evolved on the assumption that the terminal moments acting on any stanchion length are known. It is thus necessary to consider the distribution of moments at any floor level as between the stanchion lengths above and below, and the net bending moment induced in the stanchion at a joint which receives beams on opposite sides.

Distribution of moments between stanchion lengths

The method of design used for the stanchions ensures that they carry an axia load well below their Euler critical value for a pin-ended strut, particularly when the terminal moments are high. In estimating the distribution of the net beam moments at any joint as between the stanchion lengths, it is thus permissible to neglect entirely the reduction of the flexural rigidity of the stanchions by axial load. It is suggested that, in a multi-bay frame, the simple rule be adopted of dividing the net beam moment at a joint between the upper and lower stanchion lengths in the ratio of their stiffnesses (moments of inertia divided by the respective heights between floors). Whilst it has been shown [10] that this procedure can lead to very considerable errors in entirely elastic structures, its use can be more easily justified in structures in which most of the beams are plastic whilst the stanchions remain elastic. In general, the rule suggested should be sufficiently accurate provided sidesway deflexion is not appreciable. The rule will be inadequate for single-bay multi-storey frames when these are subjected to wind load—a condition which has not been investigated sufficiently to enable plastic design methods to be applied with confidence.

Determination of net beam moments at joints

The greatest moment which can be applied to an outside stanchion length at

any joint is known to be the full plastic moment of the beam concerned. For an inside stanchion the maximum moment is not known so readily except when the critical loading condition is an overall collapse mechanism, such as that shown in Fig. 4a, when at some of the joints the net moment applied by the beams is determined by the equations of statical equilibrium. As a rule, failure involving sidesway will not be the critical condition for the stanchions in multi-storey frames if the usual practice is adopted of allowing a lower load factor when wind forces are present. The maximum beam moment which can be exerted on any internal joint is then the greater of two quantities, namely:

(*a*) the difference between the full plastic moments of the two beams entering the joint on opposite sides; and

(*b*) the full plastic moment of the beam with the larger plastic modulus, less the elastic moment of resistance induced in the beam on the opposite side of the joint.

The calculation of the second quantity is difficult, since the loading condition assumed has to be that which leads to the least terminal moment in the lighter of the two beams. It thus appears that the calculation of the most critical loading condition for the internal stanchions may involve the use of Tables of end moments such as those produced by Baker and Williams,[30] which were adopted by the Steel Structures Research Committee in their Recommendations for Design, but attempts are being made to derive a simpler approach. Fortunately, the situation is not as unpromising as first appearances suggest. In design methods which are purely elastic, none of the bending moments in the members meeting at a joint are known *a priori*. In the design method here suggested, the greater beam moment is known and the smaller beam moment on the opposite side of the joint has only to be determined approximately.

Conclusion

It will be seen from this discussion that a complete design method for multi-storey buildings is not yet available. At the same time, it is hoped that discussion on the broad lines attempted in this Paper will focus attention on the essential aspects of the problem. It is believed that, with the knowledge now available, it should be possible to design multi-storey multi-bay frames provided the designer is prepared to use plastic theory in conjunction with reasonable approximations derived from elastic theory where appropriate. With growing experience of such intelligent applications of existing knowledge it should be possible to develop an economical, yet straightforward, design method.

The Paper forms part of a general investigation into the behaviour of steel structures in the plastic range being carried out in the Engineering Laboratory, Cambridge.

Notation

A	denotes	area of cross-section
B	,,	flange width
D	,,	overall depth of section
E	,,	modulus of elasticity
F	,,	a function of β (see equation (4) and Table 1)
G	,,	modulus of rigidity

I_x, I_y denote moments of inertia about the major (XX) and minor (YY) axes respectively
K denotes torsional constant of a section
M ,, bending moment
M' ,, uniform applied moment replacing initial curvature
M_A, M_B denote terminal bending moments
M_E denotes uniform moment causing lateral instability in the absence of axia load, resistance to warping being neglected
M_x ,, uniform moment applied externally about the major axis
M_x', M_x'' denote terminal moments about the major axis
$[M_x]_{\frac{l}{2}}$ denotes major-axis bending moment at mid-height
M_y ,, uniform moment applied externally about the minor axis
M_y', M_y'' denote terminal moments about the minor axis
$[M_y]_{\frac{l}{2}}$ denotes minor-axis bending moment at mid-height
N_x, N_y . . . see equations (15), (16), and (17), and Fig. 14
P denotes axial load
P_E ,, Euler crippling load for a pin-ended strut
R_x, R_y denote external restraint forces along axes OX and OY respectively (see Fig. 11)

$T = \dfrac{AGK}{Z_x^2}$

Z_x, Z_y denote section moduli about major and minor axes respectively
a_x, a_y ,, distances from the major and minor principal axes respectively to the extreme fibres
e denotes eccentricity (see Fig. 16)
f . . . see equations (15) and (18), and Fig. 13
f_L denotes yield stress
f_x, f_x', f_x'' denote extreme fibre bending stresses corresponding to major-axis bending moments M_x, M_x', and M_x'' respectively ($f_x = M_x/Z_x$ etc.)
f_y, f_y', f_y'' ,, extreme fibre stresses corresponding to minor-axis bending moments M_y, M_y', and M_y'' respectively
l denotes length of a stanchion
p ,, mean axial stress

$p' = p + \dfrac{f_x^2}{T}$

p_y denotes value of p at which yield stress is just reached in most highly stressed fibres
p_y' ,, value of p_y obtained by substituting equal terminal moments for the actual applied moments
r_x, r_y denote radii of gyration about the major (XX) and minor (YY) axes respectively
t_1 denotes flange thickness
t_2 ,, web thickness
u ,, initial deflexion in the OZX plane

u' denotes deflexion in the OZX plane under axial load and major-axis bending moments
u_0 „ value of u at mid-height
u_1 „ value of u' at mid-height
v „ initial deflexion in the OYZ plane
v' „ deflexion in the OYZ plane under axial load
z „ distance measured from one end in the longitudinal direction
β „ ratio of end moments $\left(\text{usually } \frac{M_x''}{M_x}, \text{ but sometimes } \frac{M_y''}{M_y}, \text{ or } \frac{M_y'}{M_y''}\right)$

$\gamma_x = Pl^2/\pi^2 EI_x$

$\gamma_y = \left(\frac{M_x}{M_E}\right)^2 + \frac{P}{P_E}$

ϵ denotes constant in the Perry–Robertson strut formula
θ_A, θ_B denote end rotations in a stanchion length

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The Paper, which was received on 24 September, 1955, is accompanied by twenty-three sheets of diagrams, from which the Figures in the text have been prepared.

Discussion

Dr R. H. Wood (Principal Scientific Officer, Building Research Station), after complimenting the Author, said that designers by the elastic theory had become accustomed to the alternative arrangements of loading which could be obtained on the same frame, and both he and Professor Baker before him on the Steel Structures Research Committee had tried to follow those alternatives. Dr Wood had come to the conclusion that what was good for the goose was good for the gander, and in the present case he thought that if the live load was taken off certain floors, so as to promote the critical arrangement corresponding to the lowest mode of buckling, it might be possible, even with the same frame, to change the $P_x . P_y$ case into a $P_x . E_y$ or an $E_x . E_y$ case, with the probability of a lower collapse load. He thought, therefore, that it was a question of finding the absolute minimum of various cases mentioned in the Paper.

He then referred to a stanchion that had collapsed as an $E_x . E_y$ case (Fig. 26). The Cambridge team had done well to point out that that sort of thing was possible. It would

FIG. 26.—STANCHION COLLAPSED AS AN $E_x \,.\, E_y$ CASE

be noticed that to make use of the plastic hinges at the ends it was necessary to have stiff beams. The load was not shown on the beams, but it developed the kind of collapse shown even with load on the beams. That particular stanchion would support nearly twice the Euler load about the minor axis.

Dr Wood then referred, by means of slides, to the analysis of elasto-plastic states of stanchions by means of a differential analyser,[31] and showed a set of elastic-plastic moment-rotation relationships for the integrated effects from one end to the other. Such generalized results for $P/P_E = 0$, 0·6, and 1·0 had already been obtained.[32] At 60% of the Euler load there was positive rotational stiffness of the stanchion, if still elastic, which might eventually become negative stiffness if plasticity developed.

After extracting in Fig. 27 from those known results the properties dealing with the single-curvature type of beam loading at various direct loads, the question arose of how

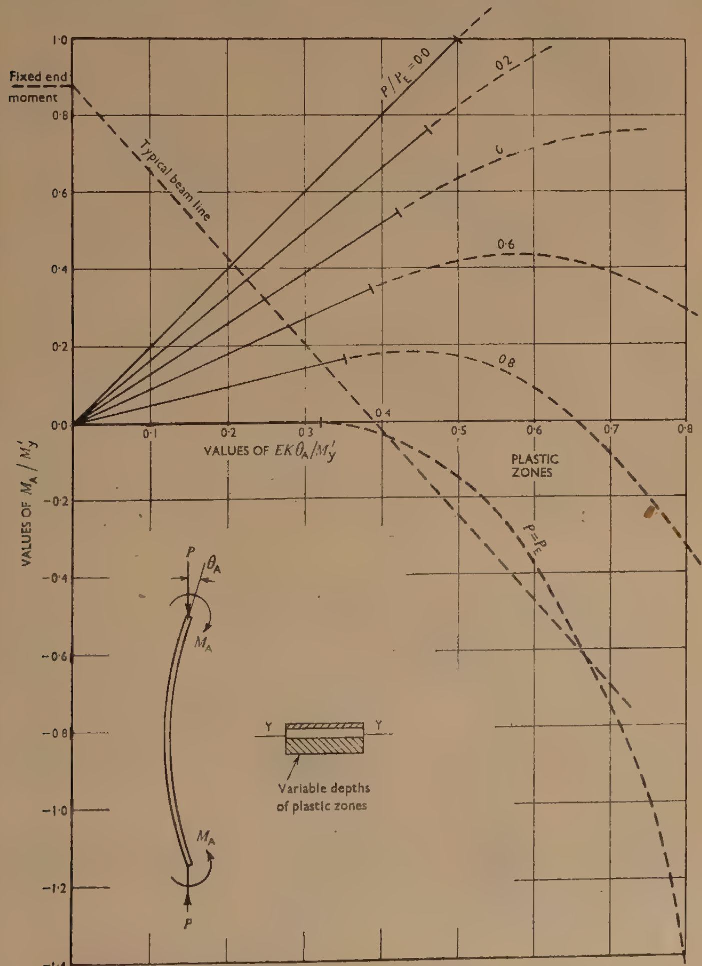
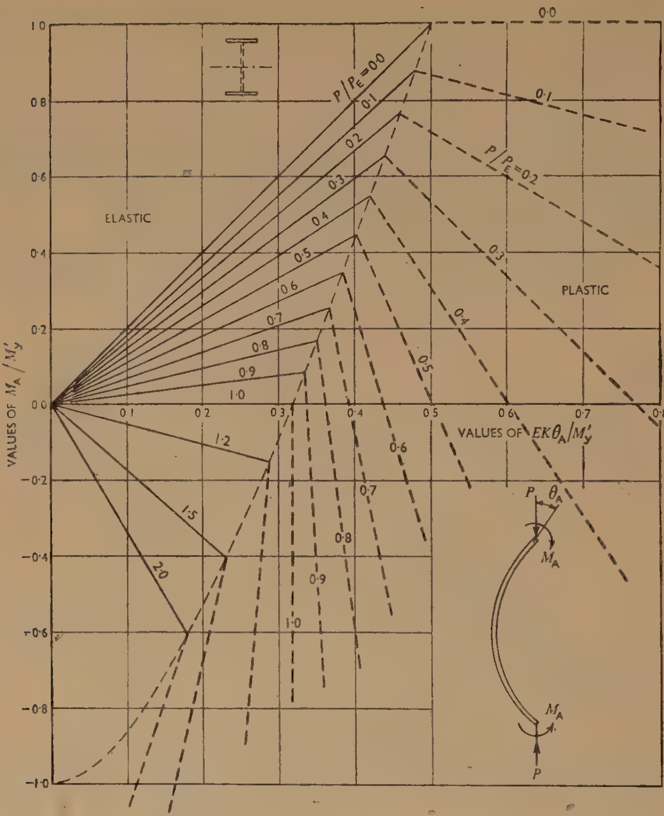


FIG. 27.—EQUILIBRIUM STATES FOR RECTANGULAR STANCHIONS WITH EQUAL AND OPPOSITE END MOMENTS



Note. Direct stress requires the following limitation:

P/P_E	Minimum values $\frac{fy}{E}\left(\frac{L}{k}\right)^2$	$\frac{L}{k}$ for $fy = 45{,}000$ lb/in^2, $E = 30 \times 10^6$
0	0	0
0·4	3·95	51·3
0·8	7·90	72·6
1·0	9·87	81·1
1·2	11·84	88·9
1·6	15·79	102·6
2·0	19·74	114·7
4·0	39·48	162·2

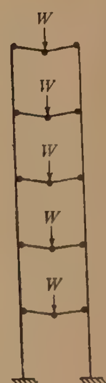


FIG. 28.—EQUILIBRIUM STATES FOR A PAIR OF PARALLEL PLATES WITH EQUAL AND OPPOSITE END MOMENTS

FIG. 29

advanced stages of plasticity corresponding to those shown in Fig. 26 could be used. It was possible to start with zero rotation and corresponding fixed-end moment on the beams, and relaxing the system would then give a "beam line" with a positive stiffness, which would be a straight line (Fig. 27). If, however, at any stage the beam went plastic the corresponding beam line would then shoot off horizontally. It would be seen at once, since the dotted lines represented plastic states and the full lines elastic states, that if the beam became plastic the chances of getting anything superior to elastic design for the stanchion were very remote, and that agreed with the Author's conclusions. To use the advanced elasto-plastic states at heavy direct loads would necessitate a beam which was still elastic.

Fig. 28 showed corresponding states for a pair of parallel plates. It was interesting, because the form factor was unity, whilst in the previous case it had been 1·5. There was positive stiffness to start with and an abrupt change to negative stiffness. At the Euler load the stiffness of such an elastic stanchion was zero, whilst plasticity resulted instantly in a stiffness of minus infinity. That showed the effect of plasticity when the form factor was very small but the effect would be limited since it applied usually only to the strong axis.

Commenting on the subject of overall frame stability, Dr Wood said that that was a feature which was probably not very important at the moment, because the Author's method applied to no-sway conditions; but Dr Wood wished to show what happened when what he would call a deterioration of the stiffness matrix took place with any kind of plasticity developing anywhere. It would depend, of course, on where it did develop. In Fig. 29 which showed vertical loads only carried by the beams and no wind loads, it was found that even in the elastic region there was a critical load for the whole frame. The best work on that subject was that by Dr Merchant and Dr Chandler at Manchester University. The corresponding stanchion loads for the overall critical load would be about twice the Euler load if side-sway was prevented. If plastic hinges developed on the various beams in Fig. 29, bearing in mind that there were only vertical loads on the frame and that side-sway was prevented, the critical load was gradually brought down until at the most there was approximately the Euler load in each stanchion length. If lasticity developed in the stanchion lengths the critical load would come lower still.

He would now assume that side-sway was not prevented. That was going to be a ifficulty in the future, though it did not apply to the present Paper. The upper critical oad for the whole frame would probably correspond to about $0{\cdot}4P_E$, i.e., about 40% of the uler load for any individual stanchion. It would be noted that that was already a very onsiderable drop of about 5:1 even if the frame stayed elastic. As the plastic zones eveloped in the beams with side-sway not prevented, the bottom limit for the critical oad in the stanchions would reduce to nearly zero, because, taking the left-hand stanchion ith the hinges as shown, the equivalent length of that stanchion was then about twice the eight of the building. It had practically no restraint left, and no stanchion would stand p to that, not even about the strong axis. It meant that the frame could never get into hat kind of collapse mode at all. The structure would collapse sideways instead. In eneral, as the moment-distribution process carried on, raising the carrying capacity, the pper critical load of the whole structure was continuously reduced by the development f plastic zones, and the collapse of the whole frame would take place when those tendencies et.

Two conclusions might be made. First, it had been a wise decision on the part of the teel Structures Research Committee to limit simple design methods to cases not involving de-sway, where the side-sway was prevented by the walls and floors. Secondly, there ere obviously various ways in which it was possible to deal with the acute problem of astic-plastic states. At the Building Research Station their terms of reference were ightly different, in that they were expected to produce the simplest possible design nsistent with increasing economy.

Dr Jacques Heyman (University Lecturer in Engineering, University of Cambridge) that the Author had adopted an excellent classification system when reducing the

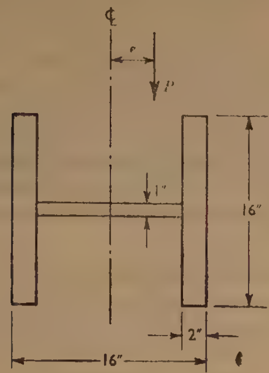
possible forty-five different cases of end conditions for stanchions to the nine shown in Fig. 5. It would probably be agreed that those covered most of the stanchion terminal conditions within the limits which the Author had imposed on himself, and Dr Heyman was pleased to hear that the Author had promised a new Paper on the plastic-hinge terminal condition.

Dr Heyman was also impressed with the efficient way in which the Author had made certain simplifying assumptions, not only to enable him to solve the problem but to give results which appeared always to be on the safe side. Dr Heyman believed that the results given in the Paper could be used with perfect confidence in design. The Author had intimated that he was not presenting a design process, but Dr Heyman believed that it was possible even at the present stage to design stanchions from the curves given in the Paper, and offered a spot check on the proposals in the form of a very simple design example.

At first glance it did not appear to fit into the Author's nine terminal conditions, but on further analysis it would be found that it could be fitted in. What he wanted to consider was a continuous heavy plate-girder system designed by the plastic theory, in which each span had a full plastic moment at the ends and also at the centre; i.e., each span became statically determinate at collapse. On the drawing-board it was possible to work out the reactions at each support and therefore find the loads transmitted to those supports. The loads coming on to the stanchions used with that plate-girder system would be known, and it should be possible to design the stanchions plastically by the methods which the Author had proposed. "On the drawing-board" the forces on the stanchions would be acting nicely on the centre-lines, but in practice a centre-line was rather an intangible thing and various codes differed in the eccentricity which could be allowed in designing, say, a capped stanchion with no true moment connexion to the beam system which it supported. In fact the reaction on the stanchion could shift by a few inches without in the least affecting the collapse system for the continuous plate girder, so that although one had a statically determinate system there was a degree of indeterminacy about where the reaction came on the stanchion.

In Fig. 30 Dr Heyman had taken as an example a very heavy plate-girder section, perhaps slightly unrealistic, 16 in. sq. with 2-in. flanges and 1-in. web and a length of 20 ft. The Author's torsion function was extremely high, 200 (over 100 there was virtually no need to worry about instability due to torsion). It was basically a 1,000-ton stanchion at collapse. The figure of 1,032, calculated by the methods proposed by the Author, corresponded to axial loading. Dividing that figure of 1,032 by 1·75, one obtained a working value corresponding to a load factor of 1·75 of 590 tons, corresponding to the value given in B.S. 449 of 475. The Code of Practice said that an eccentricity of one-third the width of the stanchion shaft should be allowed for—in the present case 5·3 in. That led to a working load of 294 on the stress system of analysis of the stanchion. The Author's method, fitting the stanchion into the $P_x \, . \, P_y$ classification, gave a load of 583 tons at which the yield stress was reached at the top of the stanchion. Dividing by 1·75 gave 333 tons, which was not very different from the figure of 294 tons. The British Standard asked for an eccentricity of half the shaft width and led to a value of 246 tons; the Author gave a value of 467 tons (or 266 tons when divided by the load factor).

Going to a still further eccentricity, the 9·38 in., which Dr Heyman had labelled (i), there was a still further reduction to 423 tons. The figure of 509 tons in the line beginning 9·38 (ii) corresponded to some unpublished work by the Author in which he had allowed a full plastic moment to develop at the top of the stanchion. It was possible to read off from a curve the axial stress of a stanchion at collapse, in the case in question 6·7 tons/sq. in., leading to the figure of 509 tons, and from that figure of axial load one could work out the reduced full plastic moment and therefore the eccentricity of loading, which came out to 9·38 in. One therefore got a measure of the safeness of the Author's work in the present Paper. The figure of 1,032 tons was reduced progressively as the eccentricity was increased, but jumped up again when full plasticity was developed. The attainment of the yield stress at the end of the stanchion corresponding to the figure of 423 tons did not represent the ultimate carrying capacity of the stanchion; there was a margin of safety, and the load could go up to 509 tons.



$l = 20$ ft $l/r_x = 36{\cdot}5$ $l/r_y = 55{\cdot}6$ $T = 202$ tons/sq. in.

Eccentricity e, (in.)	Values of P (tons) and p (ton/sq. in.)			
	B.S. 449	C.P. 113	Horne (collapse)	Horne (÷ 1·75)
0	475 (6·25)	475 (6·25)	1,032 (13·6)	590 (7·8)
5·33	—	294 (3·87)	583 (7·78)	333 (4·38)
8	246 (3·24)	—	467 (6·15)	266 (3·51)
9·38 (i)	—	—	423 (5·57)	242 (3·18)
9·38 (ii)	—	—	509 (6·7)	291 (3·8)

FIG. 30

A further interesting point came out of that. The figure of 1,032 tons for the zero ccentricity corresponded to failure of the stanchion by instability at the centre, the ondition being $p + N_x f_x < f$, the function f coming from the chart which the Author ave. At the eccentricity of 5·3 in., the collapse condition was determined by the attain- ent of the yield stress at the end of the stanchion, and the stanchion itself did not ecome truly unstable. Similarly, the greater eccentricities also corresponded to the ttainment of yield stress at the end of the stanchion. If one wrote the condition that e yield stress should be reached at the end of the stanchion and also that the Author's nction f should just be reached, that corresponded to a very small eccentricity of about in. in the present example, well below the values specified by the British Standard and y the Code of Practice. The corresponding axial load for that condition at collapse was bout 11 tons/sq. in., or 850 tons in round figures.

In practice Dr Heyman thought that what would happen with the particular design xample which he had chosen of a continuous plate-girder system was that the load might t first be applied at some fairly large eccentricity, and that would presumably cause ield at the top of the stanchion; there would then be some relaxing of the eccentricity, e point of application of the load moving in towards the centre-line; and the ultimate

condition would be reached when in the present example the load became about 2 in. eccentric and the stanchion then began to fail by true instability rather than by reaching the yield stress at the end.

Dr. Heyman did not wish to make too much of that example or conclusion, but it brought out another point of excellence in the Paper, which was that the Author had pointed out not only the load at which a stanchion collapsed but also *how* the stanchion collapsed, whether by attainment of the yield stress at the end or by overall torsional buckling combined with lateral bending instability. That might open up the way to tackle problems such as the one which Dr Heyman had presented, of eccentricity of loading, and a host of others which up to the present had been intractable.

Mr Eric Ingerslev (Director, Alderton Construction Co. Ltd) expressed some hesitation in commenting on the excellent Paper which the Author had presented, because it put the whole matter so clearly, and because of the great simplicity at which it aimed. He wished to comment, however, on the main equation in the Paper, equation (4), which was $\frac{1}{F}\left(\frac{M_x'}{M_E}\right)^2 + \frac{P}{P_E} = 1$. He had been interested to read that equation, because 10 years ago he had looked into those problems and had come to the same equation, and in 1948 he had presented a Paper in which that was mentioned.[33]

First, that equation was an approximation. For the case of a column with a constant moment in the column it was exact, but for any other moment distribution it was not, but it was on the safe side. The equation was based on the deformation due to buckling from moments only being roughly the same as the Euler deformation, and that was approximately the case when the moment distribution was symmetrical. In the case of antisymmetrical moment distribution, however, it no longer held good.

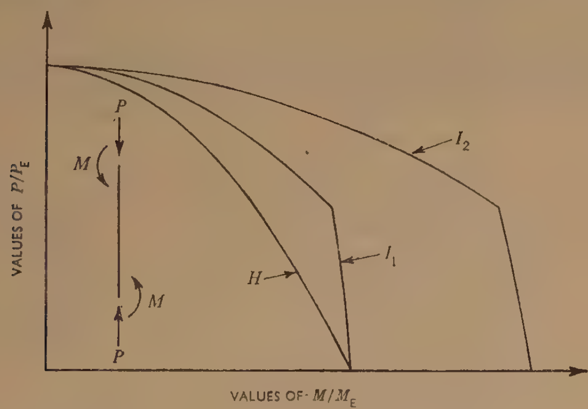
Starting with the same moment applied top and bottom turning the same way, but with no load on the column, the angle of twist would increase from zero at one end to its maximum half way and then down again to zero at the other end, i.e., describing a symmetrical curve. But since $EI_y\frac{d^2y}{ds^2} = \frac{I_x - I_y}{I_x} M\beta$ where y denoted sideways deflexion, M the moments, and β the twist; a symmetrical twist curve × antisymmetrical moments must produce antisymmetrical deflexion, i.e., the strut would deflect in an S-shape and the corresponding critical column load would be four times the Euler load and the basic equation would become $\left(0{\cdot}391\frac{M}{M_E}\right)^2 + \frac{P}{4P_E} = 1$.

If, on the other hand, one started with a column load but no moments, that would produce a symmetrical deflexion and consequently an antisymmetrical distribution of the angle of twist, i.e., the angle of twist would be zero halfway and the beam, as far as buckling from moments, would act as if consisting of two beams of half length each with triangular moment distribution, i.e., according to Table 1 on p. 124 the corresponding buckling moment became $M = \frac{2}{0{\cdot}565}M_E = 3{\cdot}55M_E$ and the basic equation turned into $\left(0{\cdot}282\frac{M}{M_E}\right)^2 + \frac{P}{P_E} = 1$. Fig. 31 showed graphically the various basic equations—in thin line the Author's equation, and in thick line the combined effect of the two abovementioned equations.

The Figure illustrated the large margin available at certain load combinations. It would be most valuable if the series of experiments now carried out at Cambridge could be made to include tests to check that curve and even more the intermediate cases, where the top and bottom movements were not equal, and where the analytical investigation was considerably more complicated.

Obviously simplicity must not be lost sight of if a general workable solution was to emerge, but there were two more factors, which contained a considerable further margin in certain cases and which could easily be included without spoiling the simple picture.

According to the Paper, $M_E{}^2 = \frac{\pi^2}{l^2}(EI_y)(GK)$ or $M_1 = \frac{k}{l}\sqrt{EI_yGK}$ in which $k = \pi\sqrt{F}$ where M_1 denoted the critical moment for $P = 0$. Now, in the literature on the subject, in the first instance in 1899 independently by Michell and Prandtl, that equation was invariably derived by investigating a narrow rectangular section where I_y was very small compared with I_x. A close investigation would show that a factor $\frac{I_x}{I_x - I_y}$ should be added to the expression for M_1 which checked with the fact, that for $I_x = I_y$ it was impossible to make a beam fail by buckling sideways. For an I-section, that factor might be quite considerablė, e.g., a 10 in. × 8 in. R.S.J. would have added as much as 23½%.



Basic equation
H according to Horne
I_1 according to Ingerslev
I_2 according to Ingerslev, but taking in resistance to warping and factor $\frac{I_x}{I_x - I_y}$

FIG. 31

Further, it was stated in the Paper that the resistance to warping was neglected, but he felt that that could be taken into account fairly easily, and by indicating the point of warping it might be possible to improve by simple means the capacity of the column very considerably. Given a beam and column subject to a constant moment, he would represent the factor of stiffness to warping by $\alpha^2 = \frac{\frac{1}{2}Dh^2}{Cl^2}$, where D denoted the stiffness of one flange, h the height of the joist, C the torsional rigidity, and l the length of the column. It would then be found that the expression for M_1 should have yet another factor, $\sqrt{1 + (\pi\alpha)^2}$. When the load varied between two equal moments and two opposite moments the factor, which for a constant moment was π, would vary within the limits of 4 and 2·9, i.e., it was reasonably constant. D was almost identical with half the rigidity of the column, so that it was possible to write $\sqrt{1 + 0{\cdot}85\frac{EI_yh^2}{4GKl^2}}$. That could be written to a fairly accurate approximation and in a form easy to use as $\sqrt{1 + \frac{3}{4}\left(\frac{hb}{lt}\right)^2}$, where h,

again, denoted the height of the section, b the width, l the column length, and t the thickness of the flange. That was a simple extra factor to put on. He had worked it out for a 10 in. × 8 in. × 55 lb.-joist with a column length of 9 ft, and it gave another 30% strength to the column. Taking in the two extra factors, they added up to 60%, so that quite a considerable amount could be saved by adding those fairly simple factors to the basic moment.

The full expression for M would therefore be:

$$M_1 = \frac{\pi\sqrt{F}}{L}\sqrt{EI_yGK} \times \frac{I_x}{I_x - I_y} \times \sqrt{1 + \frac{3}{4}\left(\frac{hb}{lt}\right)^2}$$

and the basic equation $\left(\frac{M}{M_1}\right)^2 + \frac{P}{P'} = 1$ in which either $\pi\sqrt{F} = 8{\cdot}03$ and $P_1 = 4P_E$ or $\pi\sqrt{F} = 11{\cdot}12$ and $P_1 = P_E$.

Now the last term in the formula for M_1 was based on free rotation of the flanges at the ends. In actual fact there would be a certain restraint, and that made it possible by simple means to increase the resistance to warping considerably.

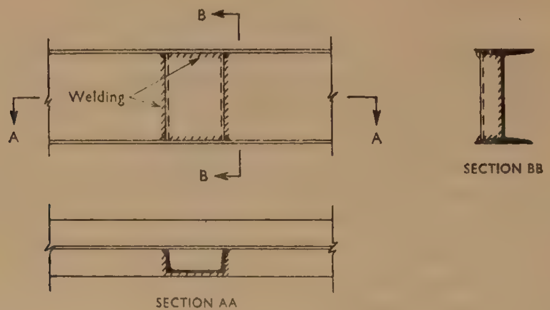


FIG. 32

Mr Ingerslev drew attention to a means of stiffening up the torsional rigidity published in Stockholm by Mr Nylander.[34] Taking an I-section, what Mr Nylander suggested was to weld in a channel between the flanges at intervals along the web (Fig. 32). If that was done the additional stiffness was enormous, as the box formed by the channel and the web prevented relative twist between the flanges, and it was obtained at very little cost.

Mr H. S. L. Harris (Lecturer in Engineering, University of Cambridge) said that when designing a structure by B.S. 449, any restraint on the beams from the stanchion was effectively being ignored. That had been shown to be quite inaccurate so far as the resulting moments coming into the stanchions were concerned, but led to the use of relatively slender stanchions. Would the Author go a little further in discussing his reasons for rejecting the stiff-beam-whippy-column approach and taking in its place the failing-beam-elastic-column condition. Anyone who had not put a great deal of work into understanding plastic design methods was bound to compare the difficulty of a proper analysis of the collapse of a stanchion with the very simple approach in B.S. 449. Mr Harris wondered whether it was possible that, of the many frames designed to B.S. 449, a few in which the joints between beams and stanchions happened to be strong had fortuitously been designed so that their collapse loads were approximately the same as if the structure had been designed on a collapse basis, but in the opposite mode to that set out in the Paper. Had the Author analysed any B.S. 449 frames, with strong joints, to find their true load factor?

A second point concerned elastic restraints coming into one side of a stanchion when he other side of that stanchion was subjected to a known moment, i.e., the beam coming nto the other side of the stanchion had become fully plastic. He was not clear what one hould assume as the least moment that might arise on the side of the stanchion remote rom the beam which had reached its fully plastic moment. Would the Author say a little nore on that subject?

Mr E. Goodwin (Experimental Officer, Building Research Station) said he had been particularly impressed by the way in which the Author had reduced his complicated nformation to simple charts. Combined flexural-torsional buckling had been given onsiderable prominence in the Paper, and it was a new feature to crop up in building lesign. Fig. 23 showed that in the particular case represented there the new method llowed far lower bending stresses than had been recommended previously, and that was ecause torsional effects had been brought into consideration, whereas before they had een ignored. The case represented in Fig. 23 happened to be a bad case from the orsional point of view.

It seemed to him that a general impression was being created that torsional effects were ikely to exert considerable influence on advanced design methods generally, but he would oint out that that was not necessarily so. At the Building Research Station they were ery interested in torsional effects and had been studying the extra stresses involved with he aid of their differential analyser. That machine was a mechanical analogue computer, nd it was very useful in studying the question of torsional-flexural buckling, because the asic equations were capable of analytical solution only in the simple case when the tanchion was pin-ended. With their machine they had studied the more complicated ases where it had a partial restraint at the ends. The extra stresses which came into the icture were due to the warping of the cross-sections and to increased bending about the veak axis. The extra stresses were much reduced if the stanchion was encastered by lastic restraints at the ends. In the particular case shown in Fig. 23, he had been able o show that at a slenderness ratio of 200, if the stanchion were encastered and prevented rom warping at the ends, the allowable bending stresses could be raised from $4\frac{1}{2}$ to 1 tons/sq. in.

With regard to the degree of restraint available at the ends of a stanchion in a rigid frame, t seemed to him likely that, with regard to bending about the weak axis, it was likely o be very nearly encastered, particularly in the case of the slenderer sections which gave rouble in that respect. With regard to restraint on warping, there seemed to be no efinite evidence on which to base an assessment, and that was something on which they vould very much like information. Mr Ingerslev had given them a very useful line to ollow there, and Mr Goodwin thought that if extra stiffeners of any kind could be mployed to prevent end warping they would be very well worthwhile.

In conclusion, it should be emphasized that the beneficial effects of end restraint applied nly to design methods or design cases where the adjoining beams had not been allowed o go wholly plastic.

*** **Mr L. G. Johnson** said that Dr Horne was to be congratulated on producing a tanchion design method with such a logical foundation and which had clearly involved tremendous amount of work. His Paper represented a real and important step forward the design of steel structures, especially as the day was rapidly approaching when ulti-storey buildings would no longer be able to rely on panel walls to resist wind forces nd ideas on beam and stanchion design would have to be revised accordingly.

The design method described in Part 2 of the Paper was used to determine the sections f the stanchions in a new 17-storey block of flats having a height-to-base ratio of about 5:1. Walls and partitions were too flimsy to stiffen the structure against the action of ind and so the framing was designed as fully rigid about the major axes of the stanchions,

*** This contribution was received in writing upon the closure of the oral discussion.—EC.

and bracing was provided along the minor axes, i.e., conditions which corresponded to those set-out in section (10) (Case $P_x . O_y$).

The following facts came to light as a result of the analysis. Axial stress p was constant for a given stanchion length and therefore $p + f_x'' < 15{\cdot}25$ did not have to be checked because $f_x' > f_x''$ by definition.

It was noted that $\frac{l}{r_x}$ was always appreciably greater than $40/\sqrt{p}$ and therefore equation (31) did not hold for that particular structure. The function N_x was obtained from Fig. 14 much more quickly than from the expression given by equation (27) and its value varied between 1·05 and 1·60, but for normal storey heights of 9 ft it did not exceed 1·10.

Partly owing to the fact that the ratio of live load to dead load was of the order of 1:4 and partly because wind forces produced double curvature bending, in no case was a stanchion length found to be bent in single curvature. That meant generally that $1/\sqrt{F}$ remained at approximately 0·4. T was almost invariably greater than 100 and, since bending stresses were smaller than the axial stresses, f_x was usually quite small and f_x^2/T could be neglected as it never exceeded 0·2. It was sufficiently accurate therefore to assume that $p' = p$.

In all but one or two isolated instances, maximum end stress was the criterion for design and even in the odd cases when stability was critical, the combined stresses were always greater than 14·6 tons/sq. in.

The value of f was never less than 13·5 and usually $> 14{\cdot}0$.

The actual design of one 17-storey stanchion, after all moments and loads had been tabulated, was achieved in about one day by one man, which speed could obviously be greatly increased when the designer became more familiar with the method.

All those facts seemed to indicate that that logical design method could be applied successfully to multi-storey structures and it soon became apparent to the designer that the calculations involved were by no means difficult; numerous short cuts simplified the method and reduced the amount of labour involved.

The Author, in reply, first dealt with the remarks concerned entirely with elastic stability, since that was an aspect which could be dealt with in isolation. Mr Ingerslev had discussed equation (4) at some length. It was not claimed as exact by the Author, who had elsewhere[26] dealt fully with the problem, comparing his own exact results, which were in agreement with results also obtained by Salvadori,[27] with equation (4), and showing conclusively that it invariably gave results on the safe side. All the factors mentioned by Mr Ingerslev increased the instability load, but Mr Ingerslev had omitted a further factor, known as the Wagner effect, which decreased that load. To take account of all the factors involved at all ratios of end moments would be impracticable. Mr Ingerslev's final equation did not take account of the Wagner effect; it only applied (after an important correction, to be dealt with later) to symmetrical and antisymmetrical end moments; and, with antisymmetrical moments, it was necessary to try the two conditions $\pi\sqrt{F} = 8{\cdot}03$ and $P_1 = 4P_E$ or $\pi\sqrt{F} = 11{\cdot}12$ and $P_1 = P_E$ to find which gave the smaller answer. Furthermore, the derivation of the elastic critical load was but one step in the formulation of a design method; the idea that it might be possible to introduce all those refinements into a method which sought to limit the maximum stress occurring anywhere in an initially imperfect member to the yield value was, the Author considered, quite unrealistic.

Mr Ingerslev had given the wrong solution for the effect of flexure about the major axis on the critical load. The factor $\frac{I_x}{I_x - I_y}$ in his final equation should be replaced by $\sqrt{\frac{I_x}{I_x - I_y}}$. Considering the 10-in. × 8-in. R.S.J. quoted by Mr Ingerslev, the increase in critical moment became 11% in place of 23½%. Even the figure of 11% was misleading, since flexural-torsional instability only became of importance in steel members when one moment of inertia was many times the other. Thus a 12-in. × 5-in. R.S.J.

howed an increase in critical moment as a result of that factor of only 2·3%, whilst a very lender member such as a 20-in. × 6½-in. R.S.J. showed an increase of only 1·4%. In ome cases it might even give unsafe results if the term under discussion were taken into ccount. It arose because of the curvature which existed about the major axis just prior o buckling; if that curvature were offset by pre-cambering, as was sometimes done, then he term could not be admitted.

The neglect of warping resistance by the Author, discussed by Mr Ingerslev, was of reater potential importance. If, however, warping resistance were taken into account, t might be necessary to allow also for the Wagner effect. The Author had shown[26] that n practical sections resistance to warping would always at least compensate the Wagner ffect, and it was therefore a very convenient simplification to neglect both. Moreover, f one did actually allow for warping rigidity in the calculation of the critical load for nstability, then logically one ought also to take account of the greater increase in extreme bre stresses in the compression flange as compared with the tension flange. The net esult would probably be that the benefit of taking warping rigidity into account was not ery great. The Author felt that the design method presented in the Paper was perhaps oo complicated, even as it stood, to be used widely in practice and it might still have to e simplified, although he hoped that the attempt to use it would be made. While, herefore, it was true that some of the effects which Mr Ingerslev had mentioned would be dvantageous if taken into account, it was impracticable, in the Author's view, to introuce them at a stage when one was trying to see one's way through a forest of difficult roblems towards a practicable design method. In the sifting process, as the attempt as made to apply the results, he thought that those additional effects might well be aken into account, perhaps in a semi-empirical and not directly obvious manner.

The Author agreed with Mr Ingerslev that the use of battens or channels to prevent arping could significantly increase the critical load. He ventured to doubt, however, hether the benefit was necessarily so great in relation to the cost as Mr Ingerslev seemed o indicate.

The Author certainly agreed with Mr Goodwin that it was important to take account f every restraint condition that could conveniently be obtained, particularly end restraints n warping. He had stated in the Paper that the case P_xE_y was of considerable imortance, and restraint against warping at the ends was one reason why that was so. In ich cases torsional failure would not be as important as might appear from the present aper. The Author had, however, placed great emphasis on torsional failure because of s observation of unclad structures which had collapsed. In full-scale tests and in tests n small-scale stanchions the potential importance of torsional failure had been evident, nd although it might ultimately be possible to return to a method in which it did not ave to be taken into account, it was unwise to neglect it in the intermediate stages of an tempt to produce an entirely new design method. In the past the chief factor which had nabled torsional failure to be neglected was the encasing of the stanchions by material fficiently rigid to prevent twisting. Where that was still done there was probably more ope of achieving economy by the use of composite action theory[35, 36] than by means of e methods suggested in the Paper. There was, however, a growing tendency to use fire sing and floor systems which contributed only slightly if at all to the strength of the ructure. Under such circumstances it might be found that existing codes were in some ses dangerous, and torsional failure ought certainly to be considered in research devoted the production of more rational and dependable design methods. That fact had been lly confirmed in some recent tests carried out in the United States.[37]

Dr Wood had quite rightly raised the question of the most critical load arrangements. he closely related problem of the safe minimum value to assume for the moment of sistance of the beam on the side of the stanchion opposite to the fully plastic beam had en raised by Mr Harris. Considering the stanchion length BE in Fig. 33a, the critical ading condition was not likely to be reached with full live loading on all four beams, but ther with conditions which led either to the most severe case of double curvature, as in g. 33b, or the most severe case of single curvature, as in Fig. 33c. The beams carrying

dead load only would probably be elastic for their entire length, but in some cases the moments of resistance in those beams at B and E might exceed the yield moment, thus reducing very considerably the bending moments applied to the stanchion length BE by the other beams. How could one hope to arrive at a simple method of assessing the moments of resistance of the beams carrying dead load only, when it was realized that they must depend on the elastic stiffnesses of the beams and stanchions in the vicinity of the joints under consideration? One might be tempted to despair if one thought of the great labour entailed in the work of the Steel Structures Research Committee, and in the methods of analysis developed by Wood,[7, 9] directed towards the solution of that same problem for entirely elastic structures. Might it not therefore be necessary to abandon the ideal of a rational and reasonably simple design method, even with the assistance of the plastic theory? The Author did not think so, for the following reasons.

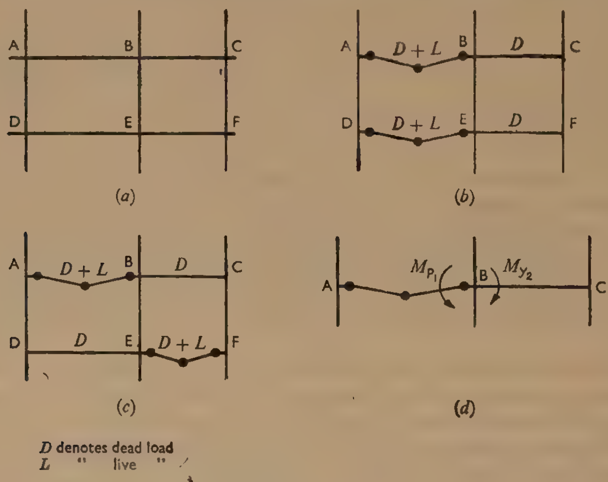


FIG. 33

In the tests[22] carried out on the $E_x \cdot E_y$ case (Fig. 8) the major axis beams applied large bending moments to the stanchion in the early stages of the test. As soon as the stanchion yielded at all extensively, however, and before it was really in its collapse condition, those bending moments were relaxed away completely and did not appear to affect at all radically the collapse load of the stanchion. Similar behaviour might well occur in the beam systems ABC and DEF in relation to the stanchion length BE in Figs 33b and c. The two beams AB and BC might be replaced, so far as their effect on the stanchion length was concerned, by a suitably loaded single elastic beam, provided one of the actual members AB or BC was just on the point of collapse, the other remaining entirely elastic. Hence, when considering any joint B, as in Fig. 33d, the maximum out-of-balance beam moment which had to be taken into account was $M_{p_1} - M_{y_2}$, where M_{p_1} was the full plastic moment of AB, assumed to be the larger member, and M_{y_2} was the yield moment of the smaller member BC. That result followed since any larger out-of-balance moment would be relaxed away before the stanchion reached its final state of incipient collapse. Further work was required to establish that approach, and it might be necessary to impose limits for its application depending on the relative elastic rigidities of the members concerned. It did, however, represent a promising line of attack on that difficult problem.

The Author was interested to see the stanchion curves presented by Dr Wood in Figs 27 and 28, and to receive Dr Wood's confirmation that, with fully plastic beams, there was

ittle likelihood of obtaining anything superior to elastic design for the stanchions. Curves similar to those in Fig. 27 had been obtained by the Author himself, and a description of their application was to be published shortly.[38] The Author agreed with Dr Wood that the question of frame stability was extremely important. It could fairly be said that no existing design methods really tackled that aspect and effectively depended for overall rigidity on the cladding. With the advent of tall lightly clad buildings the necessity of finding a solution to the frame-stability problem was becoming acute and remained a challenge to those concerned with structural research.

Mr Harris had asked why the Author had dismissed as unpromising the stiff-beam-whippy-column approach. The Author would not say that he had rejected it altogether but it had the disadvantage that it was rather intractable as a design method. In order to sustain the stanchions the beams would be continuous and elastic and the collapse loads of the stanchions would depend on the rigidities of the beams. The design of the stanchion itself influenced the bending moments which it exerted on the beam system at the point of collapse, so that there was a complex interaction between the beam design and the design of the stanchions. That made the problem intractable—it led, in fact, to a process of double trial and error. It was necessary to assume stanchion sections and the ratio of beam-to-stanchion stiffnesses, to check the adequacy of the assumed stanchion sections, to calculate suitable beam sizes to take the bending moments applied to them by the stanchions in addition to the usual floor loads, and finally to recalculate the stiffness ratios. The calculations had to be repeated where the actual ratio of beam-to-stanchion stiffness was less than that previously assumed. In the cases which the Author had considered, no great economy resulted from the process as compared with the design obtained by the direct application of the simple design method according to B.S.449, and there was thus no great incentive to pursue that approach further. So far as the Author could tell, a frame designed according to the simple method and then constructed with rigid joints would at collapse fall into the category $E_x . E_y$ for inside stanchions, into one of the categories $O_x . E_y$ or $E_x . O_y$ for outside stanchions, and into category $O_x . O_y$ for stanchions at outside corners, although the classification would be approximate only. It was probable that some of the beams—particularly those in outside bays—would be partially plastic at collapse, and the nominal B.S.449 design would not be so extreme in sacrificing beam economy in the interest of economical stanchions as would the $E_x . E_y$ approach mentioned in the Paper. The partial plasticity of the beams in a nominal B.S.449 design would make a collapse analysis extremely difficult, and it had not hitherto been attempted by the Author.

Dr Heyman had raised a problem in connexion with the use of the stanchion curves which was related to the question of what was the most critical loading condition. The beam-column interaction in his example would necessarily depend on the relative rigidities of the members up to the collapse load of the columns, but he was inclined to agree with Dr Heyman that the condition at collapse could be argued from other considerations. Dr Heyman's approach, in which he suggested that the effective eccentricity was that for which the critical load, calculated for yield at the end, coincided with that calculated on the basis of the instability behaviour of the member, was very convincing, and seemed likely to be the correct solution.

Dr Heyman had remarked that he did regard the Paper as presenting a design method for multi-storey frames. Whilst the Author hoped that, in a sense, that was so, he wished to emphasize that he had not sought to establish a well-defined method such as would be expected, for example, in a Code of Practice. The Paper could only have real value if attempts were made to apply it and if the experience so obtained were used both to simplify the process and to expand its scope to take account of factors not previously covered. It was the Author's hope that that might be done.

The Author was very grateful to Mr Johnson for placing on record his experience in the use of the stanchion design curves. He found that, in practice, f always exceeded 13·5 tons/sq. in. Should that prove to be a rule only very rarely broken, it might be profitable to replot the curves in Fig. 13 to give a wider spacing in that region. Mr Johnson was

quite correct in pointing out the redundancy of the inequality $p + f_x'' \leqslant 15\cdot25$ in the conditions (30) which governed the design when bending was about the major axis only. Mr Johnson's remarks concerning the non-appearance of single-curvature bending and his experience that, in almost all cases, the critical conditions for the stanchions in his problem were for end stress and not instability were of particular interest. Used under such circumstances the design curves would certainly show considerable economies as compared with the requirements of B.S.449, as might be seen from Figs 22 and 25. There would therefore appear to be some incentive for using the curves in the Paper, in conjunction with a suitable load factor, in elastic design methods for rigid frames, with the prospect of considerable economies without excessive complication of the design process.

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The closing date for Correspondence on the foregoing Paper was 15 March, 1956. No contribution received after that date will be printed in the Proceedings.—Sec.

WORKS CONSTRUCTION DIVISION MEETING

20 December, 1955

Mr A. C. Hartley, Vice-President, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Author.

Works Construction Paper No. 30

THE DIVERSION OF THE ANNALONG RIVER INTO THE SILENT VALLEY RESERVOIR

by

* Cyril Frank Colebrook, T.D., Ph.D., B.Sc.(Eng.), M.I.C.E.

SYNOPSIS

The Paper describes the third phase of development of the Belfast Water Commissioners' Mourne Scheme, by means of which a further 4½ m.g.d. is made available.

Records of rainfall and run-off have been analysed and an estimate made of the reliable yield of the catchment area.

The works described comprise the main and secondary river intakes and a tunnel, 2¼ miles long, for the diversion of the Annalong River into the adjacent Silent Valley reservoir.

An extensive fissure was encountered in the tunnel which caused a delay of 7 months in one heading, and the measures taken to consolidate the fissure by high-pressure grouting in order to effect a passage are described in detail.

INTRODUCTION

THE City of Belfast derives its water from a number of sources, the largest of which is in the Mourne Mountains, an important range in the south-east corner of Northern Ireland. The Mourne source comprises the catchments of two adjacent valleys in the southern section of the Mourne Mountains through which flow the Kilkeel and Annalong Rivers.

The first phase of development of this source was completed in 1901 with the construction of intakes on both rivers to divert the flow of these rivers into a conduit, about 35 miles in length, which terminates in a service reservoir of 100 m.g. capacity situated on the outskirts of the city. The conduit was constructed in tunnel or cut and cover for about two-thirds of its length and this portion has a capacity of 30 m.g.d. The remainder of the conduit is in steel and cast-iron siphon pipes, only one of which was laid initially, with a capacity of 10 m.g.d.

The second phase of the scheme involved the construction of a 3,000 m.g. reservoir in the Silent Valley[1] and the duplication of the siphons in the conduit. The construction of the dam started in 1923 and despite exceptional constructional difficulties the work was completed about 9 years later. The completion of this work increased the maintainable yield from 7 to about 16½ m.g.d.

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* The Author is a Senior Engineer, Messrs Binnie, Deacon, & Gourley.

[1] The references are given on p. 173.

The rapid growth of the City of Belfast and its continued demand for more water could only be met by still further development of the Mourne catchment area. The flow in the Annalong river, since the inception of the scheme, had been diverted into the conduit by means of a weir, but under this arrangement it was not possible to maintain an adequate dry weather flow and a considerable portion of the yield of the Annalong catchment area was lost. The construction of a reservoir in this valley would provide the storage necessary to even out the flows, and borings were put down at what appeared to be an excellent site for a dam in the lower part of the valley. These borings proved that the bed rock is at a considerable depth and that the valley is filled with glacial deposits consisting of silt, running sand, gravel, and boulders. The geological formation is, in fact, the same as that in the adjacent Silent Valley and it was evident that difficulties similar to those encountered in the construction of the Silent Valley dam would have to be overcome in order to construct a dam in the Annalong Valley.

In view of the difficulty and expense likely to be involved in the construction of a dam in the Annalong Valley, an alternative scheme was adopted which would necessitate the diversion of the Annalong river into the Silent Valley reservoir by means of a tunnel, the duplication of the conduit from the Silent Valley to the Annalong Valley, the installation of rotary screens in the Silent Valley, and the provision of additional storage by means of a reservoir to be constructed near the Salmon Leap in the upper reaches of the Silent Valley. Work on the duplicate conduit and screens was completed in 1950 and in 1952, about 5 years after the commencement of the work, the intakes in the Annalong Valley and the tunnel into the Silent Valley reservoir were brought into use, thus adding a further 4½ m.g.d., to give a total of about 21 m.g.d. The works completed to date are shown in Fig. 1, Plate 1.

Estimation of yield

The area of the Silent Valley catchment is 5,500 acres whilst the Annalong Valley catchment area is 3,500 acres in extent, although about 750 acres of this area is below the main intake and a proportion of the run-off from this lower area is at present diverted into the conduit at the old weir.

Rainfall

Rain gauges are located near the entrances to the Silent Valley and Annalong Valley catchment areas at 457·0 and 435·0 O.D. respectively, and in the mountains at Lough Shannagh (1,280·0), Slieve Lamagan (1,320·0), and Slieve Bearnagh (1,954·0). Records of annual rainfall are given in Table 1.

Table 1

Gauge	Period of rainfall records	No. of years	Mean rainfall: in.
S.V. entrance	1930–1949	20	53·22
S.V. Lough Shannagh	1933–1949	17	70·73
	1940–1949	10	72·12
S.V. Slieve Bearnagh	1940–1949	10	73·82
A.V. entrance	1900–1949	50	49·08
	1940–1949	10	49·11
A.V. Slieve Lamagan	1940–1949	10	71·86

The mountain gauges at Slieve Lamagan and Lough Shannagh are at the mean levels of the Annalong and Silent Valley catchment areas respectively.

The Mourne catchment area is at about the same mean level (average 1,300·0 O.D.) as the Elan catchment area (average 1,350·0 O.D.) and the valuable records of rainfall on the latter area extending back to 1887, which were published in 1954 by Risbridger and Godfrey,[2] may be used with confidence in making an assessment of the probable long-average rainfall on the Mourne area, in view of the similarity of the ratios of the average rainfalls for the periods 1933–1949 and 1900–1949 on the Elan catchment area and at the Annalong Valley entrance gauge (97·7% and 98·8% respectively).

The ratio of the average rainfalls for the period 1933–1949 and the standard period from 1887 to 1930 for the Elan area is 99·4% and the probable long-average rainfall at the Lough Shannagh gauge in the Silent Valley is estimated at

$$\frac{100}{99{\cdot}4} \times 70{\cdot}73 = 71{\cdot}1 \text{ in.}$$

The probable long-average rainfall at the Slieve Lamagan gauge in the Annalong Valley may be estimated by comparing the rainfall during the 10-year-period 1940–1949 at this gauge with that at the Lough Shannagh gauge. This gives a probable long-average rainfall of

$$\frac{71{\cdot}86}{72{\cdot}12} \times 71{\cdot}1 = 70{\cdot}8 \text{ in.}$$

The probable long-average rainfall for the combined catchment areas is, therefore, 71·0 in.

Evaporation and absorption

The average losses due to evaporation and absorption during the 4 years 1934 to 1937 in the Elan and Mourne catchment areas were 21 in. and 13·7 in., and the mean rainfalls during this period were 70·5 in. and 70·2 in. respectively. The long-average loss on the Elan area for a long-average rainfall of 69·4 in. is 21·6 in. and the probable long-average loss on the Mourne area is estimated to be 14·0 in.

It is reasonable to assume that the relation between loss and annual rainfall on the Mourne area can be expressed by a curve similar to Fig. 17 of Risbridger and Godfrey's Paper,[2] and the probable losses in the Mourne area for the two and three driest consecutive year periods are estimated to be 11 in. and 12 in. respectively.

Run-off

The average rainfall during the two driest consecutive years, as recorded at the Annalong Valley entrance gauge, was 38·9 in. This is 79·2% of the 50-year mean at this gauge and it is reasonable, therefore, to assume that the mean rainfall averages of 79·4% and 84·7% of the long-average rainfall determined for the Elan catchment during the two and three driest consecutive years respectively will apply to the Mourne area. Glasspoole's analysis[3] of the rainfall records covering the years 1870–1943 for a gauge at Seaforde, Co. Down, which is only 15 miles from the catchment area, shows that the corresponding averages for this gauge are as high as 81% and 86·7%. The probable run-off during these periods may thus be estimated as shown in Table 2.

The actual measured run-off in the Annalong catchment area during 1921 and 1922, the two driest consecutive years of the period 1900–1949, was 47·3 in. This is rather

TABLE 2

	Two driest consecutive years	Three driest consecutive years
Probable long-average rainfall: 71·0 in.		
Probable average rainfall: in.	56·3	60·0
Probable loss: in.	11·0	12·0
Probable run-off: in.	45·3	48·0

higher than the estimated run-off given in Table 2 and is probably due to a slightly higher rainfall than the estimated 56·3 in.

The reliable run-off of the Mourne area is estimated to be not less than 48·0 in. or 5·5 cusecs per 1,000 acres, and the reliable yields of the catchment areas are as shown in Table 3.

TABLE 3

Catchment area	Area: acres	Reliable run-off: cusecs/1,000 acres	Total run-off: cusecs	Total run-off: m.g.d.
Silent Valley	5,500	5·5	30·2	16·3
Annalong Valley (above tunnel intake) . .	2,750	5·5	14·6	7·9
	Total through screens in Silent Valley . . .			24·2
Annalong Valley (below tunnel intake) . .	750	5·5	3·3	1·8
	Total for catchments .			26·0

The estimated yields have been calculated on the following basis:—

(*a*) The tunnel diverts up to 97% of the total yearly run-off from the area above the intake in the Annalong Valley.

(*b*) The run-off below the intake in the Annalong Valley is diverted into the conduit at the old intake. To divert 80% of the total yearly run-off all flows up to 5 m.g.d. must be taken into the conduit and it will be necessary to vary the flow into the conduit at the Silent Valley accordingly.

On the completion of the works described in the Paper more than 24·0 m.g.d. became available in the Silent Valley, but owing to insufficient reservoir capacity in that valley at present the maintainable yield is reduced to about 21 m.g.d. When the dam now under construction in the Upper Silent Valley has been completed the full yield of the catchment areas will then be utilized.

Geological considerations

The geological formation of the Mourne Mountains may be briefly described as a multiple intrusion of granite magma into hard Silurian shales. This extensive intrusion covers an area of 55 sq. miles and is the largest outcrop of any Tertiary granite in the British Isles (Fig. 2, Plate 1).

There are three varieties of granite in the Eastern Mournes which, according to Richey,[4] were intruded separately under great load through a ring cavity which was formed when a large Silurian block foundered into a magma reservoir probably far below the present surface. The initial subsidence allowed feldspathic granite to flow over the top of the block and under the shale roof which remained intact and this was followed by two further injections, each below the other, which cooled and crystallized in the depth beneath the land surface of that time to form quartzose and aplitic granite.

A fourth granite, part of it still overlain by shale roof, forms the Western Mournes. The shale roof over the Eastern Mournes and much of the oldest feldspathic granite have been almost completely removed by denudation and a number of mountain ridges and valleys, including the Silent and Annalong Valleys, running roughly north and south have been carved out of the granite intrusions (Fig. 3, Plate 2).

The granite area of the Mournes is traversed by a number of andesitic and basaltic dykes which penetrated through the granite and surrounding Silurian rocks along steep planes of fracture, mainly north-east to south-west in direction.

Pari passu with the crystallization of the granite magma, fugitive constituents, mainly water and water vapour, were released. The hot, probably acid, waters became concentrated especially along planes of fracture such as joints in the consolidated granite, and these acted chemically upon the feldspar of the granite adjacent to the fissures. This led to disintegration of the rock itself in the vicinity of the fissures, and where this was complete it resulted in the formation of pink kaolin (clay). Thus, spots of pink kaolin are common throughout the Mourne granites.

During the tunnelling operations which were carried out in connexion with the present scheme, numerous bands of partly or completely decomposed rock were passed through. These bands of decomposed rock varied from a few inches up to about 80 ft in thickness and they were much more numerous in the end sections of the tunnel where the depth below the surface was least. In the first 800 yd of the inlet and outlet headings, where the cover did not exceed about 500 ft, sections requiring support totalled 80 yd and 120 yd respectively (about 50% of these sections were ribbed as the face advanced) which averages $12\frac{1}{2}$% of these lengths. In the centre length of 2,245 yd, where the cover reached a maximum of 2,000 ft, only 80 yd or $3\frac{1}{2}$% of the length (including more than 30 yd at an extensive fissure which is described in detail later) required support. The positions of the bands of decomposed rock which required support are shown in Fig. 4, Plate 2.

It was observed that about 75% of the bands of decomposed rock had a general direction approximately north-east to south-west (about 45° in plan to the centre-line of the tunnel), whilst the majority of the remainder were approximately at right-angles to this direction.

The diversion scheme

General description

The main intake on the Annalong river was constructed at the confluence of two streams; the main stream flows from the north in a deep valley to the west of

Slieve Donard, the highest mountain in the Mournes, whilst the smaller stream drains the Blue Lough and Bignian Lough on the higher slopes of Slieve Bignian. A secondary intake is necessary to divert into the main river the portion of the catchment area fringing the eastern boundary (Fig. 1, Plate 1).

The intakes are located about 1 mile above the old weir intake in the Annalong Valley to ensure sufficient fall in the tunnel between the main intake and the top water level of 500 O.D. in the Silent Valley reservoir, and before construction of the intakes and the tunnel could be started it was necessary to construct a road to provide access to the works. The outlet end of the tunnel discharges into an existing culvert of 6-ft 6-in. dia. under the road running alongside the Silent Valley reservoir.

The main and secondary intakes

The main intake in the Annalong Valley serves to divert the river flow into the tunnel, to form a stilling pool for settling out grit, and to provide an overflow weir for the discharge of flood water into the old river course.

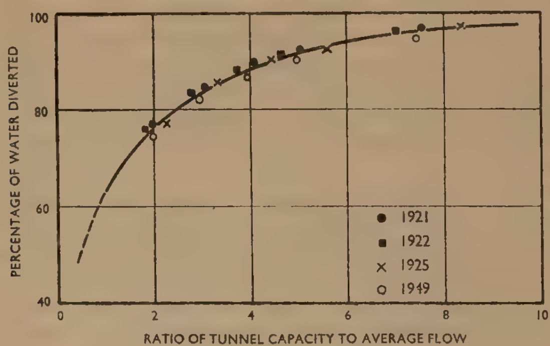


FIG. 5.

The intake is constructed of mass-concrete walls faced with granite blocks built to block-and-sneck pattern, whilst the storm overflow weir and the two bridges which give access from one side of the intake to the other are of normal mass and reinforced concrete construction. The masonry facing to the walls gives an exceptionally fine appearance to the works and is in keeping with the magnificent stonework used in the construction of the earlier works in the Silent Valley, which are of such great interest to the thousands of people who visit the waterworks each year.

The water passes from the intake into the tunnel through two orifices, each 6 ft 3 in. × 11 in. These measure and limit the flow into the tunnel to 75 m.g.d., which is about nine times the estimated average run-off during the three driest consecutive years. An analysis of the run-off in the Annalong Valley catchment area during the years 1921, 1922, 1925, and 1949 shows that a tunnel of this capacity would divert about 97% of the total yearly run-off (Fig. 5).

The sill levels of the orifices are 4 ft 0 in. above the floor of the intake and 3 ft 0 in. below the storm-overflow weir level. The orifices are shown in Fig. 6, which also gives a general view of the intake in the vicinity of the tunnel.

The storm overflow has five bays, each 14 ft 9 in. long, which will discharge a flood of 1,300 m.g.d. (900 cusecs per 1,000 acres) before the water level rises to the top of the bank. The highest recorded flood since 1900 is 640 m.g.d. and there is, therefore, a considerable margin before overtopping of the intake occurs. In addition, the intake is founded on rock so that should a flood of catastrophic magnitude overtop the works, the intake will not suffer any damage. Views of the intake looking upstream and downstream are shown in Figs 7 and 8.

A secondary intake was constructed on a stream near the eastern boundary of the catchment area which is similar in design to the main intake, although it is very much smaller having been designed to divert a maximum of only 16 m.g.d. into the main river. All excess flood water is discharged over the storm-overflow weir. The intake is founded on rock and is entirely of mass and reinforced concrete construction, the masonry facing to the walls having been omitted in this case.

The tunnel

The tunnel for diverting the run-off in the Annalong catchment area into the Silent Valley reservoir is about 3,845 yd in length and has a fall of 17½ ft from end to end. It is of horseshoe shape with an average pay width for excavation of 7 ft 6 in. below the springing level, which is 4 ft 0 in. above the finished concrete invert level at the side walls. The arched roof was excavated to a crown level of at least 3 ft 3 in. above springing level.

The tunnel was provided with a concrete lining to springing level, having a clear waterway width of 6 ft, in order to obtain the maximum discharge capacity. Where the rock was sound, the lining was constructed of precast lightly reinforced concrete slabs 4 in. thick, laid on edge and supported by precast reinforced concrete posts at 7-ft centres (Fig. 9d). Slots were provided in the slabs to facilitate handling and to equalize the water pressure on both sides of the slabs. The lining was rapid and easy to erect and on completion the sides had a smooth finish. In places where the rock was partly decomposed and liable to fall the space between the precast lining and the rock was filled in with concrete and the roof was supported by a concrete arch (Fig. 9b).

The orifices at the tunnel entrance restrict the flow into the tunnel so that the top water level in the tunnel rises no higher than springing level and ensures that the tunnel operates under open-channel conditions.

An open-ended free-flowing tunnel is subject to draughts which cause rapid cooling of the rock. This may lead to the opening-up of some of the joints in the rock and it is desirable, therefore, to reduce draughts to a minimum. This was done very simply in the diversion tunnel by means of a screen which was fixed near the outlet portal. The part of the screen above springing level was fixed to a short length of roof lining and the part below was hinged horizontally to form a flap. The end of the flap had sufficient buoyancy to ensure that the flap would swing with the rise and fall of water level in the tunnel and thus prevent draughts without causing an obstruction to the flow of water in the tunnel.

Setting-out the tunnel

The survey work involved in determining the length of the tunnel and in setting out the alignment of the two headings presented little difficulty since it was possible to line in a station at a level of 2,500·0 O.D. on the top of Slieve Bignian, the mountain separating the two valleys, with a station at each end of the tunnel, and long base lines were readily set out in each valley which enabled direct sights to be obtained

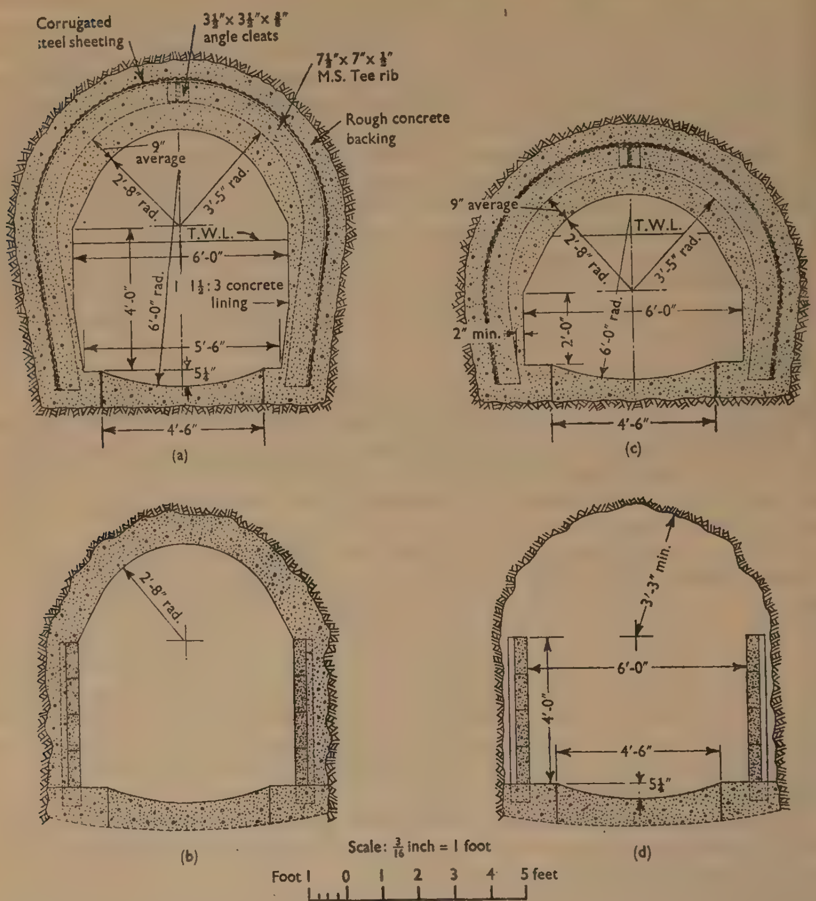


Fig. 9.—Tunnel sections

on to the station at the top of the mountain. This considerably simplified the calculations for determining the tunnel length. It was also most fortunate that, in addition to having a clear view of the hill-top station from the two stations at the ends of the tunnel, it was possible to sight directly up each heading on to lines carried on the roof stations (Fig. 10).

Tunnelling in sound rock

The tunnel was driven from both valleys almost entirely by local labour which had to be trained in the use of tunnelling machinery and equipment. Progress was not, for some time, as good as had been hoped for, owing partly to the initial lack of experience and skill of the local men and partly to the difficulty of obtaining modern tunnelling equipment immediately after the end of the war. When this deficiency

Fig. 6.—General view of main intake
(The two orifices in the concrete wall measure and control the flow into the tunnel)

Fig. 7.—The main intake (looking upstream)

FIG. 8.—THE MAIN INTAKE (LOOKING DOWNSTREAM)

FIG. 10.—THE TUNNEL ENTRANCE IN THE ANNALONG VALLEY
(The top of Slieve Bignian, which is $1\frac{1}{2}$ mile away, can be clearly seen from the station outside the tunnel)

in equipment was later made good and the men became proficient in its use, the rate of progress reached a satisfactory figure.

The equipment used in driving the tunnel included light rock drills for drilling the face, with self-adjusting pneumatic feed-legs and tipped steels of chisel pattern having tungsten-carbide inserts, tunnel rocker shovels for mucking out, and electric locos for hauling the skips. Air ducts of 11-in. dia. were taken to within 100 ft of each face and fresh air was blown to the face by large fans during drilling and mucking-out operations. The fans were reversed to suck out foul air and fumes from the face after blasting operations. Air was supplied for the four drills in each heading by a 300 cu. ft/min stationary compressor which was housed in a power house near the entrance to the heading, whilst water for the drills was piped from small intakes on the streams flowing down the mountain-sides.

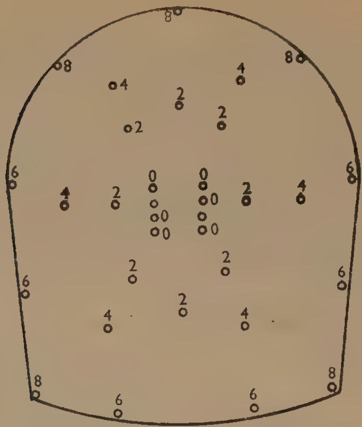
Using this equipment the rate of progress averaged 80 ft in each heading for a working week of 136 hours during the last year of construction, the record advance in one heading being 144 ft. A typical cycle time in sound rock is given in Table 4.

TABLE 4.—DATA FOR EXCAVATION IN ROCK

Operation	Average round: hrs min	Record round: hrs min
Marking face and bringing forward drilling gear	0 30	0 24
Setting up machines	0 15	0 12
Drilling 33 holes with 4 drills	5 30	2 45
Removing drills, blow-out holes, etc.	0 30	0 15
Charge face, fire	1 15	1 04
Smoke clearance, lights, and shovel to face	1 30	1 20
Mucking out, advance track	4 00	2 30
Total	13 30	8 30
Advance per week: ft	80	144
Number of rounds	10	16
Advance per round: ft	8	9
Charge per round: lb.	172	172
Explosives/cu. yd rock: lb. (based on pay area)	8·6	7·6

At the start of the tunnelling operations the wedge cut was used for blasting out the rock but the pulls were often considerably short and it was abandoned in favour of the burn cut which pulled out the rock square to the tunnel and to within about 6 in. of the full depth of the holes. About thirty-three holes were drilled including eight cut holes of which two holes were not charged. The average amount of explosive used for a pull of 8 ft was 172 lb., giving about 8·6 lb. of gelignite dynamite per cu. yd of granite based on the pay section. The layout of the holes drilled into the face is shown in Fig. 11, the numbers referring to the seconds delay on the electric detonators.

The amount of water to be dealt with in each heading was not considerable during normal tunnelling operations and an air pump at the face in the Annalong Valley heading was capable of pumping the water into a rising main which delivered it to a drain outside the tunnel. In the case of the Silent Valley heading an air pump delivered the water over a stank built across the tunnel and it then flowed by gravity



The numbers refer to the delays of the detonators

FIG. 11.—LAYOUT OF DRILL HOLES

to the outlet end of the tunnel. The volume of water dealt with in each heading was approximately 20 g.p.m., although at times this increased to about 40 g.p.m.

Tunnelling in unsound rock

Except where the ground was very decomposed or badly fissured, the face was advanced through and beyond weak sections before steel ribs of 7-in. × 7-in. × ½-in. Tee-section were erected, lagged, and backed with concrete (Fig. 9a). When the nature of the ground prevented this procedure ribs were erected near the face and the depth of the drilling was reduced to about 3 ft. The face was fired in two or three rounds so as to prevent damage to the ribs, and as the face advanced beyond the last rib the depth of pull was increased until at 10 yd normal 8-ft rounds could be fired with little risk of damage to the ribs. Finally, the sections of the tunnel which had been ribbed were completely lined with concrete.

Water was sometimes encountered in the joints at the face which, owing to its high pressure, slowed down the rate of drilling considerably and made working conditions more difficult. These wet conditions at the face sometimes persisted for a considerable distance until a transverse fissure was reached which provided a freely flowing outlet for the water.

THE FISSURE IN THE ANNALONG HEADING

A band of rock was encountered in the Annalong Valley heading at chainage 1,200 yd at a depth of about 900 ft below the surface; it was particularly troublesome to drill owing to the high pressure of the water in the joints. It was thought to be similar to other bands of rock previously encountered and no unusual difficulty was expected in passing through it. However, on the 26th September, 1949, a round was fired and when the tunnel gang advanced up the heading, after the usual time allowed for smoke clearance, they were surprised to find that water extended up the tunnel for a distance of about 400 yd and that a considerable quantity of kaolin,

silt, and decomposed rock had been brought into the heading; it reached a depth of about 6 ft at the face and extended up the tunnel for a distance of about 70 yd.

Additional pumping equipment was brought to the face to cope with the large volume of water in the tunnel and at the end of a week the tunnel was cleared out to within a few feet of the original face. The roof appeared to be quite sound up to the face but as the band of rock was intrusive in one granite and formed part of a disturbed zone it was considered advisable to erect and lag a number of steel ribs at 3-ft 0-in. to 3-ft 6-in. centres (Fig. 12, Plate 2). During the erection of these ribs a temporary bulkhead was erected near the face but the build up of pressure behind this proved too great and a week later a further considerable inrush of water, sand, etc., occurred. Almost as soon as the tunnel had been cleared out a third large inflow occurred and it became obvious that the decomposed material was not coming from a local pocket, but from a fissure of quite considerable extent. The tunnel was again cleared out to within about 20 ft of the original face and a strong timber bulkhead was erected to prevent any further inflow of decomposed material. It now became clear that this fissure was unlike anything previously encountered in either heading and that special measures would have to be adopted to effect a passage.

In order to contain the soft material in the neck of the fissure steel piles were then inserted over the top of the steel ribs and these were driven forward towards the face (Fig. 12, Plate 2). The piles, which were 25 ft in length, were well greased and the gap between the piles and the roof of the tunnel was filled in with concrete to maintain the correct direction of the piles during driving operations. No suitable driving equipment was immediately available, so the spawls were removed from a heavy rock drill and this was used successfully as a pile driver. The piles were driven towards the face as far as they would go, but on measuring up it was most disappointing to find that the piles had just about reached the neck of the fissure but had not penetrated into it. Subsequent inspection showed that the roof dipped slightly at this point and fouled the ends of the piles, thus preventing penetration.

Sealing the fissure

Consideration was now given to a scheme for sealing off the fissure by means of high-pressure grouting and a specialist firm was invited to co-operate in carrying out the work. The pressure exerted by the water and decomposed material on the timber bulkhead was considerable and a 4-ft-thick concrete bulkhead was constructed across the tunnel about 38 ft from the fissure to afford additional protection to the men. This concrete bulkhead was fitted with a steel door so that the timber bulkhead and the flow of water through it could be kept under observation.

Working behind the concrete bulkhead, twelve holes were drilled towards the fissure. The holes fanned out to a radius of about 12 ft from the centre-line of the tunnel at the beginning of the fissure, and as the plane of the decomposed rock was at about 45° to the horizontal these holes met the fissure at lengths varying from 46 ft to 75 ft. Water flowed freely from most of the holes and it was often pink in colour due to the kaolin held in suspension. Short lengths of 2-in.-dia. steel pipes were driven into each hole and isolating cocks were screwed on so that each hole could be sealed off as required.

Cement grout, with a consistency of 1 cwt cement to 70 gal of water, was injected until 1 ton of cement had been injected into each hole in turn. During the injection of the grout the pressure rose from a minimum of 100 to a maximum of 450 lb/sq. in., although the usual range of pressures was from 150 to 300 lb/sq. in. As the grout

tightened up the ground, the pressure built up more quickly and the quantity of cement grout required to build up a back pressure of 300 lb/sq. in. was progressively reduced to as little as 2 cwt in some of the holes. A further indication of the tightening-up of the ground could be observed when the holes adjacent to the hole being injected (and sometimes holes considerably removed from it) discharged grout and had to be sealed off by means of the isolating cocks.

After each hole had been injected in turn, the cocks were removed and all holes were then redrilled. During redrilling the only water discharging from the holes came from the drill rods, and it would remain clear until the furthest plane of consolidation had been reached when fissures would again be tapped. The water would then become pink in colour and frequently contained fine sand in addition to the kaolin.

The ground was thoroughly consolidated to the full depth of the holes and then all holes were redrilled to a plane about 5 ft in advance. Further grouting, followed by redrilling, then continued until the ground had been consolidated to the new plane, and the whole procedure was repeated until the ground had been consolidated to a plane about 65 ft beyond the concrete bulkhead. Three test holes were then drilled into the fissure, one in the roof and two in the invert. During the drilling of these holes no fine sand was brought out and it was evident that sufficient consolidation had been effected to permit resumption of tunnelling operations.

The concrete and timber bulkheads were then removed and the tunnel mucked out until the original face was reached. The face was now perfectly dry and the cement grout could be observed in the numerous small fissures in the face.

The material at the face consisted of a bed of red decomposed intrusive rock about 2 ft thick, a bed of block kaolinized granite also 2 ft thick, and a thin bed of soft kaolin. Beyond these beds was a face of granite in which the feldspar crystals were very decomposed and had a white greasy appearance. Apart from its colour the granite appeared to be almost normal.

Before proceeding further it was considered advisable to drill a test hole into the face, and this was drilled to a depth of 15 ft through a temporary bulkhead. No water was struck and it was clear that consolidation of the ground ahead had been effective for at least this depth.

The passage of the fissure

Tunnelling operations were then resumed and the face was advanced slowly by excavating the relatively soft material by hand. In order to hold the ground the face was advanced about 1 ft at a time and two steel ribs were erected on steel plates which prevented the ribs from sinking into the soft bottom. Bags of concrete were then packed behind the ribs to support the sides and roof. After advancing about 10 ft a timber bulkhead was erected at the face as a precautionary measure while the ribs were underpinned and a concrete raft, 18 in. thick, was placed in the floor as permanent support for the ribs. This operation was rather slow since only a 3 ft length could be safely underpinned at a time. A hole was then drilled through the bulkhead for a depth of about 15 ft to test for water and to ascertain the hardness of the ground ahead. The bulkhead was removed and the operation was repeated until good rock was reached about 45 ft ahead of the beginning of the fissure.

After reaching sound rock tunnelling operations were suspended for a time so that low-pressure grouting could be carried out thoroughly to consolidate the packing behind the ribs. A total of 133 tons of cement was injected into the fissure and a further 20 tons of cement was used in low-pressure grouting behind the ribs.

The roof of this section of the tunnel, in addition to the sides and invert, was finally lined with concrete to provide protection for the ribs and to form a smooth waterway. The soffit level was 2 ft lower than normal at this point and was only about 1 ft above the top water level in the tunnel at maximum discharge (Fig. 9c).

Costs

The final cost of the river diversion scheme, exclusive of engineering and overhead charges, etc., was £317,000, made up as follows:—

	£	£
River intakes		
Main intake	37,000	
Secondary intake	14,000	
		51,000
Tunnel		
Installation of contractor's plant, etc.	56,000	
Tunnelling in rock	76,000	
Ribbing at weak places	11,000	
Work at the fissure	23,000	
Normal concrete lining	18,000	
Roof concrete lining	16,000	
Outlet works, drainage, etc.	12,000	
Cut-and-cover section	5,000	
		217,000
		268,000
Variation in labour and materials costs		49,000
Total		£317,000

Acknowledgements

The works were constructed by the Belfast Water Commissioners to the design and under the direction of the Engineers, Messrs Binnie, Deacon & Gourley.

The Contractor for the works was Messrs A. M. Carmichael Ltd and the grouting work at the fissure in the tunnel was carried out with the co-operation of the Cementation Co. Ltd.

The Author is indebted to the Belfast Water Commissioners and to the Engineers for permission to publish this Paper. He also wishes to thank the Geological Society of London for permission to reproduce Figs 2 and 3 from Dr Richey's Paper.[4]

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2. C. A. Risbridger and W. H. Godfrey, "Rainfall, Run-off and Storage: Elan and Claerwen Gathering Grounds." Proc. Instn Civ. Engrs, Pt III, vol. 3, p. 345 (Aug. 1954).
3. J. Glasspoole, "Reliability of Rainfall over the British Isles." J. Instn Wat. Engrs, vol. 1, p. 441 (Aug. 1947).
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The Paper, which was received on 23 March, 1955, is accompanied by five photographs and eight sheets of drawings, from which the half-tone page plates, folding Plates 1 and 2, and the Figures in the text have been prepared.

Discussion

The Author introduced the Paper with the aid of a series of lantern slides.

Mr H. J. F. Gourley (Partner, Messrs Binnie, Deacon, and Gourley, Consulting Engineers) said that the Author had stated that, because there was no site available for construction of an impounding reservoir in the Annalong Valley, the Belfast Water Commissioners had to be advised that the only alternative was to divert the Annalong River through the tunnel into the Silent Valley reservoir.

Before Mr Gourley's firm were able to advise the Commissioners in that way, they had done much exploratory work in the Annalong Valley. The first site considered was the old Parliamentary site where a dam 1,680 ft long and of a maximum height of 122 ft would have given storage of about 1,700,000,000 gal. That was enough to equalize the run-off from that catchment. However, they had done borings and surveys, and concluded that on that site the geology was so unsatisfactory that any attempt to construct a reservoir would only be at ruinous cost. They had borne in mind the experience in the Silent Valley where the cut-off had to be carried on to a depth of 180 ft—the depth to rock—and to get a watertight base for the cut-off, they had to go down a further 20 ft. Those were the conditions to be anticipated on the Parliamentary site.

They thought better conditions might occur farther up the valley; they did exploratory work, but the same tale was unfolded. Finally they advised the Commissioners to adopt the tunnel and get the additional storage by a further reservoir in the Silent Valley. Fortunately, there was a suitable site upstream of the first reservoir to be constructed which would provide the necessary storage without undue foundation troubles.

Mr Gourley noticed "Ratio of tunnel capacity to average flow" below the graph in Fig. 5 on p. 166. He assumed that was the average flow of each year. Later on the same page the Author stated that that represented about nine times the estimated run-off during the three driest consecutive years. Until the Paper was written, those in his firm had never thought in terms of the run-off of the three driest consecutive years; they had always determined their catch-water capacities and tunnel capacities on the average annual run-off. If that was taken as the basis, it gave in the case under consideration about 4½ times the average. Mr Gourley's firm considered that if 4½ to 5 times the average flow on a diversion was obtained they could collect about 95% of the run-off.

The moral of the Paper was surely that nobody could predict to what depth fissues and similar troubles might extend in granite.

He wished to say something about the unusual form of lining used, consisting of concrete panels and concrete posts at intervals. That suggestion had emanated from Dr Colebrook. The contractor had very much overdriven the tunnel; in other words, the overbreak was considerable at that stage. The rise and fall clause was operating. If the contractor put in more concrete, it would have involved the Commissioners in greater cost because they were paying for more cement. A few other considerations of that sort made the design appropriate. It was economical not only in concrete but also in time required to complete the job.

With regard to the fissure it was rather interesting that the pressure that developed there showed that the surcharge almost equalled the overburden. The pressure developed down in the tunnel corresponded closely to the head of water of 900 ft. It was not surprising that when they approached that and got into it, all their troubles began.

It was fortunate that the main contractors, Carmichaels, did not object to their getting

THE DIVERSION OF THE ANNALONG RIVER INTO THE SILENT VALLEY RESERVOIR

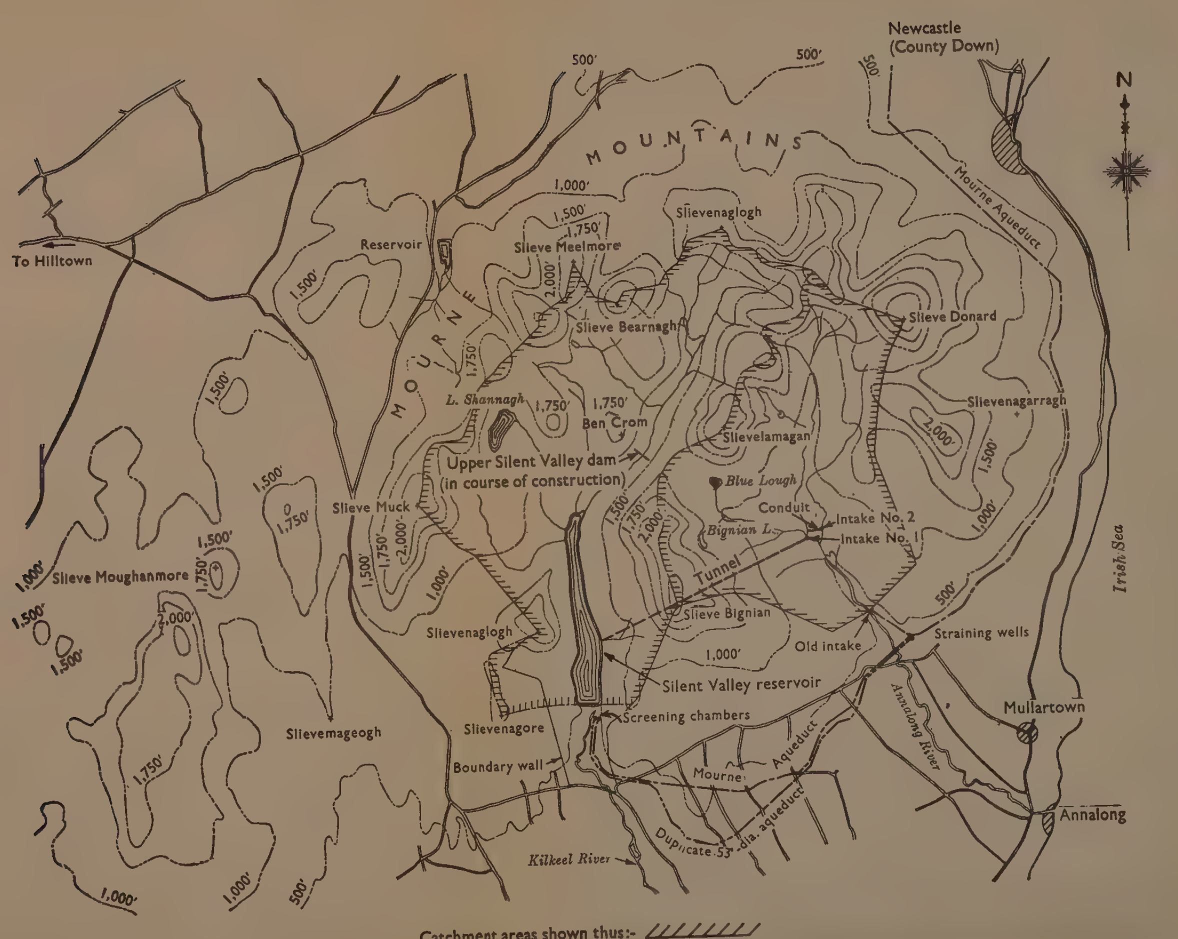

Fig. 1.—General layout of scheme

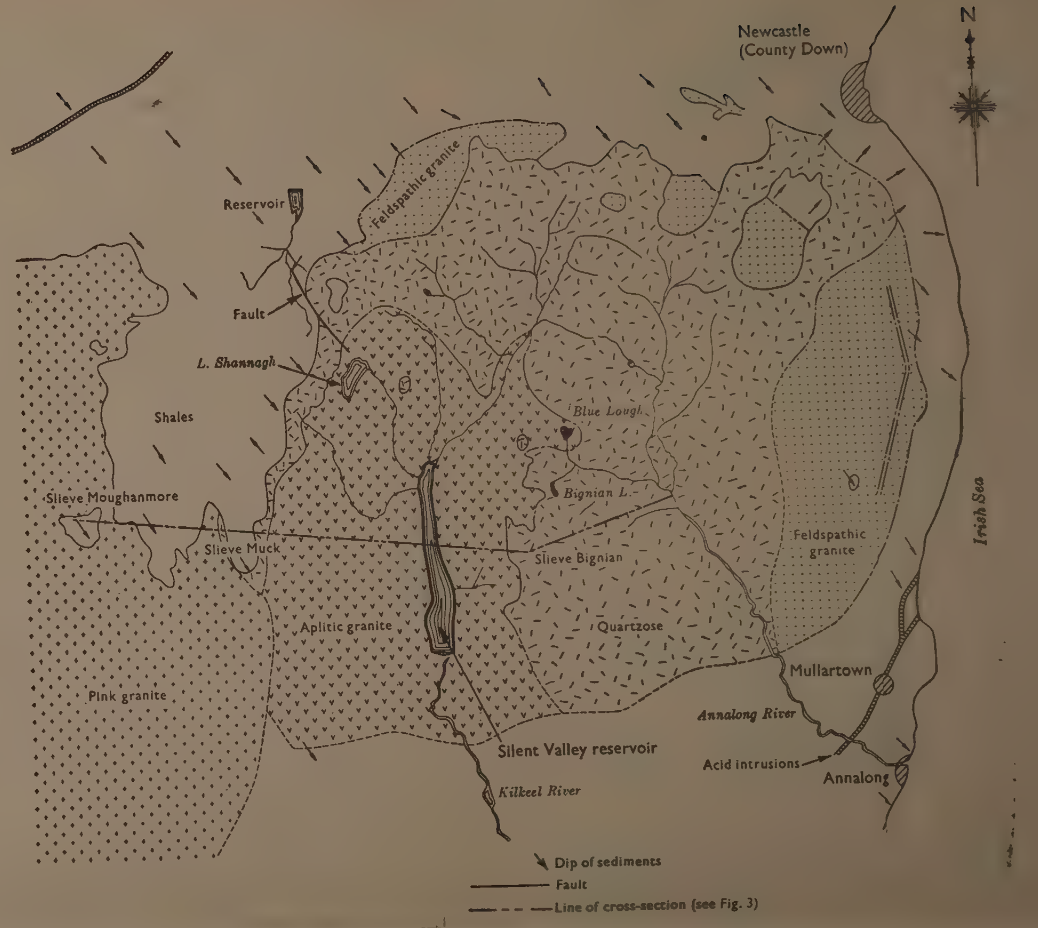

Fig. 2.—Distribution map of Mourne Mountains granites

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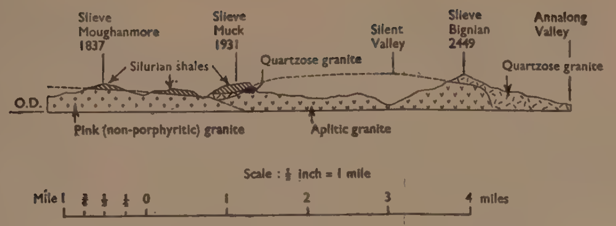

Fig. 3.—GEOLOGICAL SECTION OF MOURNE MOUNTAINS
(Along the line shown on Fig. 2)

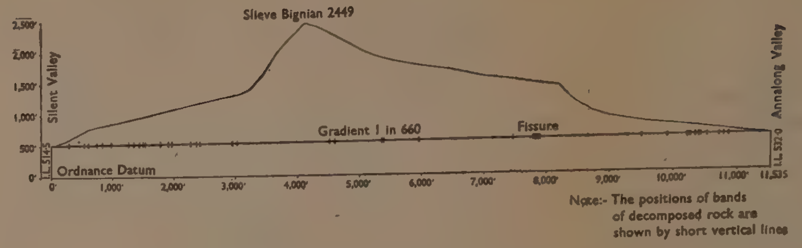

Fig. 4.—LONGITUDINAL SECTION OF TUNNEL

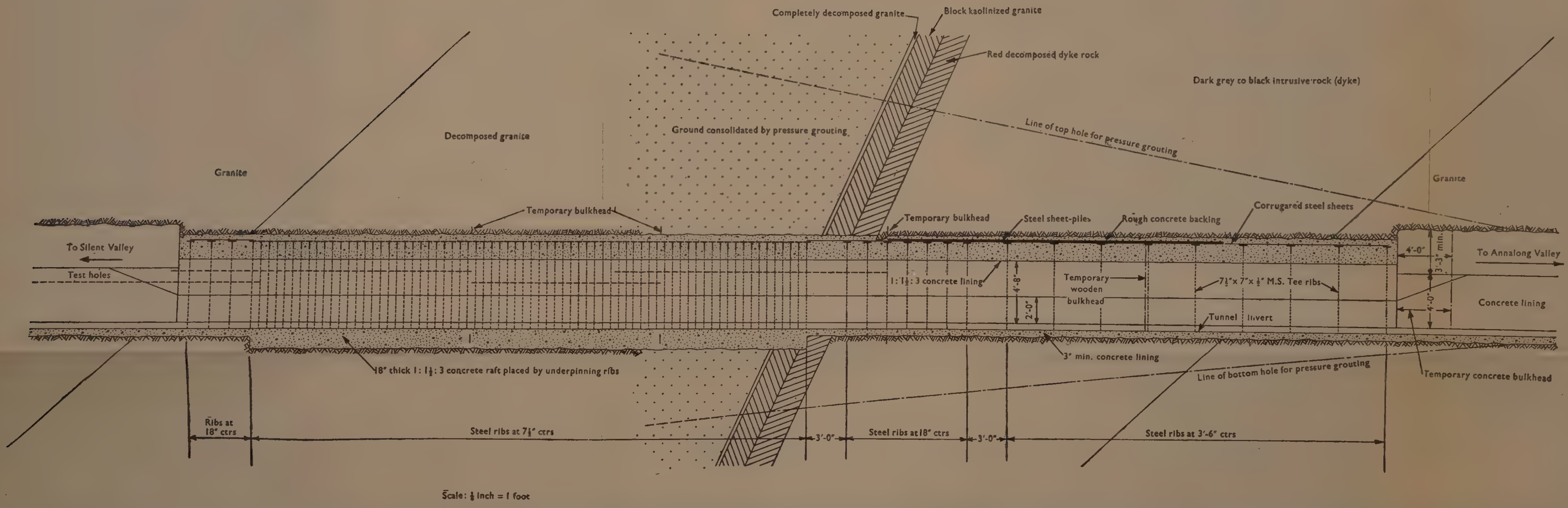

Fig. 12.—LONGITUDINAL SECTION OF TUNNEL AT FISSURE

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the help of the Cementation Company, who were specialists in dealing with the trouble. The ordinary tunnelling contractor was not so used to that sort of thing. It was largely due to the efforts of the Cementation Company that they managed to get through the fissure with a certain degree of honour. He thought the cost for that work was £22,000, or about 10% of the cost of the job.

Mr C. M. Roberts (Partner, Sir William Halcrow and Partners, Consulting Engineers) said that the placing of an intake or diversion weir across a river involved certain problems, not the least of which was siltation, which might occur in varying degrees according to the character of the river.

In small mountainous streams with relatively steep catchments, there might be very flashy flow and the periodic occurrence of torrential floods which brought with them considerable quantities of silt and gravel. Such material was trapped by any obstruction across the river and deposited when the velocity of the flow was reduced; it was possible for an intake to become completely filled within a short period. That problem arose particularly in hydro-electric schemes where, apart from the necessity for keeping intakes free from blockage, it was, of course, desirable to prevent gravel entering the power tunnels.

Much attention had, therefore, been given to the design of intakes to remain free from silting. A common form of intake was the box type in which water was collected into a channel placed across the stream bed. The channel was covered by screens, the bars of which were parallel to the line of flow and laid to a slight fall so that they might be self-cleansing. That type of intake did not exclude material finer than the spacing of the screen bars, and it was necessary therefore to have a stilling pool between the actual intake and the mouth of the tunnel or shaft, as the case might be. That, of course, necessitated the periodical cleaning-out of the stilling pool.

A further type was currently being developed in which it was hoped that gravel would be eliminated almost entirely from the intake. It consisted of a concrete-lined apron with a sloping floor, the lower or downstream end of which was turned up to form a pool. The intake openings were placed horizontally on either side of the pool, and at normal flows a hydraulic jump was formed on the apron upstream of the openings so that the water flowed sideways into them. When floods occurred and velocities were such as to cause the movement of gravel, the jump was pushed off the downstream end of the apron, and the ensuing shooting flow tended to scour any material accumulated in the pool and prevent its further deposition.

From Fig. 7 the bed of the Annalong River appeared to be movable, but no particular provision had been made in the design of the intake to avoid silting or for the removal of deposited material. That omission was presumably justified from experience of the previous intake at the lower site. Perhaps the Author would say a word on that.

The use of masonry had now become almost traditional in certain sections of the water supply industry, and it was pleasing to note that it had been used to good effect in the Annalong intake. The terminology of masonry work was varied, but could the Author describe more fully the appearance of block and sneck work? From Fig. 7 that appeared to be similar to the generally known squared rubble.

Difficulties had been experienced elsewhere in obtaining skilled masons to carry out such work, and sometimes in finding a supply of suitable material, which had tended to force up the prices. Did the Author consider that the additional cost of the masonry was justified by the final appearance of the intake?

The concrete lining to the tunnel was an interesting and, apparently, economic means of increasing the discharge—the post and slab work. The alternative would have been to leave the tunnel unlined where the rock was sound, increasing the gradient as necessary or drowning the tunnel to utilize the full cross-section. Could the Author give any information on the economic studies made and the coefficient of friction assumed for that purpose?

With regard to Table 4, the amount of explosive used per cubic yard of tunnel excavation seemed rather high, being about 8·6 lb/cu. yd excavated. Under comparable conditions elsewhere, a maximum of 6 lb/cu. yd had been used, but that was associated with a shorter

pull of 4 to 5 ft. With larger-diameter tunnels, of course, the amount of explosive per cubic yard excavated was much reduced.

The Author gave a useful account of the measures adopted to overcome the difficulties at the soft seam, or fissure encountered in the Annalong heading. Considering that cement grouting of rock was a technique widely used in the construction of dams and tunnels, there was little positive information available on such matters as the degree of penetration or extent of travel of grout for various types of rock. Therefore, it remained largely conjectural, and the technique was based upon past adequate results. In that case, the decomposed rock had broken down into very fine clay-like material, as evidenced by the kaolin, and some doubt might have been felt on the ability of the grout to have the desired penetration. Had any other means of consolidation been considered and what were the reasons for the selection of cement grouting? Did the Author consider that the use of colloidal grout would have given any advantage over that used?

Reverting to the question of positive information about the penetrating capacity of grout, that was one of the few opportunities for an engineer to excavate and make observations in a grouted zone. The Author mentioned that cement was visible in small fissures in the tunnel face. Mr Roberts wondered if anything more definite could be deduced from the spacing of grout holes, distance from the tunnel, grouting pressures, and so on.

Mr J. C. Dickinson (Senior Engineer, Messrs Preece, Cardew and Rider, Consulting Engineers) said that there were two aspects on which he wished to speak. The first was the limitation of flow to a fixed amount, such as required by a free-surface tunnel, and the second was the effects of constriction when difficulties were encountered, but he would confine himself to the question of foreseeing trouble in tunnels.

In underground work the consulting engineer had to attempt to foresee all troubles long before he went underground, if only to give his client a fair estimate of the expenditure. He seemed to have only two sources of information to assist him—geological surveys and borings.

It should be noted that the trouble referred to in the Paper occurred 1,000 ft underground, and it was expensive to bore to that depth.

Mr Dickinson quoted a case which threw some light on the comparative usefulness of borings. Some years previously he had been engaged in tunnelling about 400 ft underground, in gneiss in which there were bad patches.

One evening he had been called to the face. The roof had been sound but all over the face the rock had been falling. The men had been withdrawn and sent out of the tunnel to get timber. By the following morning the tunnel was completely full, from floor to roof, with a material of the consistency of tooth-paste, which was moving down the tunnel at 3 in/hour. It had moved 50 ft down the tunnel before it could be checked.

The work had been carried back to the face, but it had proved impossible to get through or over the trouble. In despair, they had gone back about 200 ft and driven round it. The diversion was only 100 ft from the old line. Precautions had been taken but no repetition of the trouble had been encountered. What would have been the odds against a number of borings, taken on a tunnel line 8,000 ft in length, giving some indication of that trouble?

Supposing the borings had hit the trouble: what would have been the chances of the tunnel being assessed as an impossible project? As it happened, in the case he had described, the trouble had been overcome at a little expense and a very profitable scheme had been completed. Borings were therefore perhaps a mixed blessing—if there were too many of them!

The sole function of borings was that they should be put down not necessarily to the tunnel level but in carefully selected places, named by the geologist, and in some form to amplify the information from surface observations; they should not be used as a method of finding what rock was in the tunnel line.

Of course, once the work was under way, test holes could be driven ahead, but unless trouble was anticipated the interruption of the driving procedure was such as to make that uneconomical and not to be considered as a regular feature.

In the case described in the Paper there were dikes. Would the Author say whether the dikes were visible on the surface?

He did not know whether geologists worked sufficiently closely with the engineers. In any tunnel work, they should be the engineer's best friends.

Colonel E. R. Rowbotham (of the Cementation Company, Ltd) said his special interest was in the section of the Paper dealing with difficulties caused by the fissures in the Annalong heading and the Author's description of how those difficulties were tackled and finally overcome. The Author's description of the methods he adopted formed almost a classic illustration of some of the chief principles of the injection of soil with cement or chemicals.

First, there was the principle of treatment of an extended area—treatment in depth. There was the concrete bulkhead which was set back from the fissure about 35 ft. That was probably necessary for safety; the timber bulkhead was bulging and probably did not inspire confidence. But it was also for technical reasons that the bulkhead had been set so far back. The treatment was certainly carried well beyond the fissure. In other words, the treatment was general and not local.

The Author had also described clearly the principle of treatment by stages, in that case stages of 5 ft, each stage reinforcing and strengthening the preceding stage. Thirdly, the adjustment of the consistency of the cement injection was finally fixed at about 1 : 9 by volume. Usually, the consistency was settled by experiments on site. A weak injection was started first which was progressively increased in strength until the optimum was reached. Could the Author give his personal observations on the site procedure at the concrete bulkhead—the site procedure by which the optimum strength of injection was reached?

Linked with that was the selection of the correct pressure. The pressure had ranged in that instance between 150 and 450 lb/sq. in. He would be grateful if, together with details of the experiments on consistency, the Author would include the procedure followed on site for arriving at the optimum pressure.

The selection of the correct consistency of grout and the correct pressure had been done by experiments; if those experiments were not to go on too long and become uneconomical and in fact if they were to come to a successful conclusion at all, it was necessary that experienced staff should conduct them. The matter could be summed up in the American term, "know-how." That was behind the method in which the consistency of the grout injection and the optimum pressure had been selected.

There were dangers in a bad selection of consistency in grout and there might be too much error in trial and error methods. If the injection was too weak, and pressures were too low, the ground would not be treated, or would take a great deal of time and expense to treat. If the pressure was too high, there was danger of structural damage—not in that case because at the danger point in effect there was no structure, but the grout could be driven great distances in the wrong direction and money might be spent in treating large areas of ground which were unimportant.

If the injection was too strong the ground might be treated close to the injection point with thick grout, fissures would be sealed off, but perhaps other important fissures just beyond would be left untreated—and not only untreated but unattainable, because they would be out of reach. Quite likely they would be unsuspected, too.

He had referred to the danger of grout being driven into unwanted ground. The Author had mentioned in the Paper, and in his introduction, that the original injection into his holes driven through the concrete bulkhead was made with a limited injection of 1 ton. That was a device whereby the excess travel of grout was limited. A limited injection of 1 ton was given and allowed to go off; it acted as a cushion against which subsequent injections could push, so that travel was limited and pressure could be built up safely and extra consolidation achieved.

Apparently the portion of the tunnel in the neighbourhood of the fissure was hand-excavated. When a cement injection was placed underground there was usually no direct observation of the travel of the grout—where it went and how it behaved; that

had all to be deduced by the alteration in behaviour and characteristics of the ground treated. It was largely guesswork. In that case, however, the ground had been excavated by hand. The kaolin, with which he associated the decomposed rock might be thought to be impermeable and that the cement injection would be unable to permeate. The grout therefore would travel in places where the kaolin or decomposed rock was broken down, or along water channels where water had flowed. It would block those channels and consolidate the kaolin or decomposed rock not by penetration but by pressure; it would squeeze it together.

In view of the fact that hand-excavation was used, could the Author see anything of the behaviour of the grout? Did he observe any direct penetration of the rock or of the kaolin by the grout? Was it in veins and, if so, were the veins laminated or reticulated, criss-cross or dendriform, like the roots of a tree; about how thick were the fissures, if any?

That was a case of ground treatment for work—that was to say, temporary not permanent stabilization of the soil. The only requirement was to stabilize the soil while the excavation was being carried out; after that it did not matter what happened. That called, of course, for a less strict specification and for less care. A very similar job had been widely reported recently, the case of a tunnel with an inundation in California, the Tecolote Tunnel. The problem there was overcome in precisely the same way; a series of radiating holes was drilled round the tunnel and they were pressure-grouted. He thought that the engineers who carried out the task were exceptionally pleased with the method used, but had rather more difficulty in having to contend with methane gas in addition to inflow of water. As a matter of interest, the process had also been used in shaft sinking, and in that case the pressures used were very high. The depth of ground was greater, of course. In South Africa, a pressure for cement grout injected for temporary stabilization of the soil had been recorded as 5,000 lb/sq. in.

The Chairman said he was prompted, as a result of the remark by Mr Dickinson about the need to work with geologists with a view to sensing trouble ahead, to ask whether any use had been made in the past of any modified geophysical method such as that used in oil exploration. He wondered whether any modified method might be used to indicate trouble ahead.

*** **Mr A. M. Carmichael** (Director, A. M. Carmichael Ltd) observed that the average time of 13½ hours for a pull of 8 ft in the Mourne tunnel was due, in the main, to the shortage of skilled labour, although the method of drilling used in the initial stages of tunnelling, i.e., drifters mounted on column and bar, was similar to prewar methods, but before the war the average drive per week in a tunnel of similar section was about 90 to 100 ft/week.

An advance on the column and bar method was the introduction of drifters on a Jumbo with pneumatic air feed, but spare parts had to be imported from the United States and those did not come forward in time to ensure continuous working.

Towards the end of the contract the Swedish system of tunnelling, i.e., light hammers mounted on pneumatic-feed air-legs operated by one man as compared with two men for drifters, and using only half the column of air, reduced the drilling time and consequently the average time per round, but the introduction of that equipment was too late to have any appreciable effect on the overall progress of the work.

The Swedish system as introduced in the Mourne tunnel was also introduced to tunnels on the Scottish hydro-electric schemes by us at the same time, and the tremendous advances in speed of tunnelling from that date had all been achieved with the Swedish system as the basic principle.

The record weekly advance of 144 ft achieved at Mourne was considered fast driving until 1951, when an advance of 245 ft with the Swedish method was obtained by his firm in one week in an 11-ft-dia. tunnel at the Glen Shira Hydro-Electric Project. The next

*** This contribution was submitted in writing upon the closure of the oral discussion. —Sec.

advance on that figure was 340 ft/week in a 7-ft tunnel in 1952, and from then on tunnelling records had been created at regular intervals, the latest figure being something over 500 ft obtained recently on the Breadalbane Hydro-Electric Project.

Even allowing for those isolated record weekly drives the firm had found that the average rate of progress over the whole period of tunnel driving on any contract very seldom, if ever, exceeded 150 ft/week per heading.

Present-day driving at average speed in a tunnel with similar strata to that encountered at Mourne would, using 7-ft 10-in. rods drilling holes 7 ft 3 in. long, probably record times for the various operations per round as follows:—

	Hours
Drilling	2
Charge and fire	½
Clear smoke	½
Lay rails and mucking, etc. . .	2½
	5½

That would give twenty-three rounds per week, allowing for minor unavoidable delays, at, say, 6-ft 3-in. average pull, i.e., approximately 140 ft. At Mourne Mr Carmichael's firm invariably found, owing to the very tight nature of the rock, that a 12-in. socket was left at each hole.

The Author, in reply, said that exploratory work at two sites in the Annalong Valley had indicated that the cut-off wall would have to be carried down to rock through some hundreds of feet of glacial drift as in the case of the Silent Valley dam, and that the cost of constructing a dam in the Annalong Valley would be prohibitive.

Mr Gourley had referred to the pressures at the fissure being equivalent to a 900-ft head of water, or a pressure of about 450 lb/sq. in. Whilst the pressure of the water in the fissure might well have been as high as that before the fissure was reached, the pressure had been considerably reduced when water was able to find its way into the tunnel through the opening in the roof of the tunnel. When grouting started the back pressure had been about 100 lb/sq. in. but that pressure had steadily increased to 450 lb/sq. in. owing to the fact that as the ground tightened up it became more and more difficult to force the grout under pressure into the smaller fissures. The high pressure reached during the grouting operations had, therefore, to be regarded as a local back pressure rather than as a head of water of 900 ft.

Mr Roberts had referred to the troubles experienced on schemes with which he had been connected arising from silting up of intakes as a result of the settlement of large quantities of grit, and he had made a very interesting reference to a new design of intake which appeared to be self-cleansing. He himself looked forward to hearing more about the actual practical operation of that type of intake. The problem of siltation was not a big one in the Annalong Valley since the bed and sides of the river and all the streams were lined with boulders. Indeed, the whole surface was a mass of boulders. Boulders of 2 to 4 cu. ft were numerous, and it was not uncommon to see boulders up to 2 cu. yd lying on or very near the surface. He thought for that reason that the amount of grit washed down into the intakes was perhaps not as large as in some of the schemes with which Mr Roberts was concerned. He had recently had a letter from the Water Engineer of Belfast who told him that about 150 cu. yd of grit was cleaned out per annum. The cleaning-out was done twice a year, about 75 cu. yd being removed each time. That quantity was less than 10% of the volume of the intakes and there was therefore no danger of the intakes becoming suddenly filled up.

As a matter of interest to Mr Roberts, if he wanted to compare the Annalong catchment area with those with which he was concerned, the gradient of the river for about 2 miles above the intake was about 1 in 20.

Mr Roberts had referred to the masonry work at the intakes. With the exception of

the embankment, all the earlier works visible in the Silent Valley were built in granite and they had a magnificent appearance. It was not, therefore, surprising that the engineers desired to face the main intake in the Annalong Valley with granite. The cost of the masonry was about £5,000, which was only a little over 1½% of the total cost of the scheme, and the expenditure of that comparatively modest sum was certainly justified.

Apropos the previous remark about the boulders, sufficient boulders were excavated from the river bed to provide all the stone for the masonry work in the intake.

Mr Roberts had referred to "block and sneck" masonry. That was a local term for masonry which consisted of stones of two sizes laid in a regular pattern and it was found to be quite economical to work all the stones to those two sizes.

There was no lack of masons in the Mourne area, and there was no difficulty in obtaining locally ample supplies of good stone for building purposes.

Mr Roberts had also referred to the tunnel capacity and had asked whether a study had been made of the economics of an unlined tunnel and a partly-lined tunnel. When the tunnel had been designed it was considered desirable to restrict the flow into it to ensure that the tunnel would run with a free water surface at all times. The reason was that in the tunnels which had operated under varying conditions of flow, i.e., running full bore for part of the time and part full at other times, falls of rock had occurred. It was thought that those falls of rock were due to surging of the air trapped in the roof of the tunnel as the tunnel filled up. That gave a very undesirable hammer-effect which was repeated every time the tunnel ran full. To ensure that the tunnel would run part full at all times, orifices of a suitable size were placed at the entrance to the tunnel.

The gradient of the tunnel was determined by the level of the intake and the level in the Silent Valley where it was intended to discharge the tunnel into an existing culvert. The size of the tunnel, that had an average diameter of 8 ft 3 in., was regarded as the minimum possible economic size. Confirmation that that was the minimum economic size was given in a Paper by Fulton.[5] From his analysis of tunnelling costs in Scotland, Mr Fulton found that for all tunnels of 8-ft dia. and less there was no saving in cost.

Having decided the dimensions of the tunnel, the next thing was to consider what discharge could be obtained from a tunnel of that minimum economic size. With regard to a comparison between a lined tunnel and an unlined tunnel there was, until recently, very little information available regarding the probable coefficient of an unlined tunnel. It was not unusual to assume a value of n in Manning's formula of 0·04. and for lined tunnels a value of n of 0·013. On that basis it was found that an unlined tunnel with a water level 6 in. below spring level had a discharge capacity of only half that of the lined tunnel. Referring to Fig. 5, in which the percentage of the total yearly run-off diverted by the tunnel had been plotted against the ratio of the tunnel capacity to the average flow in each of four dry years, it would seem that the lined tunnel, which would have a capacity about nine times the mean run-off during the three driest consecutive years, would divert about 97% of the run-off. An unlined tunnel for the same water level would have a capacity of about 4½ times the mean run-off during the three driest consecutive years and would divert about 90% of the run-off. By putting in a lining an increase in discharge of about 7% had been obtained at an additional cost of about 6½% and there was the additional advantage of a lined concrete invert which greatly facilitates the inspection and maintenance of a long tunnel.

Some data had recently been published on Swedish unlined tunnels in granite and gneiss. Analysing that up-to-date data he found that in the design of future schemes one might well use a value of n in Manning's formula of 0·029 for unlined tunnels, with a range of values between minimum and maximum of 0·026 and 0·032.

With regard to the amount of explosives used it was true that they had used 8·6 lb/cu.yd, which was a very high figure indeed. The reason was that the depth of pull, 8 ft, was undoubtedly too long. A more normal pull for a tunnel of the size described in the Paper

[5] A. A. Fulton, "Civil Engineering Aspects of Hydro-Electric Development in Scotland." Proc. Instn Civ. Engrs, Part I, vol. 1, p. 248 (May 1952).

would be about 6 ft, and the probable amount of explosive for that pull would be 5 to 6 lb/cu. yd. Certainly in the case of large Swedish tunnels in granite it was not uncommon to have a figure of 1·6 to 1·9 lb/cu. yd.

Mr Roberts had also asked about grouting; he would try to combine the answers in his reply to Colonel Rowbotham's observations.

Mr Dickinson had raised a very interesting point when he had discussed the value of borings along tunnels as an aid to the investigation of schemes. He agreed with Mr Dickinson's view that borings were only justified to amplify surface observations, as the odds against their bringing to light trouble spots were very great. There was also, as Mr Dickinson had pointed out, the great danger that encountering a trouble spot in the borings might cause undue worry and it might be extremely difficult not to make grossly excessive provision for tunnelling through and lining such tunnels.

With regard to the present scheme a number of trial holes were made in the Annalong Valley in the vicinity of the main intake and about 50 yd along the line of the tunnel. Those trial holes were about 10 ft square, and they had disclosed rock at about 10 ft below the surface which appeared to be only slightly weathered. It was fortunate too that there were a number of quarries in the district just at the entrances to the Annalong and Silent Valleys. They had been inspected and it had been possible to form a very good opinion as to the type of rock likely to be met in the tunnel.

In addition a geological map had been available which, of course, indicated that the mass of the mountain was granite. It had also indicated a few dikes which traversed the country rock largely in an east-to-west direction. Therefore, they had had sufficient warning that a few dikes would probably be encountered in the tunnel. On the ground there was certainly no sign of a major transverse dike. Mr Dickinson had remarked that one frequently saw the dikes standing proud above the country rock; but there were no such signs in the Annalong and Silent Valleys. He thought that in the Mournes the country rock was probably as tough as the dikes. There was no evidence of dikes on the surface, since it was almost entirely covered by a layer of peat and boulders, and he confessed that he himself had not expected to strike such a large number of dikes as had been found in the tunnel. About thirty dikes, varying in thickness from 2 in. to 80 ft, were traversed and out of a total of forty-one bands of decomposed rock which required lining, eleven were dikes.

Regarding the general question of dikes, the fact that there were dikes in the area was sufficient indication that fissuring of the country rock had taken place and that bands of decomposed rock would be met in the tunnel. Some provision had therefore been made in the contract for tunnelling through those bands of rock. However, the difficulty was to know what provision should be made, when preparing the contract, for tunnelling through bad ground. In the present scheme 8% of the whole length of the tunnel had to be fully lined, and he rather hoped that that information and other data which he had given in the Paper would be useful. He hoped too that others who had similar data would perhaps let him have them so that he might append them to the Paper.

There was no indication on the ground of a major fault in the Annalong Valley, and in the absence of such an indication he did not think that one could make any provision, either in the design or in the contract documents, for dealing with such an extensive fissure as that described in the Paper. In all tunnel schemes there was an element of risk, and it seemed to him that one could not afford to be too pessimistic.

Colonel Rowbotham had referred to the principles of grouting and the snags that might be encountered. He had pointed out the importance of grouting at the correct pressure and consistency and that for a grouting scheme to be successful and concluded in the shortest time it was very important that the personnel in charge of the work should have considerable experience of similar work. The Cementation Company had sent over their oldest and most experienced Superintendent to supervise the work and as a result of his "know-how" the fissure had been successfully sealed off.

At first a very weak grout of consistency 1 : 18 by volume was injected into two holes. It had produced only a small rise in pressure after injecting about 1 ton of cement into

each hole and the mix had then been strengthened to 1 : 9 by volume. An appreciable rise in pressure had been obtained after injecting 1 ton of cement into each hole at the thicker consistency and the technique adopted thereafter was as followed:—Each hole had been injected with a priming injection of 1 ton of cement at 1 : 9 consistency, during which the pressure had risen from 100 to as high as 450 lb/sq. in., and by the time the hole was to be re-drilled and re-injected the initial injection had completely set.

As the ground had tightened up, it was found that the quantity of cement required at each injection to increase the pressure to about 300 lb/sq. in. had reduced to as little as 2 cwt.

However, a few of the holes, which presumably passed through wider fissures, did not respond quite so well and in those cases the grout had been thickened up to about twice the normal consistency until a suitable rise in pressure was obtained. Having tightened up the local spots, the grout in all subsequent injections had been reduced to normal consistency.

The material in the open fissure, which was about 3 ft thick, was composed of a mixture of quartz, decomposed felspar, and gravel-size pieces of semi-decomposed granite together with pieces of rock. It had been thought that thin cement grout would probably penetrate sufficiently into that mixture to form a reasonable seal and after cleaning out the tunnel back to the original face it had been observed that, although there was a small cavity overhead in the fissure, the material above appeared to be cemented together and the tunnel roof and face were perfectly dry.

Open fissures in the face, of the order ⅛ to ¼ in., which appeared to follow the natural joints in the rock, had been filled with cement and four test holes drilled into the face had indicated no water until at about 15 ft there was a small seepage of clear water from the top holes. The bottom holes had been dry to 20 ft, at which depth the drilling had been stopped. A complete seal to 15 ft from the beginning of the decomposed rock had been effected and, indeed, only small drips at a few places had been observed in the advance beyond that point to the sound rock about 45 ft from the start of the decomposed rock. He had to admit that he did not examine the ground personally to ascertain the extent of the penetration of the grout as the face advanced beyond about 15 ft since, although he spent a considerable time in the tunnel during the advance through the fissure, he had at that time been more concerned with ensuring that the ribbing would be kept tight to the face and well packed to avoid the possibility of the roof coming in and undoing all their previous work.

Grouting was not an exact science, and it was doubtful whether data for the present scheme could be analysed to give a quantitative answer in terms of spacing of grout holes, distance from the tunnel, grouting pressure, etc., for the extent of the penetration of grout outside the ring of grout holes was completely unknown.

Regarding the possibility of using colloidal grout in the present scheme, it could be stated that that type of grout was much too thick for work of that nature since it would not penetrate into the decomposed material in the open fissure and it was doubtful, too, whether it would travel far enough in the small fissures in the ground ahead to effect full treatment in depth.

There was one correction he wished to make to Fig. 12. He had always been under the impression that the decomposed material was in fact decomposed granite. It looked very like kaolin. It was, of course, a zeolitized porphyry. Since he wrote the Paper the Geological Survey of Great Britain had published a Bulletin[6] which described a geological investigation carried out by one of its staff in Northern Ireland. It was apparent that between the extreme planes, about 80 ft apart, shown in Fig. 12, the rock was a composite dike. That to the right of the open fissure was a porphyrite, whilst to the left of the fissure the rock was zeolitized porphyry.

Furthermore, it was interesting to note from the Bulletin that there was evidence of a

[6] "The Slieve Binnian Tunnel, An Aqueduct in the Mourne Mountains." Bulletin of the Geological Survey of Great Britain, No. 8, 1955.

slickensided surface at the junction between the dike rock and the granite at the western margin of the dike, which suggested that at some time or other the composite dike had fractured along the plane indicated in Fig. 12 and the whole of the section to the left of the plane of fracture had moved. Following that fracture presumably acid waters or gases had come up from below along the plane and attacked the dike rock.

To complete the record, the time taken to do the work was about 29 weeks. That period included the initial 3 weeks during which the tunnel had been repeatedly mucked out and an attempt had been made to seal the fissure by means of piles. During the following 3 weeks a concrete bulkhead had been erected, and by the end of that period the specialist grouting and drilling equipment had arrived on site. During the day-time in that period the tunnel gangs had been employed on the mountain-side diverting streams that might perhaps have been feeding the fissure. Those efforts had not been successful, but he thought it had been worth attempting.

About 13 weeks had been spent in grouting the fissure and 8 weeks in advancing through the decomposed ground. It had taken another 2 weeks to advance the face 20 yd into the granite with very short pulls of a few feet at a time, so that the blast would not displace any of the ribs erected previously. During that time the tunnel had also been cleared of cement slurry and kaolin which had covered the invert for a considerable distance up the tunnel. The rising main from the pumps had been half-choked with a mixture of cement grout and kaolin and had been completely dismantled and cleaned out.

The Chairman had put forward a novel suggestion when he asked whether any use had been made in the past of any modified geophysical method to indicate trouble ahead. So far as the Author was aware there was no known geophysical method which could be applied to work in tunnels. The working space available at the face was so small that the spacing of the electrodes or geophones would be much too close to be effective.

Turning to Mr Carmichael's remarks the Author estimated that, on the basis of a 5½-hour round, an advance of about 140 ft/week in each heading could be expected in driving a tunnel in strata similar to that in the Mourne tunnel, and it should be noted that although the world record advance of 557 ft was achieved in No. 1 tunnel in October 1955 on the Breadalbane Hydro-Electric Project, the average rates of advance in the No. 2 and No. 3 tunnels had been only about 165 ft and 140 ft/week respectively.

Estimates of time for the carrying out of tunnelling schemes were based on the average rate of advance to be expected and that appeared to have increased from about 100 ft/week prewar to about 150 ft/week at the present time. The latter figure was less than one-third of the present record advance, and although even greater record advances might be achieved in the future, it was doubtful whether the average rate of advance would increase to more than 150 ft/week for many years to come.

The closing date for Correspondence on the foregoing Paper was 15 March, 1956. No contribution received later than that date will be printed in the Proceedings.—SEC.

Paper No. 6077

A DESIGN CHART FOR THE ECONOMIC SECTION FOR PRESTRESSED CONCRETE BEAMS

by

* Professor Reginald George Robertson, M.A., M.I.C.E.

(Ordered by the Council to be published with written discussion)

The design quantities

For any given beam the moment arising from superimposed load at any section may be denoted by M and the moment from the weight of the beam itself by mA, where A denotes the area of cross-section at the section considered; both M and m are known if the layout of the beam is given.

Equations will be developed connecting the area of section A, the depth d, and the cable pull P with the allowable working load stress C_2 for any specified top-flange-to-web proportions and using non-dimensional quantities by combining the required quantities with the known moments M and m as follows :—

$$\text{Area term} = \frac{mA}{M} \qquad \text{Cable-pull term} = \frac{P}{\sqrt{Mm}}$$

$$\text{Depth term} = d\sqrt{\frac{m}{M}} \qquad \text{Stress term} = \frac{C_2}{m}\sqrt{\frac{M}{m}}$$

A chart (Fig. 1) can be prepared from such equations; from it the effect of increasing the working stress C_2 is seen to give a smaller area and depth and a larger cable pull, except in long spans where the cable pull remains almost constant.

Since the stress at the time of tensioning C_1 is often specified as larger than the working stress C_2 it is necessary to state a relation between the maximum allowable values for C_1 and C_2. This relation is assumed as $C_2 = \eta C_1$ where η denotes the ratio of the cable pull at the section considered at time of full load (after relaxation) to the tension at time of tensioning (after anchoring). It is assumed that $\eta = 0{\cdot}85$ and in the first instance there is no tensile fibre stress either at time of tensioning or at full load.

Definitions of spans

The definition of a short span is that the cable position is decided by the necessity to avoid tension in the upper fibre at the time of tensioning the cables.

This condition is later shown to be: $\dfrac{m}{C_1 d} = \dfrac{d-y}{d} \cdot \dfrac{y-g}{d} - \dfrac{y}{d} \cdot \dfrac{h}{d}$

Hence the largest value of m for a short span occurs when g is at its minimum value The governing stress is C_1.

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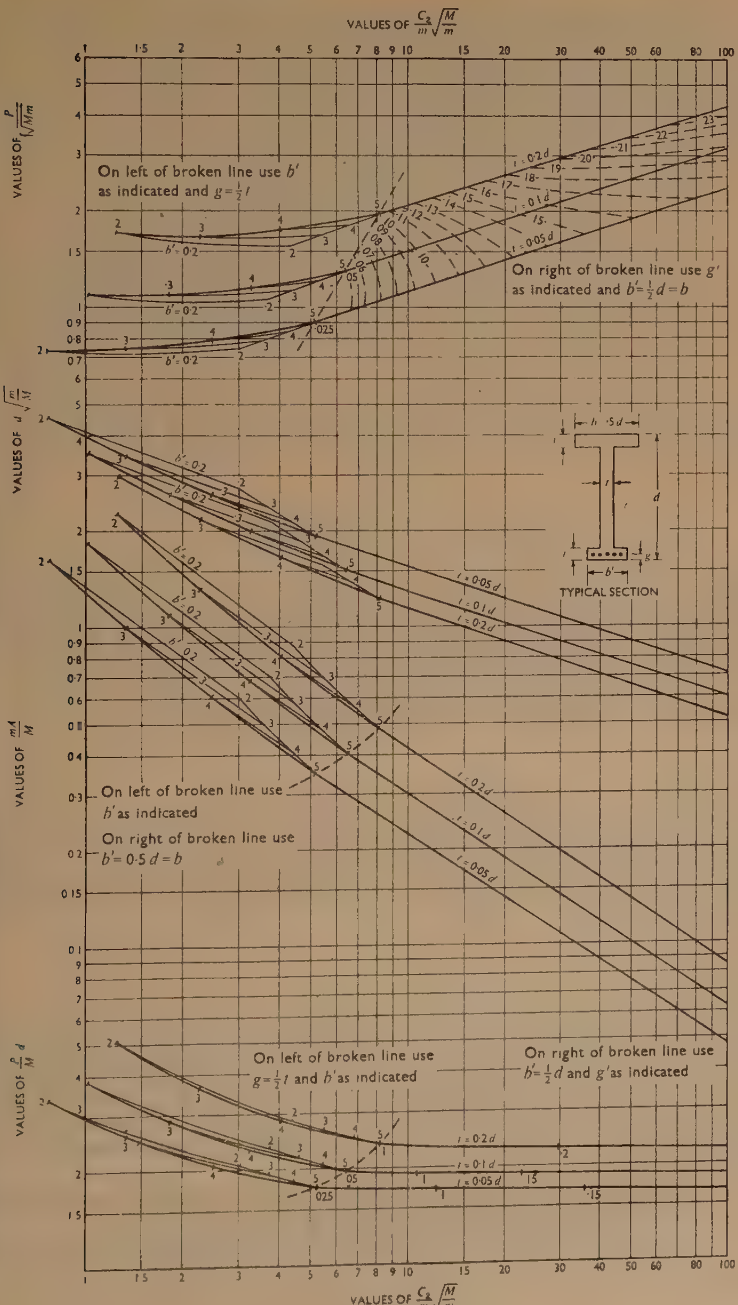

FIG. 1a.—SECTIONS WITH FLANGES AND WEB OF EQUAL THICKNESS AND WIDTH OF TOP FLANGE EQUAL TO HALF THE DEPTH

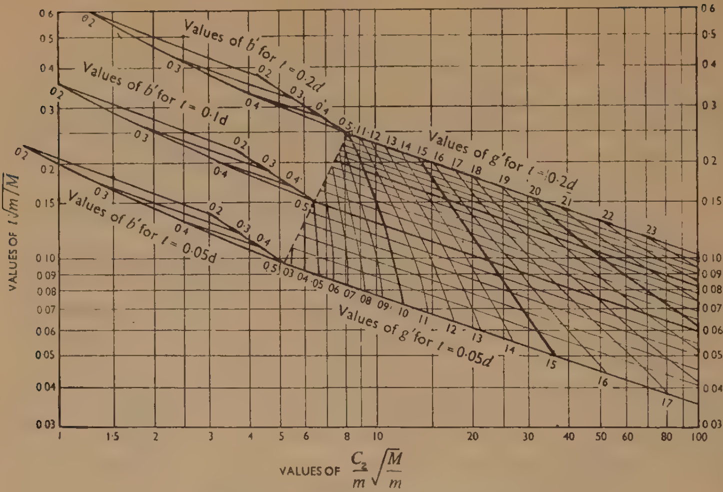

Fig. 1b.—Continuation of Fig. 1a

Long-spans

The definition of a long span is that, when the cables are at their lowest possible position and the bottom flange at its minimum size to accommodate the cables, the bottom fibre stress at the time of tensioning is less than that allowable.

This condition is later shown to be: $\frac{m}{C_1 d} > \frac{y}{d} \cdot \frac{y-g}{d} - \frac{d-y}{d} \cdot \frac{h}{d}$

Medium spans

With the cables at their lowest position there is no possibility of tensile stress in the top fibre for spans between the above limits for short or long spans. These limits coincide for a symmetrical section with equal flanges, but when the bottom flange is smaller than the top there is an intermediate range of spans for which C_1 is the governing stress.

Properties of short spans

Width of bottom flange

For a short span with any specified top-flange-to-web proportion an increase in the bottom flange reduces the area of section and the depth required without appreciably altering the cable pull, until the bottom flange becomes equal to the top flange. Further increase in the bottom flange increases the area of section for a given working stress C_2 so that the economic section for short spans has equal flanges when $C_2 = \eta C_1$ as assumed above.

For other ratios of the allowable maximum stresses the economic section for short spans is such that $\frac{d-y}{y} = \frac{C_2}{\eta C_1}$ and if $C_2 > \eta C_1$ a larger bottom flange will be economical for short spans.[1]

[1] The references are given on p. 194.

Working stress

The depth and area of section are reduced when the working stress is increased, so that if depth is important the highest stress is required consistent with the allowable deflexion of the beam.[2] The reduction in depth, however, involves increase in cable pull and if the cost of the cable exceeds double the cost of the concrete, i.e., exceeds two-thirds the cost of the whole beam, then for short spans economy in cost results if the beam is made deeper with a reduction in working stress.

Properties of medium spans

Width of bottom flange

An increase in the bottom flange reduces the area and the depth but increases the cable pull so that for economy a study of the proportions must be made.

Working stress

An increase in the working stress reduces the depth and area of section but has little effect on the cable pull so that the highest stress consistent with allowable deflexion should be used.

Properties of long spans

Width of bottom flange

The bottom flange must be as small as possible to accommodate the cables since an increase in the bottom flange increases the area, the depth, and the cable pull.

Working stress

An increase in working stress reduces the area, the depth, and the cable pull so the highest stress consistent with allowable deflexion must be used.

Minimum size of bottom flange

To accommodate the cables the area may be taken as about 5 times the area of the ducts and with Freyssinet cables this gives an area (top flange $+ \frac{1}{2}$ web) $\times \dfrac{C_2 \text{ lb/sq. in.}}{6{,}000}$

Proportions for top flange to web area to ensure failure by steel elongation

When failure occurs through elongation of the steel a crack opens and the concrete forms a plastic hinge in the upper flange, which bears the whole compression force. The steel should reach its ultimate strength first if a brittle failure is to be avoided and in this case a minimum ratio for the top flange area to web area in terms of the working stress factors could be specified as follows.

Let the area of web be $t.d$ where the thickness t is assumed constant for web and flanges.

Let area of top flange be $t.b$.

Let the concrete working stress be C_2 and the ultimate stress C_u.

Let area of steel cable be pA, working stress f, and ultimate stress f_u.

Let the factor for steel stress be $f_u/f = F$.

Let the factor for the concrete stress be $C_u/C_2 = N$.

If the bottom flange equals the top flange the greatest value of P is obtained: $P = C_2 t\left(b + \frac{d}{2} - t\right)$.

At ultimate load, to avoid a compression failure: $tbC_u > FP$

Substituting for P: $$\therefore \frac{bC_u}{F} > C_2\left(b + \frac{d}{2} - t\right)$$

$$\therefore b\frac{N-F}{F} > \frac{d}{2} - t$$

$$\therefore \frac{b}{d} > \frac{F}{N-F}\left(\frac{1}{2} - \frac{t}{d}\right)$$

If $N = 2F$ then $$\frac{b}{d} > \left(\frac{1}{2} - \frac{t}{d}\right)$$

If the bottom flange is smaller than the top, the top flange can be reduced. If there is no bottom flange: $\frac{b}{d} > \left(\frac{1}{2} - \frac{t}{d}\right) - \frac{t}{d}\left(1 - \frac{t}{d}\right)$

Equations

Let C_2' be the top fibre stress at time of full load and let this have a maximum allowable value C_2. Let C_1' be the bottom fibre stress at time of cable tensioning and let this have a maximum allowable value C_1. Let $C_2 = \eta C_1$.

Any shape of section can be used for any span and loading, but the area, depth, and cable pull required will vary according to the shape selected: also the governing stress may be the maximum initial stress C_1 or the maximum full load stress C_2.

Since there is to be no tensile stress in the concrete the stress distribution at full load is as shown in Fig. 2a.

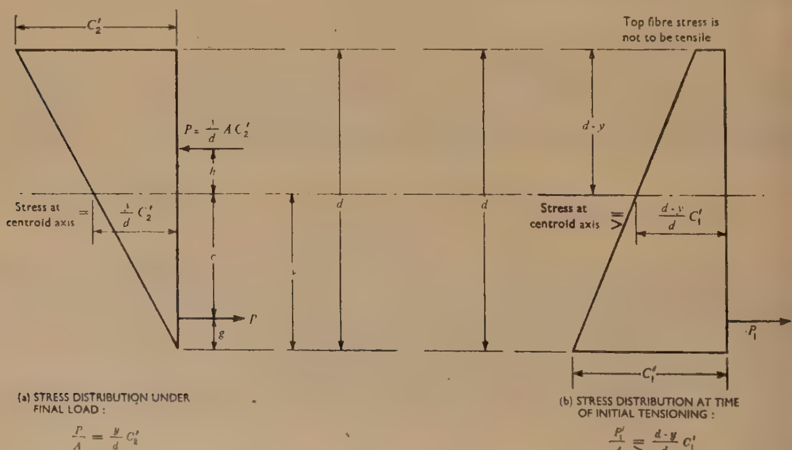

Fig. 2

Note:—Centroid is at centre of gravity at net concrete area:

From above: $$P/P_1 = \eta$$
$$P_1/P \geqq d-y/yC_1'/C_2'$$
$$\therefore C_2'/\eta C_1' \geqq d-y/y$$

It is evident that: $$P = \frac{y}{d} A C_2' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The initial dead load moment plus moment from superimposed load is

$$mA + M = P(e + h) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where h denotes the distance from the centre of gravity to the upper core point.

Substituting for P in equation (2): $mA + M = \frac{y}{d}(e + h) A C_2'$

Substituting $e = y - g$, where g is the cable cover:

$$C_2' = \frac{mA + M}{A} \cdot \frac{d}{y(y + h - g)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From Fig. 2b it is evident that if there is to be no tensile stress in the top fibre at time of tensioning: $$P_1 \geqq \frac{d - y}{d} A C_1'$$

Substituting $P_1 = \frac{P}{\eta}$ from equation (1): $\frac{y}{\eta d} A C_2' \geqq \frac{d - y}{d} A C_1'$

$$\therefore \eta C_1' \leqq \frac{y}{d - y} C_2' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

C_1' is found from consideration of the initial moment mA and initial cable pull P_1:

$$C_1' = \frac{P_1}{A} + \frac{P_1 e y}{I} - \frac{mAy}{I}$$

$$= P_1 \frac{e + h}{Ah} - \frac{m}{h}$$

$$\therefore C_1' = \frac{P}{\eta} \cdot \frac{e + h}{Ah} - \frac{m}{h}$$

Substituting for $P(e + h)$ from equation (2):

$$C_1' = \frac{M + (1 - \eta) mA}{\eta h A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Eliminating M/A from equations (3) and (5):

$$\frac{M}{A} = C_2' \frac{y}{d}(y + h - g) - m = \eta h C_1' - (1 - \eta) m \quad . \quad . \quad . \quad (6)$$

$$\therefore \frac{C_2' d}{m} \cdot \frac{y}{d} \cdot \frac{y + h - g}{d} = \eta \frac{h}{d} \cdot \frac{C_1' d}{m} + \eta \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Short spans

In short spans the cable position must be raised so that no tensile stress is present in the top fibre at time of tensioning and the section shape is given by using equation (4) as an equality so that both allowable maximum stresses are realized in full and $\frac{d - y}{y} = \frac{C_2}{\eta C_1}$

When $C_2 = \eta C_1$ as assumed above, $y = d/2$ and a symmetrical section is required. Substituting $y = d/2$: $C_2' = C_2 = \eta C_1$

From equation (7): $$\frac{\eta m}{C_2 d} = \frac{m}{C_1 d} = \frac{1}{2}\left(\frac{1}{2} - \frac{h}{d} - \frac{g}{d}\right)$$

$$\therefore \frac{g}{d} = \frac{1}{2} - \frac{h}{d} - 2\eta\frac{m}{C_2 d} \qquad (8)$$

From equation (3):

$$\frac{M}{mA} = \frac{y}{d} \cdot \frac{y+h-g}{d} \cdot \frac{C_2 d}{m} - 1 \qquad (9)$$

From equation (1):

$$\frac{P}{mb} = \frac{y}{d} \cdot \frac{A}{bd} \cdot \frac{C_2 d}{m} \qquad (10)$$

The three quantities above are evaluated for various values of $C_2 d/m$ and plotted noting that:

$$\frac{C_2}{m}\sqrt{\frac{M}{m}} = \frac{C_2 d}{m}\sqrt{\frac{A}{d^2} \cdot \frac{M}{mA}} \qquad (11)$$

$$d\sqrt{\frac{m}{M}} = \frac{d}{\sqrt{A}}\sqrt{\frac{mA}{m}} \qquad (12)$$

$$\frac{Pd}{M} = \frac{P}{mb} \cdot \frac{bd}{A} \cdot \frac{mA}{M} \qquad (13)$$

$$\frac{P}{\sqrt{Mm}} = \frac{P}{mb}\, d\sqrt{\frac{m}{M}} \cdot \frac{b}{d} \qquad (14)$$

Medium spans

In medium spans the cable is given its lowest position and $C_1' = C_1$ is the governing stress and $\eta C_1 \gtreqless C_2'$. The smallest value of C_2' is associated with zero initial top fibre stress.

From equation (4): with $C_1' = C_1$: $C_2' \gtreqless \frac{d-y}{y}\eta C_1 \qquad (15)$

Since $\eta C_1 \gtreqless C_2'$ then $y \gtreqless \frac{d}{2}$

From equation (7): substituting for $C_2' \gtreqless \frac{d-y}{y}\eta C_1$ from equation (15):

$$\therefore \frac{d-y}{d} \cdot \frac{y+h-g}{d} \lesseqgtr \frac{h}{d} + \frac{m}{C_1 d}$$

$$\therefore \frac{\eta m}{C_2 d} = \frac{m}{C_1 d} \gtreqless \frac{d-y}{d} \cdot \frac{y-g}{d} - \frac{h}{d} \cdot \frac{y}{d} \qquad (16)$$

Equation (16) used as an equality gave the deepest beam with zero initial top fibre stress, and minimum cable size.

From equation (7) substituting $C_2' \lesseqgtr \eta C_1$:

$$\therefore \frac{y}{d} \cdot \frac{y+h-g}{d} \gtreqless \frac{h}{d} + \frac{m}{C_1 d}$$

$$\therefore \frac{\eta m}{C_2 d} = \frac{m}{C_1 d} \lesseqgtr \frac{y}{d} \cdot \frac{y-g}{d} - \frac{h}{d} \cdot \frac{d-y}{d} \qquad (17)$$

Equation (17) used as an equality gave the shallowest beam with maximum allowable

ull load stress C_2: any values of $\frac{m}{C_1 d}$ within the above limits could be used for a "medium span."

From equation (5):
$$\frac{M}{mA} = \frac{h}{d} \cdot \frac{C_2 d}{m} - (1 - \eta) \quad \ldots \quad (18)$$

From equations (1) and (7), eliminating C_2':
$$\frac{P}{mb} = \frac{A}{bd} \cdot \frac{d}{y + h - g}\left(\frac{h}{d} \cdot \frac{C_2 d}{m} + \eta\right) \quad \ldots \quad (19)$$

he two quantities (18) and (19) above are evaluated for various values of $C_2 d/m$ etween the limits given by equations (16) and (17) and converted for plotting by sing equations (11) to (14).

ong spans

In long spans the cable is given its lowest position and $C_2' = C_2$ is the governing tress and $C_1' < C_1$.

From equation (7):
$$\frac{C_2 d}{m} \cdot \frac{y}{d} \cdot \frac{y + h - g}{d} \lesseqgtr \eta \frac{h}{d} \cdot \frac{C_1 d}{m} + \eta$$

$$\therefore \eta \frac{m}{C_2 d} = \frac{m}{C_1 d} \gtreqless \frac{y}{d} \frac{y - g}{d} - \frac{h}{d} \frac{d - y}{d} \quad \ldots \quad (20)$$

From equation (3):
$$\frac{M}{mA} = \frac{C_2 d}{m} \cdot \frac{y}{d} \cdot \frac{y + h - g}{d} - 1 \quad \ldots \quad (21)$$

From equation (1):
$$\frac{P}{mb} = \frac{y}{d} \cdot \frac{A}{bd} \cdot \frac{C_2 d}{m} \quad \ldots \quad (22)$$

he two quantities of equations (21) and (22) may be evaluated for values of $m/C_2 d$ reater than the limit of equation (20) and converted by equations (11) to (14) to ive the quantities plotted.

It is found that the spans given by equation (20) are longer than normally conemplated; these are not included in the design chart.

Tensile stress

When tensile stress is allowed the section may be designed by the chart and the able pull may be reduced as follows. Two cases arise.

hort spans

If the cable is not at its lowest position it may be lowered by (Δg) (see Appendix),

here:
$$\frac{\Delta g}{d} = -\frac{\frac{C'}{C_1}\left(1 + \frac{mA}{M}\right) + \eta \frac{C''}{C_2} \cdot \frac{mA}{M}}{\left(\frac{P}{\sqrt{Mm}}\right)\left(d\sqrt{\frac{m}{M}}\right)\left(1 + \frac{C'}{C_1} + \frac{C''}{C_2}\right)} \quad \ldots \quad (23)$$

here $-C'$ denotes the initial tensile stress allowed (negative)
$-C''$,, the full load tensile stress allowed (negative)

Also
$$\frac{\Delta P}{P} = \frac{C'}{C_1} + \frac{C''}{C_2} \text{ (negative)} \quad \ldots \quad (24)$$

The maximum stresses are reduced and become:

$$C_1\left(1+\frac{C''}{C_2}\right) \quad \text{and} \quad C_2\left(1+\frac{C'}{C_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where C' and C'' are negative.

If the value of Δg given above is excessive then the tensile stress C'' may be reduced accordingly and a further reduction in cable pull is allowable for the remainder of the allowable tensile stress C'', as below.

Long, medium, and short spans as noted above

With the cable at its lowest position then a reduction in cable pull $-\Delta P$ causes a tensile stress $-C''$ at time of full load.

There is also a slight increase in maximum stress C_2 but this increase is much smaller than C''.

The initial stress C_1 is reduced to $C_1\left(1+\dfrac{C''}{C_2}\right)$ . . . . (26)

and $$\frac{\Delta P}{P}=\frac{C''}{C_2}\cdot\frac{\dfrac{mA}{M}}{1+\dfrac{mA}{M}}\cdot\frac{\dfrac{C_2}{m}\sqrt{\dfrac{M}{m}}}{\dfrac{P}{\sqrt{Mm}}}\left(1+\frac{mA}{M}-\frac{y-g}{d}\cdot\frac{P}{Mm}\cdot d\sqrt{\frac{m}{M}}\right) \quad . \quad (26)$$

This expression requires the value of y/d; otherwise the quantities are those given on the chart.

Example

See reference 4, pp. 47–49. Span: 50 ft; load: 1,000 lb/ft; web thickness: 4 in.

Assume stress $C_2 = 1{,}700$ lb/sq. in. ($C_{tf} = 1{,}652$ lb/sq. in., ref. 4, p. 48).
($Cb_i = 1{,}839$ lb/sq. in., ref. 4, p. 49).

$$m = 150 \times \frac{50^2}{8\times 12}\ \text{lb/in.}$$

$$\therefore\ m = 3{,}920\ \text{lb/in.}$$

$$M = 3{\cdot}75 \times 10^6\ \text{in. lb.}$$

$$\therefore\ \frac{C_2}{m}\sqrt{\frac{M}{m}} = 13{\cdot}4 \quad \text{and}\ t\sqrt{\frac{m}{M}} = 0{\cdot}13$$

Interpolating on the chart (Fig. 1b) for $t\sqrt{m/M}$ it is seen that $g = 0{\cdot}122d$; $t = 0{\cdot}113d$; and $b' = b = 0{\cdot}5d$

Since $t = 4$ in., then $d = 35{\cdot}5$ in.

From the chart $P/\sqrt{mM} = 1{\cdot}74$

$$\therefore\ P = 211{,}000\ \text{lb.}$$

From the chart $mA/M = 0{\cdot}27$; this is a short span.

Allowing tensile stress 150 lb/sq. in.

$$\frac{C'}{C_1} = -\frac{150}{2{,}000} = -0{\cdot}075;\ \frac{C''}{C_2} = -\frac{150}{1{,}700} = -0{\cdot}08.$$

From equation (23): $$\frac{\Delta g}{d} = \frac{0{\cdot}075\times 1{\cdot}27 + 0{\cdot}85\times 0{\cdot}08\times 0{\cdot}27}{1{\cdot}74\times 1{\cdot}13\times 0{\cdot}845} = 0{\cdot}068$$

New value of $g/d = 2{\cdot}122 - 0{\cdot}068 = 0{\cdot}54 \therefore g = 1{\cdot}9$ in.
From equation (24):

$$\Delta P/P = -0{\cdot}155$$

Therefore new value of $P = 178{,}000$ lb. (Walley: 184,000) and C_2 is reduced to 1,570 lb/sq. in.

The depth and the cable pull are very similar to but less than the example quoted using the same stresses and area of section; this is due to a lower cable position.

Additional graph for depth times cable full

Since the same depth is required for a thick beam with a small bottom flange and for a thin beam with a larger bottom flange, the graphs showing depth for different thicknesses overlap each other and for design purposes the depth can best be found from the added graph giving the value of Pd/M since this is little affected by the size of the flanges. Thus P is found from the first and d from the last graph.

Conclusion

It is seen that small depth is achieved by thick and wide sections (the smallest depth is given by a wide rectangular beam[3]) but the cable pull is in inverse proportion to the depth.

The graphs submitted indicate the method by which an economic section for any beam can be found for any circumstances specified.

The three proportions shown illustrate the method and for other proportions similar graphs can readily be constructed from the equations given.

It was found that both the area of section and the cable pull remained practically unchanged when the ratio b/d was altered within wide limits if t/d remained constant.

In the case of "short" spans the depth changed in inverse ratio to b/d such that a 10% increase in b/d gave a 10% decrease in depth, whereas in "medium" spans the depth remained practically unchanged while the bottom flange was reduced by approximately the same amount that the top flange was increased.

This had already been shown in an example taken from practice.[5]

In comparison with the method for designing an economic section as given previously[2] the present method is much simpler and saves considerable labour but requires separate charts for various ratios of b/d for its full application. The minimum width given on p. 188 as $b = \left(\frac{d}{2} - t\right)$ gives the most economic use of materials but gives a rather larger depth (a few %) than that given by the present chart for $t = d/2$.

The method now submitted clearly illustrates the effect of varying the working stress and also allows for some saving in cable pull when tensile stress in the concrete is permissible.

Notation

A denotes net area of cross-section at section considered; add area of ducts to obtain gross area of section.
b ,, width of top flange.
b' ,, width of bottom flange.
C_1 ,, allowable maximum concrete stress at time of tensioning.
C_2 ,, allowable maximum concrete stress time of full load.
C' ,, allowable minimum compressive concrete stress at time of tensioning.
C'' ,, allowable minimum compressive concrete stress at time of full load.
C_1' ,, actual maximum concrete stress at time of tensioning.
C_2' ,, actual maximum concrete stress at time of full load.
C_u ,, ultimate compressive concrete fibre stress.
d ,, depth of beam at section considered.
e ,, distance of cable centroid below centroid of net concrete cross-section area.
F ,, factor of safety for cable steel working stress.
f ,, cable steel working stress.
f_u ,, cable steel ultimate stress.
g ,, distance of cable centroid above bottom fibre of beam.
Δg ,, change in g permitted by a change in concrete stress.
h ,, distance of upper core point above centroid of net concrete cross-section area $= I/Ay$.
I ,, moment of inertia of net concrete cross-section area.
M ,, moment at section considered owing to superimposed load added after cables are tensioned.
m ,, moment at section considered per unit area of section considered at time of cable tensioning.
N ,, factor of safety for concrete working stress $= Cu/C_2$.
P ,, cable pull at section considered at time of full load after relaxation.
P_1 ,, cable pull at section considered at time of cable tensioning after anchoring.
ΔP ,, change in cable pull permitted by a change in concrete stress.
$p = P/Af$.
t denotes thickness of web and flanges when these are equal.
y ,, distance of centroid of net area of concrete cross-section above bottom fibre of beam.
$\eta = P/P_1$
$\phi = A/d^2$

References

1. R. G. Robertson, "Prestressed Concrete Beams, a Rational Design Method." *Struct. Engr*, vol. 30, p. 259 (Nov. 1952).
2. R. G. Robertson, "Prestressed Concrete Beams: the Economical Shape of Section." Proc. Instn Civ. Engr, Pt III, vol. 3, p. 242 (Apr. 1954).
3. R. G. Robertson, "Design charts for prestressed rectangular beams and slabs." Proc. Instn Civ. Engrs, Pt III, vol. 4, p. 155 (Apr. 1955).
4. F. Walley, "Prestressed Concrete Design and Construction." H.M.S.O., 1953.
5. A. J. Harris, "Hangars at London Airport." *Struct. Engr*, vol. 30, p. 226 (Oct. 1952). *See* discussion by Professor Robertson, *Struct. Engr*, vol. 31, p. 66 (Feb. 1953).

APPENDIX

TENSILE STRESSES: SHORT SPANS WITH EQUAL FLANGES

The section designed by the chart has an upper core point at a distance equal to the lever arm $M + mA/P$ above the cable position and a lower core point given by $\eta\frac{mA}{P}$ above the cable.

A compression force ΔP_1 applied at the upper core point produces a compression stress $C' = 2\Delta P_1/A$ in the upper fibre and does not change the lower fibre stress.

A compressive force ΔP applied at the lower core point produces a compressive stress $C'' = 2\Delta P/A$ in the lower fibre and does not change the upper fibre stress.

If the stresses C' C'' are specified the final cable pull is the resultant of P, ΔP, and $\eta \cdot \Delta P_1$.

The maximum initial stress is amended by C''/η and the maximum full load stress is amended by $\eta C'$ owing to change in cable pull of $\eta\Delta P_1 + \Delta P = \frac{A}{2}(\eta C' + C'')$.

The cable position is amended by the amount:

$$\Delta g = \frac{\eta\frac{\Delta P_1}{P}(M + mA) + \eta\frac{\Delta P}{P}mA}{P + \eta\Delta P_1 + \Delta P}$$

Substituting

$$\Delta P_1 = \tfrac{1}{2}AC'$$

$$\Delta P = \tfrac{1}{2}AC''$$

and

$$P = \tfrac{1}{2}AC_2$$

$$\therefore \Delta g = \eta\frac{M}{P}\,\frac{C' + (C'' + C')\frac{mA}{M}}{C_2 + \eta C' + C''}$$

Note that C' C'' are to be taken negative if tensile so that the cable is lowered; the cable pull and the maximum stresses are reduced by the use of tensile stresses.

Long spans

If a change in cable pull P is made creating a full load bottom fibre stress C'':

$$C'' = \frac{\Delta P}{A} + \frac{\Delta P}{Ah}(y - g)$$

$$\therefore \Delta P = \frac{AC''h}{y + h - g}$$

This will also alter the initial maximum stress by C''/η, and cause a change in full load maximum stress which, however, is much smaller than C''.

Note: $y + h - g = (M + mA)/P$

$$\therefore h = \frac{M + mA}{P} - (y - g)$$

$$\therefore \frac{\Delta P}{P} = \frac{AC''}{P} \cdot \frac{M + mA - P(y - g)}{M + mA}$$

Using the non-dimensional quantities plotted:

$$\frac{\Delta P}{P} = \frac{C''}{C_2}\,\frac{\frac{mA}{M}}{1 + \frac{mA}{M}}\,\frac{\frac{C_2}{m}\sqrt{\frac{M}{m}}}{\frac{P}{\sqrt{Mm}}}\left(1 + \frac{mA}{M} - \frac{y - g}{d} \cdot \frac{P}{\sqrt{Mm}} \cdot d\sqrt{\frac{m}{M}}\right)$$

The Paper, which was received on 20 February, 1955, is accompanied by two sheets of diagrams from which the Figures in the text have been prepared and by the foregoing Appendix.

CORRESPONDENCE on this Paper should be forwarded to reach the Institution before 15 August, 1956. Contributions should not exceed 1,200 words.—SEC.

Paper No. 6085

THE EFFECT OF UPLIFT ON GRAVITY-DAM PROFILES

by

* **Eric Hugh Brown, Ph.D., B.Sc.(Eng.), D.I.C., A.M.I.C.E.**

(*Ordered by the Council to be published with written discussion*)

SYNOPSIS

Graphs and simple formulae are given to show the effect of internal hydraulic uplift forces with any disposition of drains on the stresses and profile of a solid gravity dam, assuming there is a linear distribution of vertical stress in the concrete. The effect of different values and distributions of uplift pressures on the volume of medium-height dams can be seen immediately.

The results of two philosophies of design are compared, and it is found that for medium-height dams the difference in total weight is not great. A later Paper will show that this is not true when earthquake loading is involved.

The arrangement of internal drains and their effect upon the distribution of vertical stress in the concrete is discussed. It is shown that the customary assumption of linearity of this distribution may be unsatisfactory when there are internal drains, but the conclusions of the Paper are still valid provided that certain parameters are redefined.

Introduction

The problem of uplift in gravity dams is twofold. There is the assessment of the forces to which a dam will be subjected by internal hydraulic pressure, and the design of a profile to allow for such forces. The Paper is concerned only with the second problem, that of structural design. It will be assumed that the hydraulic problem can be solved, and that the designer has satisfied himself as to all the forces which his dam has to resist.

The classical principles of design were set down by Wegmann,[1, 2] who assumed a linear distribution of vertical stress in the material and considered the dam in five sections, starting at the top with a fixed freeboard and top width. This linearity assumption will be discussed later, for it is far less satisfactory than is commonly supposed.

Stage I is a rectangular profile, ending at such a depth as will give zero vertical stress at the water face with the reservoir full; to continue the rectangular section any deeper would introduce tensile stresses.

Stage II maintains this condition of no tension by batter of the downstream face and ends with the beginning of tension in the downstream face with the reservoir empty.

In stage III both faces are battered to satisfy the no-tension criterion with the reservoir full or empty.

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[1] The references are given on p. 206.

This is the classical philosophy, referred to here as a "stage III design." However, the need to ensure no tension with the reservoir empty has often been questioned, the dissentients suggesting that horizontal cracks penetrating some way into the dam from the downstream face will not lead to collapse and may be tolerated. If this philosophy were accepted, batter of the upstream face in stage III could be discarded and the profile could be continued down using the stage II criterion, giving a "stage II design."

It is not proposed to enter into controversy here on the rival merits of these philosophies. Both will be considered so that the resulting profiles can be compared.

Wegmann's stages IV and V increase the batter of the faces to keep the compressive stresses within a safe limit. In the Paper the discussion is confined to "medium-height dams," defined as dams which do not enter stage IV. With permissible concrete stresses of 500 lb/sq. in. this allows discussion of dams up to 400 or 500 ft high.

The Paper presents and discusses the physical results obtained; mathematical derivations are given in the Appendices.

Principal notation

a denotes freeboard.
b ,, top width.
f_c ,, maximum permissible compression stress in concrete.
H ,, depth below water surface.
l ,, width of dam.
$m = l/H$.
p denotes vertical compressive stress at downstream face (reservoir full).
$\bar{p}$,, maximum permissible value of p.
q ,, vertical compressive stress at upstream face (reservoir empty).
$\bar{q}$,, maximum permissible value of q.
w ,, density of water.
$y = l - z$.
z denotes batter of upstream face, measured horizontally from the upstream face at the water surface.
β ,, proportion of the area of a cross-section on which uplift is assumed to act.
γ', γ'' denote uplift coefficients. (γ' and γ'' may need to be redefined, as explained on p. 200.)
ρw denotes density of concrete.
ϕ ,, angle of batter of downstream face to the vertical.

Other notation will be defined in the text as required.

Uplift forces

Let it be assumed that the hydraulic problem has been solved, taking account of drains and the effective area over which uplift is assumed to act. Then the mean of pressures along an appropriate length of the dam gives a distribution diagram at any particular depth resembling Fig. 1.

Since dams are designed solely on the principles of equilibrium this pressure distribution may be replaced by any statically equivalent system without affecting the ultimate profile, and the system shown in Fig. 2 may be adopted. γ' and γ'' can always be selected to make this system equivalent to the system in Fig. 1.

In general both the shape and the size of the distribution diagram of Fig. 1 will vary with H, so that γ' and γ'' are both functions of H. If, however, the drains are so arranged as to keep the *shape* of the diagram constant throughout the height of the dam—or if a constant shape can be considered a fair approximation—then the uplift pressures will be in simple proportion to H, and the total uplift force in proportion to Hl, so that γ' and γ'' will be constants. This will be so when there are no drains, for then the shape of the diagram at all heights is a triangle, and $\gamma' = 1$,

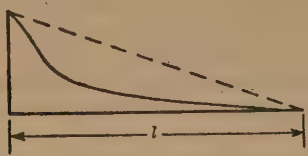

FIG. 1

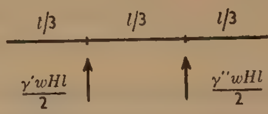

FIG. 2

$\gamma'' = 0$ if uplift is assumed to act over 100% of the area of horizontal cross-sections, (i.e., $\beta = 1$).

In this Paper γ' and γ'' will be considered as constant throughout each stage of the design, and it will be assumed that there is no tail-water.

THE EFFECT OF DRAINS ON THE STRESS DISTRIBUTION

The weight of dam above any horizontal section is carried by two systems of vertical forces acting across the section. They are the hydraulic-uplift pressures and the material-contact stresses. It is customary to consider the material stresses as varying linearly from one face of the dam to the other, and when there is no uplift the analyses of Brahtz[3] and Zienkiewicz[4] have shown this to be a good approximation except in the lower parts of the dam.

Brahtz in another Paper[5] showed the effect of uplift pressures on the material stresses. He discussed a homogeneous elastic dam of uniform permeability, and considered only the case when both stress distribution and hydraulic flow are strictly two-dimensional. For this case Brahtz proved rigorously that the material stresses in a given profile when uplift is acting are equal to the corresponding stresses when there is no uplift minus the uplift pressures. This applies only to the direct stresses; the shear stresses are not changed by uplift. When there are no drains, since the uplift pressure will vary approximately linearly across the dam, the linear distribution assumption for material stress is thus still a good one.

In a strictly two-dimensional flow and stress system the only possible drains are horizontal galleries running along the length of the dam. When there is no uplift such a gallery will cause a stress concentration, and hence a local variation from the linear stress distribution, shown in Fig. 3 for a section passing through the gallery.

The uplift pressure distribution will resemble Fig. 4a, and if the dam was designed on the assumption of linear material stress the *assumed* total pressure distribution resembled Fig. 4b, and was statically equivalent to the distribution in Fig. 3.

Brahtz's solution shows that in fact the *total* pressure distribution is still the same as in Fig. 3, and the material stress is found by subtracting the uplift pressures from it, giving the shaded distribution diagram in Fig. 5.

The horizontal gallery has thus not only resulted in a non-linear material stress distribution but has failed in its object of preventing tension in the upstream face.

Had there been no gallery the same profile would have produced the diagram in Fig. 6, from which it appears that the only beneficial effect of a horizontal gallery is to reduce the width of the tension zone and to introduce a region of high compressive stress adjacent to it. This will effectively limit the penetration of tension cracks, should the tensile strength of the concrete be insufficient to carry the load. Fortunately it is usually unlikely that short cracks of this kind will precipitate a catastrophic failure of the dam, even allowing for the resulting increased uplift pressures.

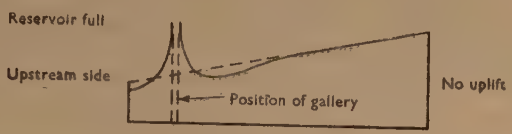

Fig. 3

Fig. 4

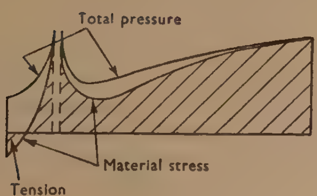

Fig. 5

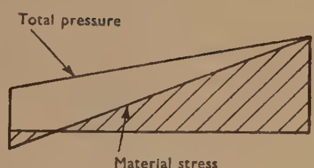

Fig. 6

Other methods of reducing the uplift forces are not so easily discussed. Vertical drains, for instance, will not induce the sharp stress concentration of the horizontal gallery, but will provoke a three-dimensional stress system and a three-dimensional hydraulic-flow pattern. Again, a region of denser concrete at the upstream face gives a non-uniform permeability. In either case Brahtz's treatment is not valid and, so far as the Author is aware, there is no published guidance on the effect of uplift on the *distribution* of the material-contact stresses.

If Brahtz's result is in fact generally applicable it will follow that in order to ensure no tensile material stress the total pressure must not be less than βwH at the upstream face, where β denotes the proportion of area over which uplift is assumed to act (see Lévy[6]). Drains and dense concrete will not then affect the width of the profile; for since the total pressure will be equal to the material stress with no uplift, for which

a linear distribution is reasonable, the profile must in any case be designed for a total pressure equivalent to a linear material stress distribution and a linear uplift distribution with $\gamma' = \beta, \gamma'' = 0$.

This severe condition is quite possibly not true except in the case of horizontal galleries already discussed. It may well be that vertical drains do in fact justify an economy of section, and that the customary assumption of a linear distribution of material stresses is not far from the truth. More research is required. Meanwhile, in this Paper a linear distribution will be assumed, and the effect of varying γ' and γ'' will be studied.

If later investigations disprove the linearity assumption, all that will be required to re-establish the validity of the results of the Paper will be a redefinition of γ' and γ'', such that $\gamma' wHl/2$ and $\gamma'' wHl/2$ are statically equivalent to the total pressure diagram minus a triangular diagram varying from zero at the upstream face to the maximum material stress at the downstream face. In other words, the total upward pressure must be equivalent to $\gamma' wHl/2$ and $\dfrac{pl}{2} + \dfrac{\gamma'' wHl}{2}$ acting at the third-points of the section. This is why, if Brahtz's result is generally true, γ' should be taken as β and γ'' as 0 whatever the uplift distribution, for the total pressure then varies linearly from βwH at the upstream face to p at the other.

Stage I

The width of the dam is constant, and it is only required to find the depth H_1 at which stage I ends. This is given by the only real and positive root of:

$$H_1^3 - (\rho - \gamma')b^2 H_1 - \rho a b^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(see Appendix I). Taking ρ, conservatively, as 2·25 (implying a concrete density of 140 lb/cu. ft), the solution is plotted in Fig. 7 for values of γ' and of r ($= a/b$) between

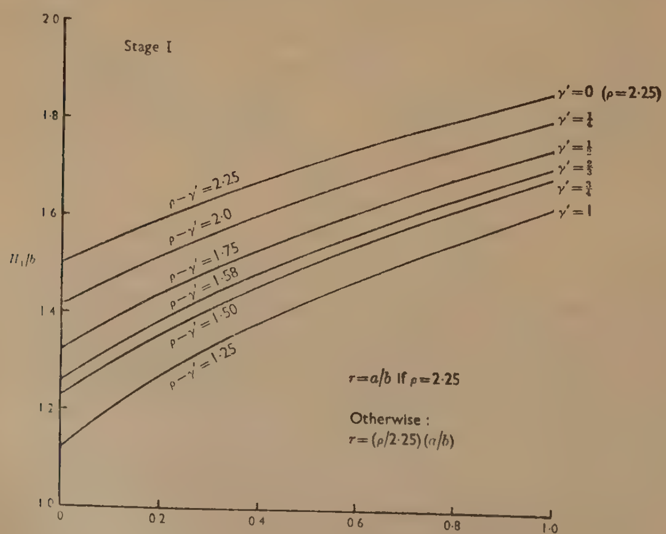

FIG. 7

0 and 1. For other values of ρ Fig. 7 can still be used by redefining r as $(a/b)(\rho/2{\cdot}25)$ and using the curve for the appropriate value of $(\rho - \gamma')$ instead of γ'.

Note that the equation for H_1 is independent of γ''.

Stage II

The best method for designing stage II is probably the standard step-by-step method. Let the width of the dam at a section H_0 below the water surface be l_0, the area of the profile above this section A_0, and the distance of its centre of gravity from the upstream face $\bar{x}_0$. Then a quadratic equation gives the width of dam at another section distant h below the first:

$$l^2\left(\frac{\gamma'}{\rho}H - h\right) - l(4A_0 + l_0h) + \left(6A_0\bar{x}_0 + l_0{}^2h + \frac{H^3}{\rho}\right) = 0 \quad . \quad . \quad (2)$$

where H, the new depth, $= H_0 + h$.

The new value of A is found from:

$$A = A_0 + \frac{h}{2}(l + l_0)$$

and of $A\bar{x}$ from:

$$A\bar{x} = A_0\bar{x}_0 + \frac{h}{6}(l^2 + ll_0 + l_0{}^2)$$

These equations give sufficient information for the usual, rather laborious, step-by-step calculations. In a stage III design the end of stage II occurs when tension appears at the downstream face with the reservoir empty, i.e., when $\bar{x} = l/3$. In a stage II design the end of stage II comes only when the compressive stresses reach their permissible limit. As in stage I, the equations do not contain γ''.

An alternative and more accurate approach would be to set up the differential equation expressing the relation between H and l according to exactly the same physical principles as before, and from its solution calculate l at any particular depth directly, without reference to the value one "step" higher up. The equation is derived in Appendix II, but so far it has defied general solution. It should be possible, however, and probably less tedious, to design stage II by one of the many numerical methods for solving such equations. The result of course would be the same, provided that h was kept small enough in the standard method.

Although the general solution of this differential equation is not known, a particular solution has been found, and it will be discussed under the heading "The stage II design."

Stage III

Stage III may be designed from the exact solution of the differential equation relating l and H. It is:

$$H^{3\mu} - (\rho - \gamma')l^2H^{3\mu-2} - cl^{\mu} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $\mu = 2(\rho - \gamma')/(\rho - 2\gamma')$ and the constant c must be chosen to fit the initial conditions at the end of stage II. When $\gamma' = 0$, μ is 2; at the other extreme, assuming maximum uplift, $\gamma' = 1$, and if $\rho = 2{\cdot}25$, μ is 10. Once again the solution does not depend upon γ''.

For numerical calculation the solution is best expressed in parametric form:

$$\left.\begin{aligned} H &= \left[\frac{1 - cx^{\mu}}{(\rho - \gamma')x^2}\right]^{\frac{1}{2}} \\ l &= H^3x. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By choosing any numerical value for the parameter x corresponding values of H and l can be found.

However, as H and l increase a great simplification becomes possible in the original form of the solution. The term cl^{μ} will be important at the beginning of stage III, but it soon becomes insignificant in comparison with the first term, which has H raised to the power 3μ and thus increases much more rapidly. When it is possible to neglect the last term the equation can be divided by $H^{3\mu-2}$ and the following is obtained:

$$l = mH = \frac{H}{\sqrt{\rho - \gamma'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This approximation becomes valid quite quickly when $\gamma' = 0$, and very quickly indeed when $\gamma' = 1$, for then the first and last terms are respectively H^{30} and cl^{10}. The values of m are plotted in Fig. 9.

Next it is necessary to discover how the batter is distributed between the two faces of the dam. It will be shown in Appendix III that dz/dH contains a factor $\{H^2 - (\rho - \gamma')l^2\}$, so that as soon as the approximate formula is valid the upstream face becomes vertical again. Thus the shape of the dam from here downwards is part of a triangular profile, *independent of the top width and freeboard.* Fig. 8 shows

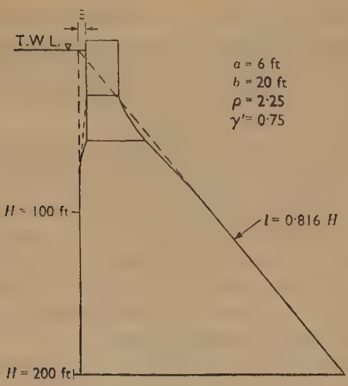

FIG. 8.—THEORETICAL PROFILE FOR A STAGE III DESIGN

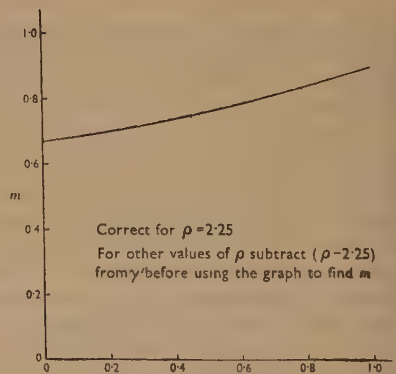

FIG. 9.—VALUES OF m FOR THE FORMULA $l = mH$

such a theoretical profile, from which it can be seen that the effect of the term cl^{μ} is to adjust the top portion of the particular dam as it were to the most comfortable position to be carried on the standard triangle.

The total upstream batter $\bar{z}$ is awkward to calculate, but a first approximation is:

$$\bar{z} = \frac{(\rho - \gamma')^{\frac{1}{2}} H_2 \{1 - (\rho - \gamma')(l_2/H_2)^2\}}{3\rho(l_2/H_2)} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where H_2 and l_2 refer to the end of stage II. The formula for a closer approximation is given in Appendix IV.

The rather sudden change of section implied by the theoretical solution and shown in Fig. 8 is not acceptable. It would provoke stress concentrations and thus

lestroy the basis of the solution, which postulates a linear distribution of vertical tress. A safe and more practical profile is given by the dotted line.

The stresses are not excessive, whatever the uplift conditions, before the triangular rofile is reached. Thereafter they follow the simple formulae:

$$\left.\begin{aligned} p &= (\rho - \gamma' - \gamma'')wH \\ q &= \rho wH. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

These are vertical stresses at the faces, and since the upstream face is now vertical is a principal stress. It may thus increase up to the full permissible compressive tress for the material, f_c.

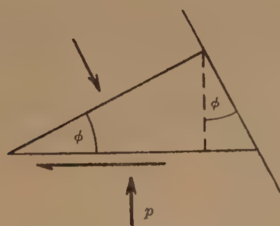

FIG. 10

The downstream face has a constant slope at depths where p becomes large, and ts inclination $\tan\phi$ is given by dy/dH (Fig. 10). Since $dz/dH = 0$,

$$dy/dH = dl/dH$$

nd

$$\tan\phi = dl/dH = \frac{1}{\sqrt{\rho - \gamma'}}.$$

The principal stress, parallel to the surface, may increase to f_c, and if the maximum ermissible value of p be written $\bar{p}$ it follows that:

$$\begin{aligned} \bar{p} &= f_c \cos^2\phi \\ &= f_c/(1 + \tan^2\phi) \\ &= f_c(\rho - \gamma')/(\rho - \gamma' + 1) \end{aligned}$$

Similarly:

$$\bar{q} = f_c$$

Then

$$\left.\begin{aligned} p/\bar{p} &= \frac{(\rho - \gamma' - \gamma'')(\rho - \gamma' + 1)}{\rho - \gamma'} \cdot \frac{wH}{f_c} \\ q/\bar{q} &= \rho \cdot \frac{wH}{f_c} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (8)$$

Stage III ends as the first of these ratios reaches unity, when $H = H_3$ given by the maller value of:

$$\left.\begin{aligned} H_3 &= (\rho - \gamma')f_c/w(\rho - \gamma' - \gamma'')(\rho - \gamma' + 1) \\ H_3 &= f_c/w\rho. \end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

nd

It is interesting to note that for an uplift pressure distribution varying linearly from wH to zero at the two faces, $\gamma' = 1$ and $\gamma'' = 0$, so that $H_3 = f_c/w\rho$ by either formula, both stresses reaching their permissible limit at the same depth.

The stage II design

If tension in the downstream face can be tolerated the criteria of stage II govern the design until the stresses become excessive, by which time H is quite large. It is therefore worthwhile to assume that when H is large the solution for l will once again become linear, and to see whether this gives a possible solution of the differential equation. It is shown in Appendix II that the only possible linear solution is:

$$l = \frac{H}{\sqrt{\rho - \gamma'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

the same as the ultimate solution in a stage III design.

How quickly this formula will become valid cannot be foretold, but the theory implies that in the design of a high dam there will be a value of H beyond which the saving of weight arising from a stage II instead of a stage III design will be negligible. (It will be shown in a later Paper that this is not the case if earthquake forces have to be considered.)

To test this theory, and see how quickly the linear solution becomes valid, the example in Fig. 8 was recalculated as a stage II design. The step-by-step calculation gave the profile shown in Fig. 11. It does not settle down to the predicted slope

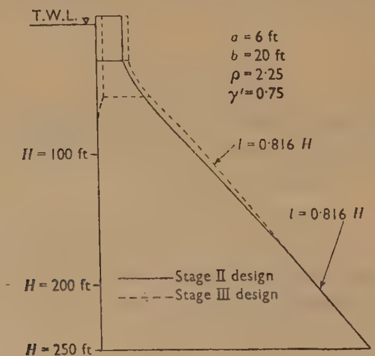

Fig. 11

until about $H = 200$ ft, compared with 100 ft in the stage III design, but the difference in area of the two profiles is not great when considered as a proportion of the total area down to $H = 250$ ft. In this particular example the difference is about 2%.

No rule has been given to indicate the depth at which the profile becomes sensibly triangular in any particular case. It will be greater for a stage II than for a stage III design, but in either case it may be considerably affected by the choice of top width. All that is asserted here is that for any given top width and freeboard the profile will ultimately conform to the one standard triangle ($l = H/\sqrt{\rho - \gamma'}$) if it is continued far enough.

Arrangement of drains

It will have been noticed from the foregoing formulae that γ'' affects only the stress p and the depth at which stage IV must begin; the inclination of the faces, and hence

the volume of the dam, are dependent only on γ' and ρ. The volume per foot length of a medium-height dam is approximately $mH^2/2$ where $m = 1/\sqrt{\rho - \gamma'}$, so that the effect of uplift on volume is summarized by Fig. 9. Clearly it is desirable to arrange the drains so as to keep γ' as low as possible.

On the assumption that the material stresses vary linearly the drains will thus be particularly helpful if they reduce the uplift pressure in the upstream third of the section. Since $\gamma' wHl/2$ and $\gamma'' wHl/2$ are statically equivalent to the uplift pressure they may be thought of as simple supports to a beam on which the uplift pressure is applied as a distributed load (cf. Fig. 2); the γ' support is greatly increased by load on the upstream overhang, whereas load on the downstream overhang actually reduces γ'.

For dams high enough to reach the maximum permissible stresses it is desirable to make γ'' as *large* as possible, in order to keep down the value of p, for once stage IV is reached the section spreads very rapidly. By the simply supported beam analogy, load on the downstream overhang is therefore an advantage for γ'' also. Uplift pressure would thus be a welcome phenomenon if only it could be made to act in the right place.

This prompts the suggestion that uplift might be deliberately introduced in the downstream third of the dam by an interconnected system of perforated pipes, say five-sixths of the way through from the upstream face. A single impermeable pipe would connect the system with the water in the reservoir, and there would need to be a fairly efficient series of drains round about the downstream middle-third line so that no new uplift would be introduced in the middle-third.

Such an arrangement would certainly both reduce γ' and increase γ'', but since its effectiveness would for both purposes depend entirely upon the material stress distribution remaining linear, the device cannot be recommended until more is known regarding the linearity assumption.

Conclusions

Graphs and formulae have been given for the stresses and profile of a solid non-overflow gravity dam whose design is governed solely by no-tension criteria. It appears that all such dams, if they are high enough, will ultimately conform to straight-line profiles independent of the top width and freeboard, and the slopes of the faces have been predicted from the differential equations of equilibrium for any uplift conditions.

The speed with which the faces achieve constant slopes will depend upon the top width and whether tension in the downstream face is to be tolerated when the reservoir is empty. If it is not (a stage III design) the slopes are soon constant; if it is tolerated (a stage II design) the constant slopes are more delayed.

Once the profile has assumed straight faces the ratio of total width/head of water is plotted in Fig. 9. The ratio for a stage II design is the same as for a stage III design, and the only saving in weight by the stage II design is a small percentage before the constant slopes are reached. A later Paper will show that a considerable saving can be made when there is earthquake loading.

As to the effect of uplift, until the stresses reach their permissible limit the profile is governed by the parameter γ' only, and the smaller γ' is the lighter will be the dam. But it may be, if Brahtz's[5] conclusion for a two-dimensional system is more generally valid, that γ' must be taken equal to β, and that internal drains are

of no avail to reduce it. Brahtz's work definitely proves this to be the case when the only drains are horizontal galleries.

γ'' concerns only the stresses, and hence the depth at which stage IV starts; the stresses are reduced by *increasing* γ''.

The analysis has been based on the assumption of a linear distribution of material contact stress, but this seems improbable. If, as may be expected, future work asserts some other condition the results of this Paper will still be valid if the parameters γ' and γ'' are redefined as explained on p. 200.

This differential equation analysis of dam profiles is given here only for solid-section non-overflow dams, but there should be no particular difficulty in extending it to certain other types—for example to buttress dams, which may often show economy of weight compared with the type discussed here.

Acknowledgement

The Author would like to express his gratitude to Mr P. O. Wolf, B.Sc.(Eng.), A.M.I.C.E., also of the Civil Engineering Department at Imperial College of Science and Technology, for many extremely helpful discussions and suggestions during the course of the work.

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APPENDIX I

ANALYSIS FOR STAGE I

The profile is shown in Fig. 12, together with the forces acting on a unit length of the dam. The problem is to find the limiting height H_1 at which the resultant of the vertical material stresses $pb/2$ will, as shown, act through the third-point X.

Taking moments about X:

$$\frac{wH_1^3}{6} + \frac{\gamma' w H_1 b^2}{6} = \frac{\rho w b^2 (H_1 + a)}{6}$$

or,

$$H_1^3 - (\rho - \gamma')b^2 H_1 - \rho a b^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

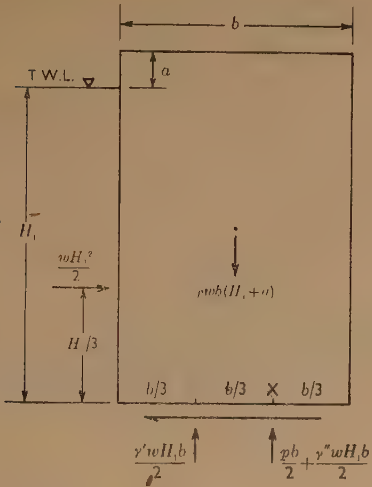

FIG. 12

APPENDIX II

ANALYSIS FOR STAGE II

) *Step-by-step method*

Fig. 13 shows the profile down to a section in stage II at a depth H below the water ırface, together with the forces acting on a unit length of the dam. Let A denote the ɔtal area of the profile so far, and take moments about X:

$$\frac{wH^3}{6} + \frac{\gamma wHl^2}{6} = \rho wA\left(\frac{2l}{3} - \bar{x}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

et the suffix $_0$ indicate values of a section at a small height h above the first one, and ɔnsider both faces as straight between the two sections. Then:

$$A = A_0 + \frac{h}{2}(l + l_0)$$

$$A\bar{x} = A\bar{x}_0 x + \frac{h}{6}(l^2 + ll_0 + l_0{}^2)$$

ıbstituting these values in equation (12):

$$l^2\left(\frac{\gamma'}{\rho}H - h\right) - l(4A_0 + l_0 h) + \left(6A_0\bar{x}_0 + l_0{}^2h + \frac{H^3}{\rho}\right) = 0 \quad . \quad . \quad (13)$$

ɪis is the required equation for l in terms of H and the previously determined values at a ghtly higher section.

) *The differential equation*

Fig. 14 shows the forces acting on an infinitesimal element of the dam at a depth H. Vertical resolution of forces gives:

$$(\gamma' + \gamma'')\frac{w}{2}d(Hl) + \frac{d(pl)}{2} = \rho wldH$$

$$(\gamma' + \gamma'')w\frac{d(Hl)}{dH} + \frac{d(pl)}{dH} = 2\rho wl \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

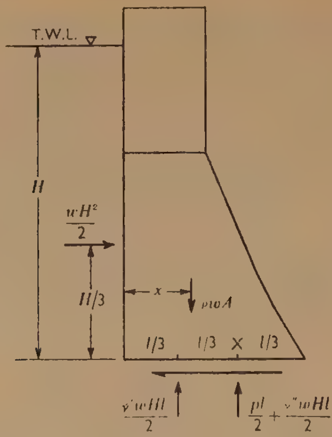

FIG. 13

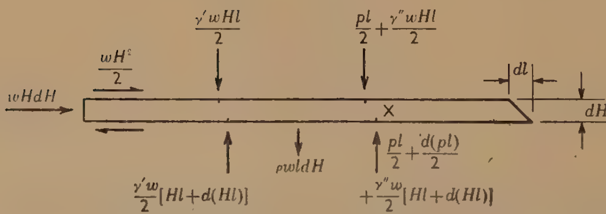

FIG. 14

Taking moments about X, and ignoring products of infinitesimals:

$$\frac{wH^2}{2}dH + \gamma'wl[Hl + d(Hl)] + \frac{\gamma'wHl}{2}\cdot\frac{dl}{3}$$
$$= \frac{\gamma'wHl}{2}\left(\frac{l}{3} + \frac{2dl}{3}\right) + \left(\frac{pl}{2} + \frac{\gamma''wHl}{2}\right)\frac{2dl}{3} + \frac{\rho wl^2}{6}dH$$

This simplifies to:

$$2pl\frac{dl}{dH} = 3wH^2 - (\rho - \gamma')wl^2 - 2\gamma''wHl\frac{dl}{dH} \quad . \quad . \quad . \quad . \quad (15)$$

If equation (15) is used to eliminate p from the left-hand side of equation (14) the required equation for l in terms of H results:

$$\{(\rho - \gamma')l^2 - 3H^2\}\frac{d^2l}{dH^2} + 2\frac{dl}{dH}\{\gamma'H\left(\frac{dl}{dH}\right)^2 - (3\rho - 2\gamma')l\frac{dl}{dH} + 3H\} = 0 \quad . \quad . \quad . \quad (16)$$

A general solution of equation (16) has not yet been found. When H is large, however, a solution in this form may be tried:

$$l = mH + n$$

This gives:

$$3H\{1 - (\rho - \gamma')m^2\} - (3\rho - 2\gamma')mn = 0$$

which must be true for all large values of H, so that the two terms must individually vanish. Thus:

$$1-(\rho-\gamma')m^2=0$$
$$n=0$$

The only possible linear solution is therefore:

$$l=\frac{H}{\sqrt{\rho-\gamma'}} \quad \ldots\ldots\ldots \quad (17)$$

APPENDIX III

ANALYSIS FOR STAGE III

Fig. 15 shows the forces acting on an infinitesimal element of the dam at a depth H, when the reservoir is full. The vertical component of the water pressure on the upstream face is ignored. Vertical resolution of forces gives

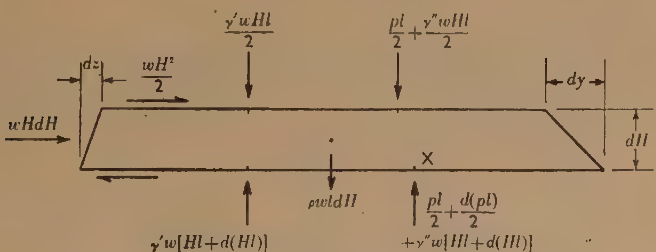

FIG. 15

$$(\gamma'+\gamma'')w\frac{d(Hl)}{dH}+\frac{d(pl)}{dH}=2\rho wl \quad \ldots\ldots \quad (18)$$

Moment equilibrium about X when products of infinitesimals are ignored gives:

$$\frac{wH^2}{2}dH+\frac{\gamma'w}{2}[Hl+d(Hl)]\left(\frac{l+dy+dz}{3}\right)=\frac{\gamma'wHl}{2}\frac{(l+2dy-dz)}{3}$$
$$+\left(\frac{pl}{2}+\frac{\gamma''wHl}{2}\right)\left(\frac{2dy-dz}{3}\right)+\frac{\rho wl^2}{6}dH$$

which simplifies to:

$$pl\left(2\frac{dy}{dH}-\frac{dz}{dH}\right)=3\gamma'wHl\frac{dz}{dH}-\gamma''wHl\left(2\frac{dy}{dH}-\frac{dz}{dH}\right)-(\rho-\gamma')wl^2+3wH^2 \quad \ldots (19)$$

Fig. 16 shows the forces when the reservoir is empty. Resolving vertically:

$$\frac{d(ql)}{dH}=2\rho wl \quad \ldots\ldots\ldots \quad (20)$$

Moment equilibrium about Y:

$$-\frac{ql}{2}\left(\frac{2dz-dy}{3}\right)+\frac{\rho wl^2}{6}dH=0$$

That is

$$ql\left(\frac{dy}{dH}-2\frac{dz}{dH}\right)=\rho wl^2 \quad \ldots\ldots\ldots \quad (21)$$

Equations (18) to (21) contain the five unknown quantities l, p, q, y, and z; y can be eliminated immediately, since $y=l-z$.

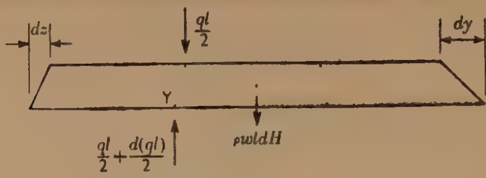

FIG. 16

If equation (20) be subtracted from equation (18) and the resulting equation be integrated then:

$$q = p + (\gamma' + \gamma'')wH \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The constant of integration must be zero, for both sides of equation (22) if multiplied by $l/2$ are equal to ρwA (cf. Fig. 17).

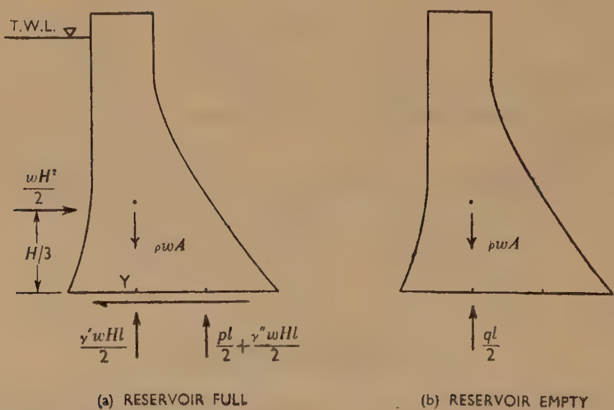

FIG. 17

Equation (22) is now used to eliminate q from equation (21); subtracting equation (19), and writing $y = l - z$:

$$\{p - (\gamma' - \gamma'')wH\}l\frac{dl}{dH} = 3wH^2 - (2\rho - \gamma')wl^2$$

Add $l \times$ equation (18):

$$pl\frac{dl}{dH} + l\frac{d(pl)}{dH} + \gamma''w\left\{Hl\frac{dl}{dH} + l\frac{d(Hl)}{dH}\right\} = 3wH^2$$

Integrating:

$$pl^2 = wH^3 - \gamma''wHl^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Again the constant of integration is zero, for this is 6 $\times$ the equation of moment equilibrium about Y (Fig. 17a). Finally, equation (23) is used to eliminate p from equation (22), and the resulting expression for q is substituted in equation (20):

$$H(H^2 - \gamma'l^2)\frac{dl}{dH} - 3H^2l + (2\rho - \gamma')l^3 = 0 \quad . \quad . \quad . \quad . \quad (24)$$

This is the required differential equation for l in terms of H, and as usual it contains γ'

but not γ''. It can be solved by making the substitution $l^2/H^2 = u$. Then $l = Hu^{\frac{1}{2}}$, $\frac{dl}{dH} = u^{\frac{1}{2}} + \frac{H}{2u^{\frac{1}{2}}}\frac{du}{dH}$, and equation (24) becomes:

$$H(1 - \gamma' u)\frac{du}{dH} - 4u + 4(\rho - \gamma')u^2 = 0$$

which can be written:

$$4\frac{dH}{H} = \frac{du}{u} + \frac{(\rho - 2\gamma')du}{1 - (\rho - \gamma')u}.$$

Integrating, and putting $\mu = 2(\rho - \gamma')/(\rho - 2\gamma')$:

$$H^{2\mu} = \frac{cu^{\mu/2}}{1 - (\rho - \gamma')u}$$

That is:

$$H^{3\mu} - (\rho - \gamma')l^2H^{3\mu-2} - cl^{\mu} = 0 \quad . \quad . \quad . \quad . \quad (25)$$

Since this contains one arbitrary constant it is the required general solution of equation (24). For numerical work it is best expressed in parametric form, by putting $l = H^3x$:

$$1 - (\rho - \gamma')x^2H^4 - cx^{\mu} = 0$$

or:

$$H = \left[\frac{1 - cx^{\mu}}{(\rho - \gamma')x^2}\right]^{\frac{1}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The simple form of the solution when cl^{μ} in equation (25) may be neglected is discussed in the main part of the Paper. Substituting it in equations (23) and (22) gives the simple formulae for p and q. It only remains to investigate z, the batter of the upstream face.

Equation (21) can be written:

$$3\frac{dz}{dH} = \frac{dl}{dH} - \frac{\rho wl}{q}$$

or by equations (22), (23), and (24):

$$3\frac{dz}{dH} = \frac{l(3H^2 + \gamma' l^2)\{H^2 - (\rho - \gamma')l^2\}}{H(H^4 - \gamma'^2 l^4)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

This contains a factor $\{H^2 - (\rho - \gamma')l^2\}$, so that z reaches a constant value when the approximate formula for l is applicable. This value will be called $\bar{z}$, and its evaluation is given in Appendix IV.

APPENDIX IV

CALCULATION OF $\bar{z}$

It was shown in Appendix III that after the early part of stage III z takes a constant value, which it would retain however far stage III continued. It is mathematically convenient to call $\bar{z}$ the value of z when $H = \infty$. Then since $z = 0$ when $H = H_2$ (H_2 refers to the end of stage II):

$$\bar{z} = \int_{H_2}^{\infty}\frac{dz}{dH}\,dH \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

or since $x = 0$ when $H = \infty$:

$$\bar{z} = -\int_{0}^{x_2}\frac{dz}{dx}\,dx$$

from equations (26) and (27):

$$\frac{dz}{dx} = \frac{dz}{dH}\cdot\frac{dH}{dx} = -\frac{l(3H^2 + \gamma' l^2)\{H^2 - (\rho - \gamma')l^2\}}{3H(H^4 - \gamma'^2 l^4)}\cdot\frac{2 + (\mu - 2)cx^{\mu}}{4(\rho - \gamma')H^3x^3}$$

or, putting $l = H^3x$ and eliminating H by equation (26):

$$\frac{dz}{dx} = -\frac{(\rho-\gamma')^{\frac{1}{2}}}{6(\rho-2\gamma')}\cdot\frac{cx^{\mu-3/2}(3\rho-2\gamma'-\gamma'cx}{(1-cx^{\mu})^{\frac{1}{2}}(\rho-\gamma'cx^{\mu})}$$

The integration can be simplified by putting $cx^{\mu} = t$. When $x = 0$, $t = 0$, since $\mu > 0$:

$$\bar{z} = -\int_0^{t_2}\frac{dz}{dt}\,dt$$

$$\frac{dz}{dt} = \frac{dz}{dx}\cdot\frac{dx}{dt} = \frac{dz}{dx}\cdot\frac{1}{\mu c^{1/\mu}t^{1-1/\mu}}$$

The integral now transforms to:

$$\bar{z} = \frac{(3\rho-2\gamma')c^{1/2\mu}}{12\rho(\rho-\gamma')^{\frac{1}{2}}}\int_0^{t_2}\frac{(1-\theta t)dt}{t^{1/2}\mu\left(1-\frac{\gamma'}{\rho}t\right)(1-t)^{\frac{1}{2}}} \quad . \quad . \quad . \quad (29)$$

where $\theta = \gamma'/(3\rho - 2\gamma')$

This cannot be directly evaluated, but if t and $\gamma' t/\rho$ are both less than unity it can be integrated in a binomial expansion. $t_2 = cx_2{}^{\mu} = 1 - (\rho - \gamma')(l_2/H_2)^2$, by equation (25), and since $l_2 < H_2/\sqrt{\rho - \gamma'}$ it follows that $0 < t_2 < 1$; and hence $0 < t < t_2 < 1$. Furthermore, $\gamma'/\rho < 1$. Binomial expansion of the integrand therefore gives:

$$t^{-1/2}\mu\left[1 + \left(\frac{\gamma'}{\rho} + \frac{1}{4} - \theta\right)t + \ldots\right]$$

After integration and simplifying, from the first term alone:

$$\bar{z} = \frac{(\rho-\gamma')^{\frac{1}{2}}H_2\{1-(\rho-\gamma')(l_2/H_2)^2\}}{3\rho(l_2/H_2)^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

The second term adds:

$$\frac{(3\rho^2+6\gamma'\rho-8\gamma'^2)(\rho-\gamma')^{\frac{1}{2}}H_2\{1-(\rho-\gamma')(l_2/H_2)^2\}^2}{12\rho^2(7\rho-6\gamma')(l_2/H_2)^{\frac{1}{2}}}$$

Further terms are unlikely to be required, but can be calculated quite simply by continuing the binomial expansion.

The Paper, which was received on 25 April, 1955, is accompanied by four sheets of graphs from which the figures in the text have been prepared, and by four Appendices.

CORRESPONDENCE on the Paper should be forwarded to reach the Institution before 15 August, 1956. Contributions should not exceed 1,200 words.—SEC.

Paper No. 6089

THE DETERMINATION OF THE COLLAPSE LOADS OF RIGIDLY JOINTED FRAMEWORKS WITH MEMBERS IN WHICH THE AXIAL FORCES ARE LARGE

by

* Noel W. Murray, B.E.

(Ordered by the Council to be published with written discussion)

SYNOPSIS

A structure does not, in general, fail when the maximum stress reaches the yield stress of the material. Frameworks with rigid joints collapse when a sufficient number of plastic hinges exist to form a mechanism. In limit design a load factor is applied to the estimated collapse loads to give the working loads and methods are available for determining the collapse loads of frameworks in which the axial forces of the members are small. However, when these axial forces are appreciable the bending moments arising from them and the distortion of the members become the dominant factor in determining the behaviour of the frame. The available methods are not then applicable.

The Paper suggests a method of determining the collapse loads of such frameworks. It uses test results from three entirely different types of framework to show that consistent results are obtained by this method.

Introduction

In the past, structures have been designed and analysed starting from the premise that they are unsafe once the maximum stress has passed a predetermined value at any point in the structure. This reasoning, although leading to safe designs, does not allow the designer to make the best use of his available material. Recently a new method of design has been developed and it is based on a different definition of the unsafe load of a structure. The unsafe load of a structure is defined as the collapse load divided by a load factor, and the collapse load is the load at which the deflexions of the structure increase for no increase in applied loads. The choice of the load factor is left to the designer but 1·75 is suggested by the British Constructional Steelwork Association.[1] It is readily seen that the basis of limit design is more logical that the older one used in the design by maximum stress. Research has shown that lighter structures are obtained by using limit design than by using design by maximum stress.

In using limit design the designer is faced with the problem of determining the collapse load of the structure. Methods are available for doing this in the case of structures whose members do not carry appreciable axial loads.[2, 3, 4] In general, the structures which fall into this category are simple and continuous beams and rectangular-type frameworks. These structures derive their rigidity from the power of

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[1] The References are given on p. 228.

their members to resist the transverse bending moments and axial loads play only a secondary part. Collapse of these structures occurs when sufficient plastic hinges exist to form a mechanism and the collapse load is determined from the values of the fully plastic moments at the hinges and the location of the hinges. The method of analysis is rapid and simple.

Bridge and roof trusses and certain types of rectangular building frames are examples of frameworks in which the externally applied loads are resisted largely by the axial loading of the members of the framework. As external loads are applied the members bend, thereby introducing at each point of the framework an additional moment whose value is the product of the distortion at that point and the axial load of the member. These moments are important in determining the behaviour of the structure and produce effects usually called "stability effects."

Distortion of the members can be caused in a number of ways. The changes in lengths of the members caused by axial forces must be accommodated by the bending of the members in a rigidly jointed structure. Initial curvatures of the struts are magnified as the applied loads increase but the opposite occurs in the case of ties and eccentricities of the members at the joints have some effect on the bending of the members. The Author has shown[5] that in the elastic range an accurate description of the behaviour of the frame can only be given provided that the effects of initial curvature, joint eccentricity, changes in member length, and joint-block or gusset-plate size are taken into account. Once the yield point is reached at one point of the structure the analysis of its behaviour becomes much more complicated and does not warrant the labour involved.

The method described below makes use of the analogy between frameworks and isolated columns as suggested by Merchant.[6] The way in which an isolated column fails is studied first and then the analogy between frameworks and isolated columns is discussed. The mechanism of the failure of three types of structure is put into mathematical terms and applied to test results.

The behaviour of pin-ended struts with initial curvature and eccentric loads

Consider* a pin-ended strut with an axial load P and a first Euler load of P_E. If the shape of the unloaded strut (Fig. 1) is given by the Fourier series:

$$y_0 = \sum_{n=l}^{\infty} a_n \sin \frac{n\pi x}{l}$$

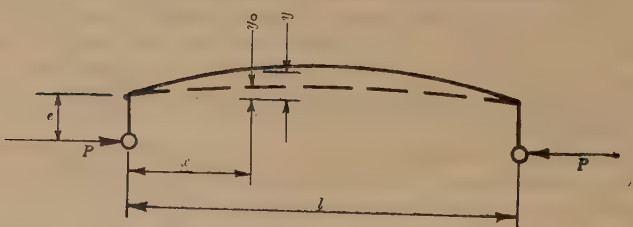

Fig. 1

* The notation is given on p. 197.

and the load is applied with eccentricity e then the expression for the bending moment at any point x can be written as:

$$EI\frac{d^2y}{dx^2} + Py = -Pe \quad \ldots\ldots\ldots \quad (1)$$

This equation may be divided into two parts, the first dealing with the initial curvature and the second with the eccentricity of the load. It is shown elsewhere[7] that the solution to the first part is:

$$y_1 = \frac{a_1 \sin\frac{\pi x}{l}}{1-\frac{P}{P_E}} + \frac{a_2 \sin\frac{2\pi x}{l}}{1-\frac{P}{4P_E}} + \ldots\ldots\ldots \quad (2)$$

Therefore at the centre of the strut the deflexion is:

$$y_{c1} = \frac{a_1}{1-\frac{P}{P_E}} - \frac{a_3}{1-\frac{P}{9P_E}} + \frac{a_5}{1-\frac{P}{25P_E}} \quad \ldots\ldots \quad (3)$$

All terms after the first are usually neglected because the coefficients, a_3, a_5, $\ldots a_n$, etc., decrease in magnitude with increasing n and also because the denominators do not approach zero as P approaches P_E as is the case for the first term of this series. Thus for all practical struts the deflexion at the centre, due to an initial deflexion at the centre of magnitude a_1, is given by the expression:

$$y_{c1} = \frac{a_1}{1-\frac{P}{P_E}} \quad \ldots\ldots\ldots \quad (4)$$

The second part of equation (1) gives an expression[7] for the deflexion at the centre of the strut as *

$$y_{c2} = e \sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}} \quad \ldots\ldots\ldots \quad (5)$$

Thus the total deflexion at the centre of an eccentrically loaded and initially pin-ended strut is given by:

$$y_c = y_{c1} + y_{c2} = e \sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}} + \frac{a_1}{1-\frac{P}{P_E}} \quad \ldots\ldots \quad (6)$$

If e is small compared with a_1, or if P/P_E is small then it is permissible to write equation (6) as:

$$y_c = \frac{e + a_1}{1-\frac{P}{P_E}} \quad \ldots\ldots\ldots \quad (7)$$

Thus the graph of P against deflexion at the centre of the strut y_c approximates to a hyperbola which has asymptotes at $P = P_E$ and $y_c = 0$, and which passes through the

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* In the case of a strut which has unequal eccentricities e_A and e_B at its ends A and B respectively it is easily shown from equation (5) of reference 5 that:

$$y_{c2} = \frac{(e_A + e_B)}{2} \sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}$$

point $y_c = (e + a_1)$ when $P = 0$. This graph is sketched in Fig. 2 for a 1-in. × ½-in. strut, 10 in. long, bending about its minor axis. The curve shown in Fig. 2 is valid until the yield point is reached at some point of the strut. The load P_h at which equation (7) ceases to be valid can be found by solving

$$f_y = \frac{P_h}{A} + \frac{P_h(e + a_1)}{Z\left(1 - \frac{P_h}{P_E}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

for P_h; where f_y is the yield stress of the material, A is the sectional area, and Z is the section modulus.

If the deflexion at the centre of the strut is increased beyond that of point h (Fig. 2) then conditions at the centre of the strut are partly elastic and partly plastic.

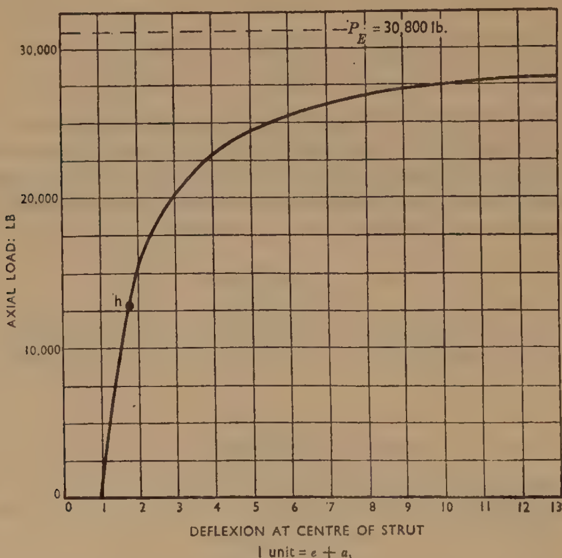


FIG. 2.—LOAD/DEFLEXION CURVE FOR A 1-IN. × ½-IN. STRUT, 10 IN. LONG, ASSUMING ELASTIC CONDITIONS EXIST EVERYWHERE IN THE STRUT

Collapse will certainly occur when a plastic hinge has formed at this point. It is now necessary to look at the conditions which are required to form a plastic hinge.

To illustrate the method a strut of rectangular section $b \times d$ $(b > d)$ will be considered. A plastic hinge will be formed in a strut of rectangular cross-section when the stress distribution in Fig. 3b exists. It is readily shown from Fig. 3b that the plastic moment M_p' when an axial force P is acting on the section is given by the expression:

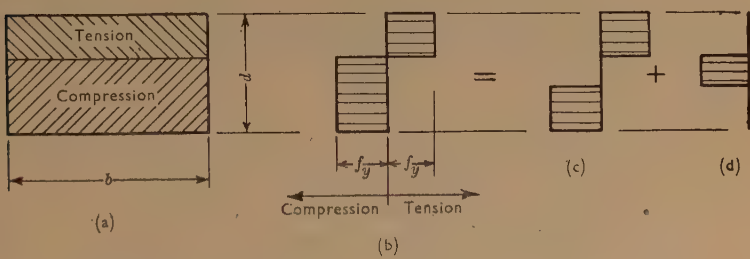
$$M_p' = M_p\left(1 - \frac{P^2}{P_y{}^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where M_p denotes the fully plastic moment when no axial force acts $\left(= f_y \frac{bd^2}{4}\right)$

P_y denotes the plastic load when no moment acts $(= f_y bd)$

The relation (9) is graphed in Fig. 4. The relation between the axial load, P, the central deflexion of the strut, d_c, and the properties of the strut, b, d, and f_y can be found by considering the equilibrium of the collapse mechanism shown in Fig. 5a. For equilibrium:

$$M_p' = Pd_c \quad \text{. . . . .} \quad (10)$$



(a) Cross-section of strut at plastic hinge
(b) Stress distribution
(c) Stress due to bending moment M_p'
(d) Stress due to axial load P

FIG. 3

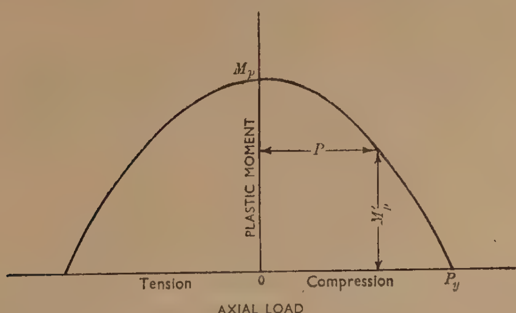


FIG. 4

On eliminating M_p' from (9) and (10) and substituting for M_p and P_y in terms of f_y, b, and d:

$$\frac{dP^2}{4P_y} + d_cP - f_y\frac{bd^2}{4} = 0$$

whence

$$P = f_ybd\left[\sqrt{\frac{4d_c^2}{d^2} + 1} - \frac{2d_c}{d}\right] \quad \text{. . . . .} \quad (11)$$

In Fig 5b this relation is graphed for a 1-in $\times \frac{1}{2}$-in. strut whose yield stress is 40,000 lb/sq. in.

Figs 2 and 5 can now be combined, and this is done in Fig. 6 for a strut in which $(e + a_1) = 0{\cdot}025$ in. If only elastic conditions exist then the load/deflexion curve

follows curve ab and the load/deflexion relation is given by curve de if plastic collapse is occurring. However, at some load P_h, the yield stress is reached at a point in the strut. This point is shown as h in Fig. 6, and it occurs before the intersection g of ab and de. Thus the interrupted line is followed after h rather than the path a to g to e. The maximum load occurs at f and is called the collapse load (see definition earlier).

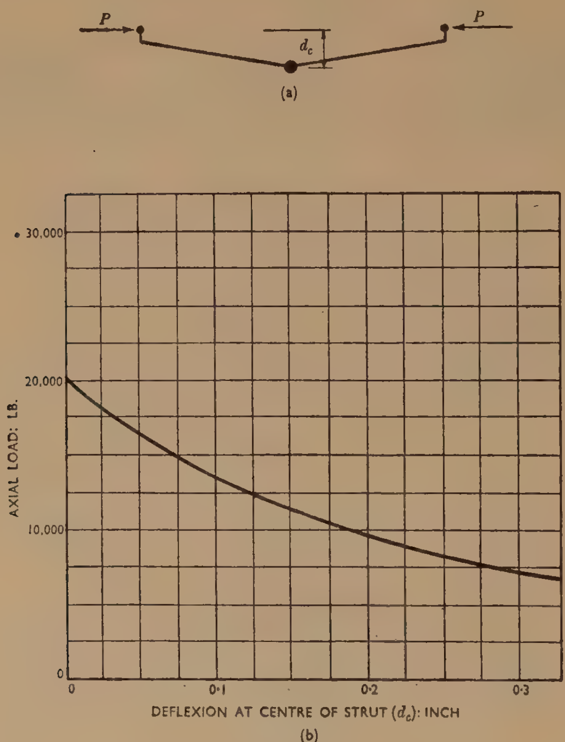


FIG. 5

For a given strut of rectangular cross-section of known initial curvature, eccentricity, and material properties the curves agb and dge can be readily calculated from equations (7) and (11). For struts of sections other than rectangular shape expressions similar to (11) may be derived.

Analogy between isolated struts and frameworks

It has been shown in the case of a pin-ended strut the initial deflexion at the centre $(e + a_1)$ is magnified according to the following law in the elastic range:

$$y_c = \frac{e + a_1}{1 - \frac{P}{P_E}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which $P_E\left(= \frac{\pi^2 EI}{l^2}\right)$ is the Euler crippling load.

Similarly, a structure has a critical load W_c which can be determined in a number of ways.[6,8,9] By the analogy suggested by Merchant,[6] the expression for the deflexion at the centre of a member is given by:

$$y_c = \frac{e + a_1}{1 - \dfrac{W}{W_c}} \qquad (12)$$

This allows the ab line (Fig. 6) to be plotted, but first the value of $(e + a_1)$ has to be established for the structure. Suppose that the critical member has been chosen, then a_1 is the value of the distance at the centre of the member from the straight line

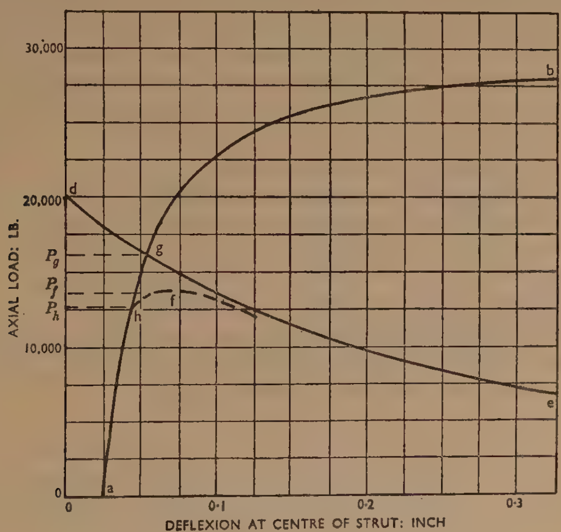


FIG. 6

joining the ends of the member to the central line of the member, as was the case of the strut in Fig. 1. The equivalent eccentricity e depends not only upon the eccentricities at the ends of the critical member but also upon the eccentricities at those ends of the surrounding members. The equivalent eccentricity at a joint where n members meet can be written as:

$$e = \frac{\sum_{k=1}^{n} P_k e_k}{P} \qquad (13)$$

where P_k is the axial load in the kth member at the joint
e_k is the eccentricity of the kth member at the joint
and P is the axial load in the critical member.

Account must be taken of the signs and directions of the axial loads and eccentricities.

Thus, assuming elastic conditions prevail throughout the framework, the deflexion at the centre of the critical member will be approximately equal to the deflexion at

the centre of an equivalent strut whose initial central deflexion is the same as that of the critical member and whose joint eccentricities are given by equation (13). The resulting load/deflexion curve agb (Fig. 6) is hereafter called the elastic stability line.

The load/deflexion line which corresponds to plastic collapse (dge in Fig. 6) will be called the plastic collapse line and can be plotted by considering the collapse mechanism of the structure. In the case of a framework the expressions can be derived by using the principle of virtual displacements and the displacement diagram.[10] The following examples will illustrate the method.

Test results and accompanying curves as obtained by the equivalent strut theory

The Author has tested a number of frames of isosceles triangular shape. They were loaded symmetrically so that the two equal members were in tension and the other was incompression. The method of testing is described elsewhere.[5] Measurements were made of:

(*a*) the load *W* at the apex C of the frame;
(*b*) the joint rotations at the ends of the strut AB;
(*c*) the deflexion of the centre of the strut relative to its ends;
(*d*) the compression of the strut along its axis;
(*e*) the curvatures of the strut at three points along its length;
and (*f*) the deflexion of the apex C relative to A and B.

Prior to testing, initial out-of-line of the members was measured along their lengths. Tensile specimens were given the same heat treatment as the frames and then used to estimate the yield stress and Young's modulus value.

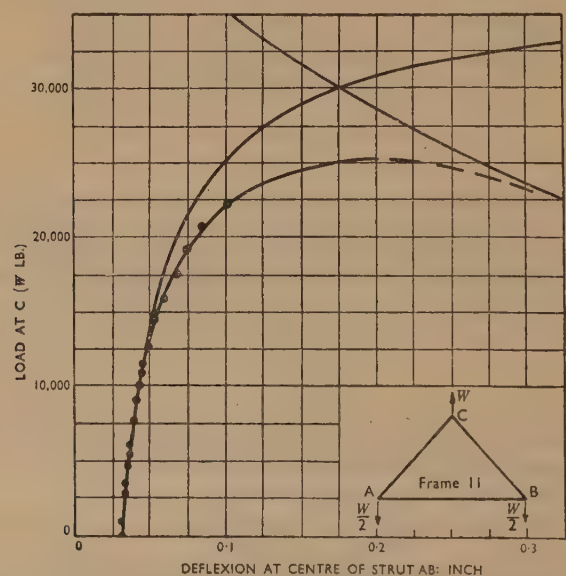


Fig. 7

The test results are analysed in Table 1 and are compared with the theory outlined in the previous section. In Fig. 7 the experimental curve of load applied at the apex of Frame 11 against deflexion at the centre of the strut AB is compared with the elastic stability line and the plastic collapse line.

The elastic stability line was obtained by applying equation (12) to the calculated values of equivalent eccentricity and initial out-of-line. The equivalent eccentricities at joints A and B were found by using equation (13). In Fig. 7 the deflexions at the centre of the strut are referred to the base line formed by joining the centres of joints A and B.

Evaluation of the critical load, W_c, involved the use of s and c Tables which were modified to account for the size of the joint block. The resulting stiffness and carry-over factors are called $\bar{s}$ and $\bar{c}$ (defined in Fig. 8a) and have been tabulated by Chandler.[11]

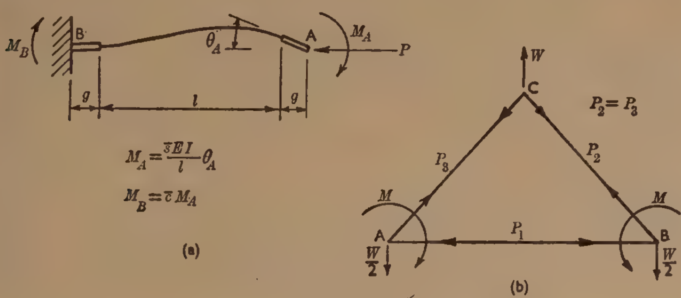


Fig. 8

Suppose a load W is applied to the frame at C (Fig. 8b), then equal and opposite moments M are applied at A and B. The relation between M and the resulting rotations at A and B (θ), is given by:

$$M = \bar{s}_1 \frac{EI_1}{l_1}(1 - \bar{c}_1)\theta + \bar{s}_3 \frac{EI_3}{l_3}\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

At the critical load, however, the moment need only be infinitesimal, whence:

$$0 = \bar{s}_1(1 - \bar{c}_1) + \bar{s}_3\left(\frac{I_3 l_1}{I_1 l_3}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Equation (15) was solved graphically to obtain W_c.

The plastic collapse line was obtained by considering the collapse mechanism in the same way as was done for the isolated strut. Failure of the frames was brought about by the three plastic hinges which formed in the strut at A′, D, and B′ (Fig. 9a) and at the apex C. Fig. 9a shows the collapse mechanism for a typical frame and Fig. 9b shows the displacement diagram[10] for the mechanism when it is in the position shown in Fig. 9a. On applying the principle of virtual displacements it was found possible to neglect the work done on the plastic hinge at C since its displacement was small for the magnitude of d_c being considered here. Hence the values of the plastic moments at A′, D, and B′ are given by:

$$M'_p = \frac{Wd_c}{4} \cot a_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

TABLE 1.—AUTHOR'S TESTS ON TRIANGULAR FRAMES

Frame No.	Joint eccentricities		Initial def. at centre of strut AB	Equivalent strut		Yield stress: lb/sq. in.	Strut size: $l \times b \times d$: inches	Base angle of frame	W_g: lb.	W_f: lb.	$\frac{W_g - W_f}{W_g} \times 100$	$0{\cdot}8 W_g$: lb.	$\frac{W_f - 0{\cdot}8 W_g}{W_f} \times 100$
	Joint A Member AB AC	Joint B Member AB BC		Initial shape	W_c: lb.								
1	+0·006 +0·002	+0·004 +0·001	−0·030	0·001 0·030 0·006	25,960	53,100	$27 \times \frac{3}{4} \times \frac{1}{2}$	48°11′	24,300	19,720	18·8	19,400	1·6
1′	−0·005 0	+0·003 −0·004	−0·044	0·005 0·044 0·003	25,960	63,000	,,	48°11′	23,400	19,270	17·7	18,700	3·0
1″	+0·016 +0·014	+0·016 +0·004	−0·050	0·037 0·050 0·022	25,960	65,000	,,	48°11′	24,800	20,280	18·2	19,800	2·4
2	−0·010 −0·023	−0·012 0	+0·012	0·036 0·012 0·012	11,240	53,100	,,	27°16′	10,550	8,650	18·0	8,440	2·1
2″	−0·026 +0·005	−0·037 −0·054	+0·054	0·020 0·054 0·098	11,240	65,000	,,	27°16′	11,000	8,850	19·5	8,800	0·6
3	+0·003 −0·002	+0·013 −0·010	−0·043	0 0·043 0·002	44,570	56,000	$21 \times \frac{3}{4} \times \frac{1}{2}$	48°11′	33,600	24,900	25·8	26,900	−8·0
8	+0·003 +0·006	+0·005 −0·021	−0·0345	0·012 0·0345 0·026	88,370	56,000	$15 \times \frac{3}{4} \times \frac{1}{2}$	48°11′	42,300	35,130	17·0	33,800	3·8
5	−0·001 +0·004	+0·006 +0·009	−0·048	0·005 0·048 0·019	17,740	66,400	$21 \times \frac{3}{4} \times \frac{3}{8}$	48°11′	16,300	15,000	8·0	13,000	13·3
11	+0·001 −0·042	−0·020 +0·040	−0·200	0·062 0·020 0·040	36,660	70,000	$15 \times \frac{3}{4} \times \frac{3}{8}$	48°11′	30,200	25,300	16·2	24,200	4·3

Note.—Eccentricities are considered positive if the end of the member is outwards from the centre of the frame relative to the centre of the joint block. Initial deflexion at the centre of the strut AB is measured from the straight line joining the ends of the member to the centre-line of the member. It is considered positive if the members present a concave surface to the centre of the frame.

TABLE 2.—TESTS BY BAKER AND RODERICK ON STANCHIONS IN DOUBLE AND SINGLE CURVATURE

Frame No.	Equivalent strut: Initial shape	Equivalent strut: W_c: tons	Yield stress: tons/sq. in.	Strut size $l \times b \times d$: inches	W_g: tons	W_f: tons	$\frac{W_g - W_f}{W_g} \times 100$	$0{\cdot}8\ W_g$ tons	$\frac{W_f - 0{\cdot}8 W_g}{W_f} \times 100$
F2B15	0·032	33·6	20·32	$10 \times 1\frac{1}{4} \times \frac{3}{8}$	8·50	6·86	19·3	6·83	0·4
F2B18	0·062	33·6	20·32	$10 \times 1\frac{1}{4} \times \frac{3}{8}$	7·75	6·17	20·4	6·21	—0·7
F2B26	0·062	33·6	20·56	$10 \times 1\frac{1}{4} \times \frac{3}{8}$	7·75	6·05	21·9	6·21	—2·7
F2B19	0·032	10·8	20·56	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	5·12	4·30	16·0	4·10	4·7
F2B22	0·077	10·8	20·56	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	4·21	3·15	25·2	3·36	—6·7
F2B23	0·032	10·8	20·56	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	5·12	4·30	16·0	4·10	4·7
F1B17	0	33·6	18·19	$10 \times 1\frac{1}{4} \times \frac{3}{8}$	8·50	7·87	7·4	6·80	13·6
F1B15	0·0050	33·6	18·19	$10 \times 1\frac{1}{4} \times \frac{3}{8}$	8·35	7·63	9·8	6·68	12·2
F1B5	0·0009	33·6	19·02	$10 \times 1\frac{1}{4} \times \frac{3}{8}$	8·90	8·03	9·9	7·12	11·3
F1B10	0·0034	33·6	19·02	$10 \times 1\frac{1}{4} \times \frac{3}{8}$	8·80	7·32	16·7	7·05	3·7
F1B18	0·0027	10·8	20·26	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	6·11	4·38	28·3	4·89	—11·6
F1B8	0·0020	10·8	16·79	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	5·20	4·50	13·5	4·16	7·6
F1B6	0·0013	10·8	16·79	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	5·20	4·56	12·3	4·16	8·8
F1B14	0·0078	10·8	20·26	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	5·72	4·69	18·0	4·58	2·4
F1B19	0·0127	10·8	20·26	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	5·62	4·44	21·0	4·50	—1·4
F1B11	0·0065	10·8	20·26	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	5·80	4·72	18·6	4·64	1·7
F1B13	0·0109	10·8	16·79	$10 \times 1\frac{1}{4} \times \frac{1}{4}$	4·82	3·62	24·9	3·86	—6·6
F1B21	0·0003	33·6	18·19	$10 \times 1\frac{1}{4} \times \frac{3}{8}$	8·50	7·61	10·5	6·80	10·6

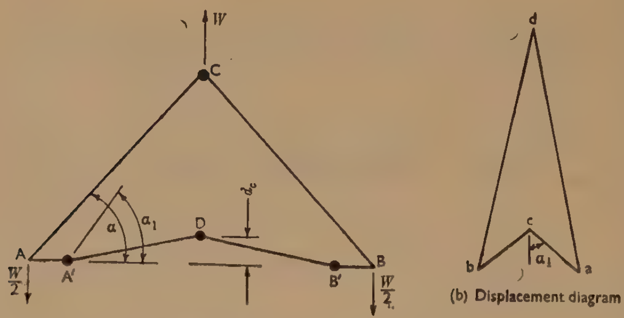
Note.—All loads contained in this Table are total axial loads in the stanchions.

Substituting from equation (9):

$$W = 2P_y \tan \alpha^2 \frac{\tan \alpha}{\tan \alpha_1}\left[\sqrt{\frac{d_c^2}{d^2} + \frac{\tan^2 \alpha_1}{\tan^2 \alpha}} - \frac{d_c}{d}\right] \quad . \quad . \quad . \quad . \quad (17)$$

where P_y is the yield load of AB (= area of AB × yield stress) and d is the depth of section AB.

In all frames the plastic hinges formed close to the centre of the strut and just



(a) Collapse mechanism

(b) Displacement diagram

Fig. 9

inside the strut away from the joint blocks at A and B. The plastic collapse line is plotted in Fig. 7 for Frame 11.

Experimental values of central deflexion agree very well with the elastic stability line until the applied load is about 22,000 lb. After that load, experimental deflexions are larger and the experimental curve becomes asymptotic to the plastic collapse line. Break-away from the elastic stability line should start at the point where the yield stress is reached at some point in the structure; in this case yield first occurs at D (Fig. 9a).

While tabulating the results of all of the frames it was noticed that there appeared to be a simple relation between the loads W_g (Fig. 6) at the intersection of the two calculated curves and the experimental failure loads W_f. It was found that W_f was approximately equal to 80% of W_g. From Table 1 this is seen to be so to within 5% for all except two of the frames, namely, 3 and 5. It was decided to see if a similar simple law applied in the case of other frameworks. Test results were available from two other sources.

Professor Baker's and Dr Roderick's work [12, 13] at Cambridge on rigid frames bent in single and double curvature is so well known that it requires little introduction. A number of stanchions, all of which had two loaded beams attached, were loaded axially to failure. The beam loads were arranged to bend the stanchion into either single curvature (this test series was given the code No. F.2.B.) or double curvature (code No. F.1.B.). The set-up for bending the stanchion into single curvature is shown diagramatically in Fig. 10.

To find the critical load equal and opposite moments, M, are applied at A and B (Fig. 10). The relation between the applied moment M and the rotations θ at A and B is given by:

$$M = 3\frac{EI_1}{l_1}\theta + s(1 - \bar{c})\frac{EI_2}{l_2}\theta$$

And the crippling load $M = 0$, whence

$$\bar{s}(1 - \bar{c}) = -3\frac{I_1 l_2}{I_2 l_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

which can be solved to find the axial critical load, W_c.

The expression which corresponds to (17) for the plastic collapse condition is readily derived and is

$$W = P_y\left[\sqrt{\frac{d_c^{\,2}}{d^2} + 1} - \frac{d_c}{d}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

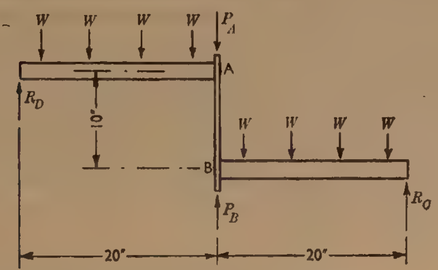


FIG. 10

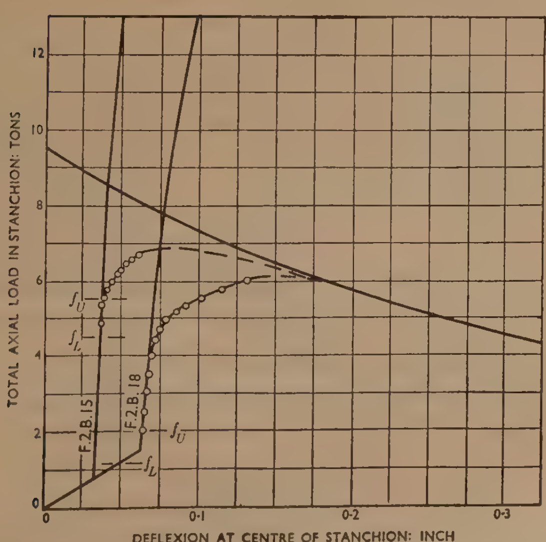


FIG. 11

Equations (12) and (19) were then used to draw the elastic stability and plastic collapse lines for the frames contained in Table 2. These were frames chosen at random from references 12 and 13. However, in the case of the frames with stanchions bent into single curvature the beam loads caused the initial deflexion. In each of these cases the hyperbola of equation (12) was made to pass through the point corresponding to the deflexion at the beam loads alone. In Fig. 11 the results of frames F.2.B.15 and F.2.B.18 are plotted.

From Table 2 it is again seen that the failure loads, W_f, are approximately equal to 80% of W_g. Agreement is best in the case of those frames whose stanchions are bent into single curvature (series F.2.B.). When the stanchion is bent into double curvature (series F.1.B.) there is a tendency for a much larger variation from this rule. However, only in the case of F.1.B.18 does this rule seriously overestimate the collapse load. It can only be suggested that perhaps some eccentricity in loading or variation of yield stress (an average value is given for each bar from which the frames were made) has caused this discrepancy.

Stevens[14] tested nine panels, each 12-in. square, which could have formed part of a typical bridge truss. The panels and method of testing are shown in Fig. 12. It is unfortunate that the joints were not frictionless but gave some measure of support to the structure by partially preventing rotation at the joints. This resulted in the collapse loads being larger than expected. Table 3 summarizes the results obtained

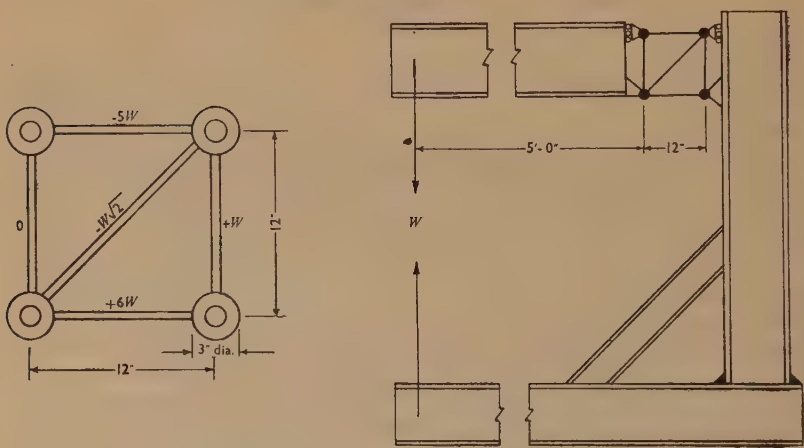


FIG. 12

TABLE 3.—STEVENS'S TESTS ON ISOLATED TRUSS PANELS

Panel No.	Detail of strut in.: AB	W_c: lb.	Yield stress: lb/sq. in.	W_g: lb.	W_f: lb.	$0{\cdot}8\ W_g$: lb.
1	12 × 1 × 0·738	40,000	45,900	5,600	5,550	4,480
9	12 × 1 × 0·368	5,000	45,900	2,790	2,770	2,230

by applying the equivalent strut theory to only two of Stevens's panels; they are the most and least rigid panels of his series. Eccentricities and initial out-of-line were small (less than 0·010 in.) and, since the critical loads were large compared with the yield loads, stability effects were small in all of Stevens's panels.

Discussion of results

In frames tested by the Author the load/deflexion curves obtained experimentally agreed closely with those predicted by the equivalent strut theory in all except one frame, namely, frame 1″. This is probably because of some error in measuring the eccentricities and initial curvatures of the members. Also, since the initial deflexions of the ties were both negative (see note, Table 1) in this frame but are neglected in the theory they would cause slightly larger deflexions of the strut than anticipated. However, this effect was not serious. The general tendency is for the estimated collapse load ($= 0{\cdot}80\ W_g$) to be conservative.

Agreement between the equivalent strut theory and the results of the frames tested in single curvature by Baker and Roderick is very good. All estimated collapse loads lie within 7% of the experimental values. The corresponding variation in those frames whose stanchions were bent into double curvature is larger, but in all cases the estimated collapse load lies within 13·6% of the experimental collapse load.

The results of Stevens's tests give a qualitative idea of the behaviour of these panels. Because of the friction at the joints additional support was given to the frame. This tended to raise the collapse load above that estimated by the equivalent strut theory. In all frames stability effects were small with the result that the experimental collapse loads almost coincided with W_g.

The equivalent strut theory shows that the direction in which the strut fails is of no great importance since it will have little effect upon the collapse load. In all frames tested by the Author failure was in the direction anticipated by the equivalent strut theory. Frame 2″ is particularly interesting because the centre-line of its strut was outside the base line formed by joining the centre of joint blocks A and B. An outwards failure of the strut might have been expected. However, by taking account of joint eccentricity by the equivalent strut theory an inwards failure was predicted and did occur. The behaviour of F.1.B.21 is contrary to that predicted by the equivalent strut theory in respect to the direction of strut failure. Whether this behaviour was caused by some eccentricity of loading or not, it was seen from the equivalent strut theory that for the relative magnitudes of initial imperfection, crippling load, and yield stress occurring in this frame, that a failure load of about 4·64 tons was to be expected.

It was shown by Baker and Roderick that collapse loads estimated by the method set out in B.S.449 (1948) could be many times less than the experimental collapse loads. The formula of Appendix C of B.S.449 (1948) was applied to two of the Author's frames using the true yield stress of the material. The estimated collapse load of frame 1 was 12,600 lb. and that of frame 8 was 15,400 lb. It is seen from Table 1 that the experimental collapse loads are respectively 1·56 and 2·28 times greater.

Conclusions

The test results are limited in that they only cover a small number of tests and are restricted to frameworks having members of rectangular section. However, they do cover a wide range of framework types whose rigidity and yield strengths

vary considerably. Experimental collapse loads are consistent with those obtained by the equivalent strut theory outlined in the Paper. The method is simple and easily applicable to rigidly jointed frameworks in which the axial forces are large.

Collapse loads of structures estimated by B.S.449 (1948) appear to bear little relation to the actual collapse loads. B.S.449 (1948) grossly underestimates the collapse loads of some structures and does not lead, in these cases, to the most economical structure. It makes no statement about initial curvature and eccentricity of the members. Both of these can have a considerable effect upon the collapse load and, therefore, some effort should be made to limit them to practical values. These values could then be used by the designer to determine the worst combination of eccentricity and initial curvature prior to calculating the collapse load of the structure.

Further work is required before the equivalent strut theory could be used with any measure of confidence. The Author is conducting further tests on triangular frames similar to those listed in the Paper. The method of choosing the critical member warrants further investigation since it depends upon a number of variables, the chief of which are the P/P_E values and the imperfections in the members. Sideways buckling of the critical member also requires investigation and it is suggested that this phenomenon can be treated in a similar way to the method described. This work is in hand and a number of simple three-panel Warren trusses are being constructed for this purpose.

Acknowledgements

The work described in the Paper is being carried out in the Whitworth Laboratories of the University of Manchester. The Author would like to thank Professor J. A. L. Matheson for his encouragement in this work.

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APPENDIX I

Equivalent strut theory applied to frame 11

The methods by which the curves of Fig. 7 were obtained are described below. These curves apply to frame 11 tested by the Author. The dimensions of this frame are shown in Fig. 13. The strut AB was made from a heat-treated bright drawn steel which had a yield stress of 70,000 lb/sq. in. and a Young's modulus value of 28×10^6 lb/sq. in.

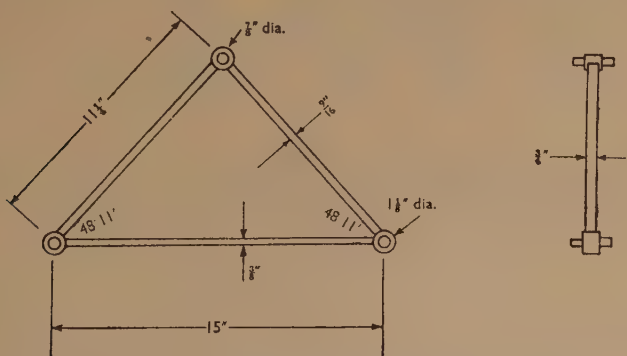

Fig. 13.—Dimensions of frame 11

To draw the elastic stability line it is first necessary to know the critical load, W_C, of the frame.

Determination of the Critical Load (W_C)

By referring to Fig. 8b it is seen that from frame 11, whose base angle is 48° 11′:

$$P_1 = 0{\cdot}4473\ W \text{ and } P_3 = 0{\cdot}6709\ W$$

$$\text{Also } \frac{I_1}{l_1} = \frac{0{\cdot}75 \times 0{\cdot}375^3}{13{\cdot}875 \times 12} = 0{\cdot}2375 \times 10^{-3} \text{ in.}^3$$

$$\text{and } \frac{I_3}{l_3} = \frac{0{\cdot}75 \times 0{\cdot}5625^3}{10{\cdot}25 \times 12} = 1{\cdot}085 \times 10^{-3} \text{ in.}^3$$

$$\text{whence } \frac{I_3 l_1}{I_1 l_3} = 4{\cdot}570$$

The first Euler load of AB (P_{E1}) = 4,729 lb.

and the first Euler load of AC (P_{E3}) = 29,260 lb.

Now let the ratio of the axial load in AB to its first Euler load (P_1/P_{E1}) be denoted by x and the ratio of axial load in AC to its first Euler load (P_3/P_{E3}) by y. Then,

$$y = \frac{P_3}{P_1} \cdot \frac{P_{E1}}{P_{E3}} x = -0{\cdot}2424x$$

The negative sign arises here because the compressive load P_1 is taken as positive and the load P_3, being tensile, is negative.

By substitution, equation (15) becomes:

$$\bar{s}_1(1-\bar{c}_1) + 4{\cdot}570\,\bar{s}_{-0{\cdot}2424x} = 0 \qquad (20)$$

The value of the stiffness and carry-over factors $\bar{s}$ and $\bar{c}$ depend upon the ratio of the length of the rigid joint to the length of the member between the joints. In the case of the strut AB the rigid joints are of equal length, namely, 0·563 in., and this ratio (0·563/13·88) is 0·0406. In the case of the ties, however, the rigid joints are of unequal length and the average value, namely, 0·5 in., is taken. The ratio of the length of joint to length of the member between the joints in member AC is (0·5/10·25) 0·0488. Values of $\bar{s}$ and $\bar{c}$ are obtained from Chandler's tables.[11]

Equation (20) is solved graphically by assuming a value of $x = P_1/P_{E1}$ and by plotting the two curves $\bar{s}_1\,(1-\bar{c}_1)$ and $4{\cdot}570\,\bar{s}_{-0{\cdot}2424x}$ against x. This is done in Table 4.

TABLE 4

x	$0{\cdot}2424x$	$\bar{s}_{-0{\cdot}2424x}$	$4{\cdot}570\,\bar{s}_{-0{\cdot}2424x}$	$\bar{s}_1$	$\bar{s}_1\bar{c}_1$	$\bar{s}_1(1-\bar{c}_1)$
3·44	0·8340	7·107	27·91	−13.248	13·221	−26·469
3·48	0·8437	6·127	28·00	−14·429	14·288	−28·717

The two curves have equal and opposite ordinates at the point where $x = 3{\cdot}467$ and equation (20) is then satisfied. When $x = 3{\cdot}467$ the strut AB has become so flexible that it cannot be supported by the ties AC and BC. At this point, the axial load in AB is 3·467 × 4,729·5, i.e., 16,400 lb.

Thus the critical load of the frame, $W_c = \dfrac{16{,}400}{0{\cdot}4473} = 36{,}660$ lb.

Plotting of elastic stability line

Having now obtained the value of the critical load, W_c, it is possible to draw the elastic stability line by substituting in equation (12). The value of $(e + a_1)$ is found from the equivalent strut illustrated in Table 1. In the case of frame 11 the value of $(e + a_1)$ is 0·031. In Table 5 values of y_c are obtained from equation (12).

TABLE 5

W/W_c	W: lb.	$y_c\left(=\dfrac{0{\cdot}031}{1-W/W_c}\right)$	Deflexion at centre of strut AB: in.
0	0	0·0310	0·0295
0·2	7,330	0·0387	0·0372
0·4	14,670	0·0517	0·0502
0·6	22,000	0·0775	0·0760
0·8	29,300	0·1550	0·1535
0·9	33,000	0·3100	0·3085
1·0	36,660	∞	∞

The deflexion which is plotted in Fig. 7 is the distance from the straight line joining the centres of the joint blocks A and B to the centre of the strut AB.

The plastic collapse line

The plastic collapse line for frame 11 is obtained by direct substitution in equation (17). By assuming that the plastic hinges at A′ and B′ (Fig. 9a) are $1\frac{1}{4}$ in. from A and B respectively, then $\tan \alpha_1 = 1{\cdot}340$. Also, since $\tan \alpha = 1{\cdot}118$ and the yield stress f_y is 70,000 lb/sq. in. then equation (17) becomes:

$$W = 2 \times 70{,}000 \times \tfrac{3}{4} \times \tfrac{3}{8} \times 1{\cdot}118 \times \frac{1{\cdot}118}{1{\cdot}340}\left[\sqrt{\frac{d_c{}^2}{d^2} + \frac{1{\cdot}340^2}{1{\cdot}118^2}} - \frac{d_c}{d}\right]$$

$$= 36{,}750\left[\sqrt{\frac{d_c{}^2}{d^2} + 1{\cdot}435} - \frac{d_c}{d}\right]$$

Values of W for corresponding values of d_c are calculated in Table 6 and then plotted in Fig. 7, as the curve called the plastic collapse line.

TABLE 6

d_c: in.	$\frac{d_c}{d}$	W: lb.
0	0	44,000
0·05	0·133	39,400
0·1	0·266	35,300
0·2	0·533	28,600
0·3	0·800	23,550
0·375	1·000	20,600

APPENDIX II

FURTHER TESTS ON TRIANGULAR FRAMES

Since the Paper was written the Author has completed the series of tests on triangular frames and the results are given in Table 7. It is seen that these results confirm the conclusions reached earlier. However, further points do emerge.

A study of the results of frames Nos 13, 14, 15, and 16 shows that the failure load, W_f, is not approximately equal to $0{\cdot}8 W_g$ as originally assumed but is more nearly equal to $0{\cdot}95\ W_g$ for these frames. These frames all had short stocky members and this seems to indicate that the factor of 0·8 could be increased for frames of this type.

The values of W_g could not be found for Frames 6, 12, and 19 because the Equivalent Strut Theory predicted an outwards failure of the strut and the intersection point, g (Fig. 6), could not therefore be located. The strut of all of the frames tested failed inwards. This apparent anomaly arises because the Equivalent Strut Theory neglects the effect of changes in axial lengths of the members.

The Paper, which was received on 4 May, 1955, is accompanied by eight sheets of diagrams from which the figures in the text have been prepared, and by two Appendices.

CORRESPONDENCE on this Paper should be forwarded to reach the Institution before 15 August, 1956. Contributions should not exceed 1,200 words.—SEC.

TABLE 7.—AUTHOR'S LATER TESTS ON TRIANGULAR FRAMES

Frame No.	Joint eccentricities: Joint A Member AB/AC	Joint eccentricities: Joint B Member AB/AC	Initial def. at centre of strut AB	Equivalent strut: Initial shape	Equivalent strut: W_c: lb.	Yield stress: lb/ sq. in.	Strut size: $l \times b \times d$: inches	Base angle of frame	$\frac{W_g}{W_f}$: lb.	$\frac{W_g - W_f}{W_g} \times 100$	$0{\cdot}8W_g$: lb.	$\frac{W_f - 0{\cdot}8W_g}{W_f} \times 100$	W_f from B.S.S. 449: lb.
6	−0·006 +0·039	−0·001 +0·042	−0·005	0·038 — 0·005 — 0·046	8,690	68,000	$21 \times \frac{3}{4} \times \frac{3}{8}$	27°16′	— 8,150	—	—	—	4,040
9	+0·008 −0·040	−0·014 +0·012	−0·021	0·037 — 0·021 — 0	43,400	30,600	$15 \times \frac{3}{4} \times \frac{1}{2}$	27°16′	11,200 9,250	17·4	8,960	3·1	8,730
10	−0·033 +0·009	0 0	−0·012	0·015 — 0·012 — 0	61,300	30,600	$15 \times \frac{3}{4} \times \frac{3}{8}$	60° 0′	26,800 21,000	21·6	21,400	−2·0	17,500
12	−0·063 −0·055	+0·007 +0·044	−0·020	0·001 — 0·020 — 0·056	19,980	70,000	,,	27°16′	— 12,380	—	—	—	7,600
13	−0·010 −0·006	−0·002 +0·009	−0·012	0·022 — 0·012 — 0·016	500,000	30,600	$9 \times \frac{3}{4} \times \frac{1}{2}$	60° 0′	38,500 36,500	5·2	30,800	15·6	36,000
14	−0·006 0	−0·009 −0·045	0	0·006 — 0 — 0·076	295,000	30,600	,,	48°11′	25,200 23,500	6·8	20,200	14·0	23,500
15	−0·004 +0·002	0 +0·008	−0·011	0·003 — 0·011 — 0·009	155,500	30,600	,,	27°16′	11,700 11,100	5·1	9,350	15·7	10,800
16	−0·006 0	−0·007 +0·004	−0·009	0·006 — 0·009 — 0·001	191,300	30,600	$9 \times \frac{3}{4} \times \frac{3}{8}$	60° 0′	28,700 27,150	5·4	23,000	15·2	23,300
17	−0·014 +0·005	+0·003 +0·011	−0.005	0·007 — 0·005 — 0·019	115,000	30,600	,,	48°11′	19,000 16,600	12·6	15,200	8·4	15,100
18	−0·075 +0·009	−0·083 +0·018	0	0·065 — 0 — 0·063	63,400	30,600	,,	27°16′	8,780 7,030	20·0	7,030	0	6,960
19	−0·007 +0·015	−0·011 +0·010	−0·010	0·023 — 0·010 — 0·009	52,300	66,200	$9 \times \frac{3}{4} \times \frac{1}{4}$	60° 0′	— 27,200	—	—	—	19,300
20	−0·006 +0·018	−0·014 +0·002	−0·007	0·021 — 0·007 — 0·011	32,100	66,200	,,	48°11′	25,400 20,200	20·4	20,300	0	12,500
21	−0·010 +0·003	−0·011 −0·007	0	0·007 — 0 — 0·019	14,200	66,200	,,	27°16′	10,900 9,070	16·8	8,720	3·9	5,750

Paper No. 6100

THE CANALS OF THE GEZIRA CANALIZATION SCHEME AND THE DESIGN OF THE GONEID PUMP SCHEME IN THE SUDAN

by

*** Ian Stanley Gordon Matthews, M.A., M.I.C.E.**

(*Ordered by the Council to be published with written discussion*)

SYNOPSIS

The greater part of the Gezira canalization scheme was constructed between 1923 and 1935, and comparatively small extensions were built in 1938 and 1950. Throughout this period the types of canal and field channel have been developed to suit the earth-moving equipment and satisfy the conditions required for irrigation. These standard sections and their method of construction are described in this Paper. A large pump scheme of about 30,000 acres was built on the banks of the Blue Nile in 1954. The Paper describes the design of the layout of the canalization and surface-water drainage using the standard Gezira channel sections. It also deals with setting-out the canal lines on the ground, the design of the canal and drain longitudinal sections, and the preparation of the information required for the earthwork excavation. Mention is made of the types of regulator and the procedure for deciding the capacity and size of regulators required.

INTRODUCTION

THE Gezira canalization scheme irrigates nearly 1,000,000 acres in the triangle of land between the Blue and White Niles, south of Khartoum, the capital city of the Sudan. The early construction of the scheme has been described by Prowde,[1] Johnston,[2] and Russell.[3] An extension to the canalized area of about 100,000 acres was completed in 1952, and this was the last extensive area of land which could be irrigated from the main canal running northwards from Sennar (Fig. 1). Preliminary reconnaissance on the east bank of the Blue Nile had shown a suitable area of about 30,000 acres which could be irrigated by pumping from the Blue Nile. Although the cost of plant greatly increases both the capital and running costs of the irrigation works, various complementary factors favoured the installation of the Goneid pump scheme. The scheme is in an area of comparatively poor rainfall (12 in.) that results in near-famine conditions as the rule rather than the exception. This poverty is the more noticeable in that the inhabitants are so close to the prosperous cultivators of the Gezira scheme. The soil survey of the area was most favourable, and in view of the high selling-price of the cash crop, cotton, it was considered that the construction of such a costly scheme was justified.

This Paper describes the canalization of the Gezira scheme, and, in detail, the

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* The Author, at the time of writing the Paper, was Divisional Engineer, Projects, and Chief Designer, Ministry of Irrigation, Sudan Government.

[1] The references are given on p. 261.

Fig. 1.—General map of the Gezira

design of the canalization of the Goneid pump scheme, which incorporates many features of the Gezira scheme. The design of the pumping plant, pump station, and all works required to take the water from the river to the discharge basin at the head of the pump channel were executed by Sir Alexander Gibb & Partners in their capacity as consulting engineers to the Sudan Irrigation Department. The Paper deals with the canalization and structures required to distribute the water on the scheme. These were designed and built by the staff of the Sudan Irrigation Department.

Gezira canalization scheme

Main and major canals

The main canal (Fig. 1) runs northwards from the Sennar Dam for nearly 200 km,† and is the main artery for irrigation. After 91 km, the Tabat branch takes off and

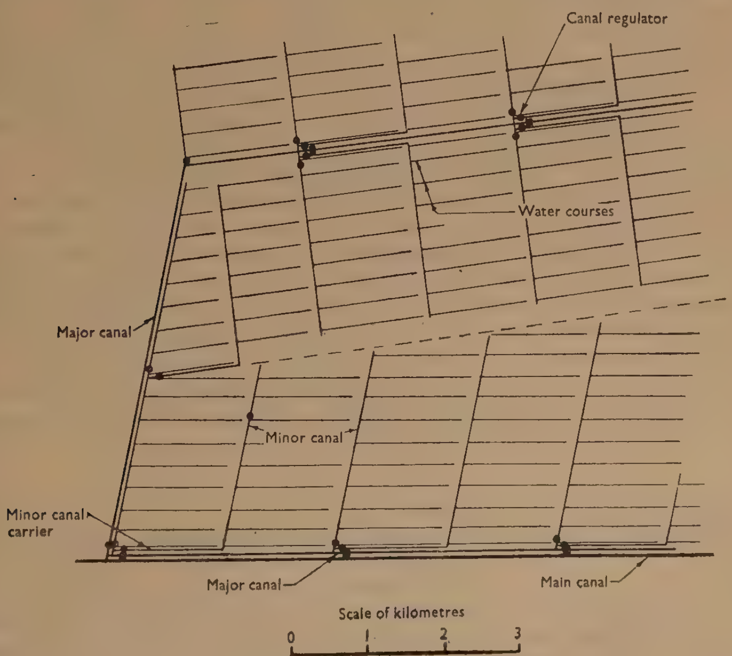


Fig. 2.—Layout of canalization

runs northwest; it is similar to the main canal. The first cross regulator on the main canal is 57 km from Sennar, and below this point the canal is divided into reaches which vary in length from 6 to 22 km. These regulators are the control points for major canal off-takes. A typical layout of canals is shown in Fig. 2; for application to the Goneid pump scheme, the pump channel corresponds to a major canal.

† A Table of conversion units is given on p. 261.

The major canals are divided into reaches of about 3 km by cross regulators. The off-takes are so far as possible grouped at these points and measured discharges through them are passed to the minor canals. Normally there is no direct irrigation from major canals or pump channels (Fig. 2).

Minor canals

With the exception of about 80,000 acres on the Gezira scheme, the minor canals (Fig. 2) are designed to store the night flow which is not passed to the fields. This water is held by providing banks on the minor canals which can hold an additional 20 cm depth of water, known as "storage depth." The canals are divided into reaches, their length varying between about 1 and 4 km. The intermediate regulators in these canals are "night storage weirs" (Fig. 11), which consist of a pipe controlled by a door and a weir of brick or concrete. The sill of the weir is at that level which allows the full discharge of the canal to flow over the weir, when the water level upstream of the regulator is at the night-storage level. With these arrangements the pipe regulators and the pipes to the watercourses are closed at sundown. The first reach of the canal fills up to the night-storage level, and the full discharge then flows over the weirs to the next reach. The process is repeated in the second reach, and so on until the tail reach of the canal is filled. At sunrise the pipe-regulator doors are opened and the watering of the fields begins. During the daylight hours the field channels draw off water at twice the daily rate of discharge and the level in the canal falls steadily to reach the design watering level at sundown.

In the case of the Goneid pump scheme, this refinement was not provided. The pumps will work for up to 18½ hours per day, and when necessary the watering of the fields will continue for a corresponding period. There will be a certain lag each day between opening up the pump channel and filling the minor canals, so that in fact irrigation on the fields will not usually exceed more than about 12 hours per day. To produce the regular layout, and reduce cross regulators in the major canals, alternate minor canals are fed by a carrier channel adjacent to the major canal. This type of channel is known locally as a "gannabia" (Fig. 2), an Arabic word which signifies that the canal runs alongside another. The characteristic of the gannabia is that only one bank has to be provided.

Field channels—watercourses and laterals

The main field channel or "watercourse" normally supplies water to a field of about 90 acres, which in the Gezira scheme is about 1,400 m long by 300 m wide (Fig. 3). Each watercourse is fed through a 14-in. steel pipe, 12 m long, passing under the canal bank. The flow to the watercourse is controlled by a chopper-type valve. The discharge is at the rate of 10,000 cu. m/day, and the watercourse normally runs for 12 hours and so delivers 5,000 cu. m to the fields. A standard watering is reckoned to be 400 cu. m per acre, and therefore the 90-acre field is watered in about a week.

From the watercourse take-off subsidiary field channels, or laterals, run parallel to the minor canal. In a Gezira field of 90 acres there are nine laterals and the resulting plots cover 10 acres each. These plots are the agricultural unit, and each cotton plot is cultivated by one tenant. In the Goneid scheme, the plots were reduced to 5 acres each, to share the advantages of an irrigation scheme among a correspondingly larger number of people in an area of low rainfall.

The tenant is responsible for the maintenance of the watercourses and laterals

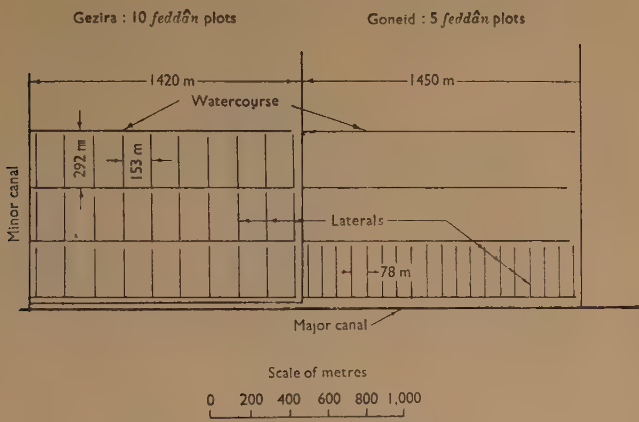


FIG. 3.—LAYOUT OF FIELD CHANNELS

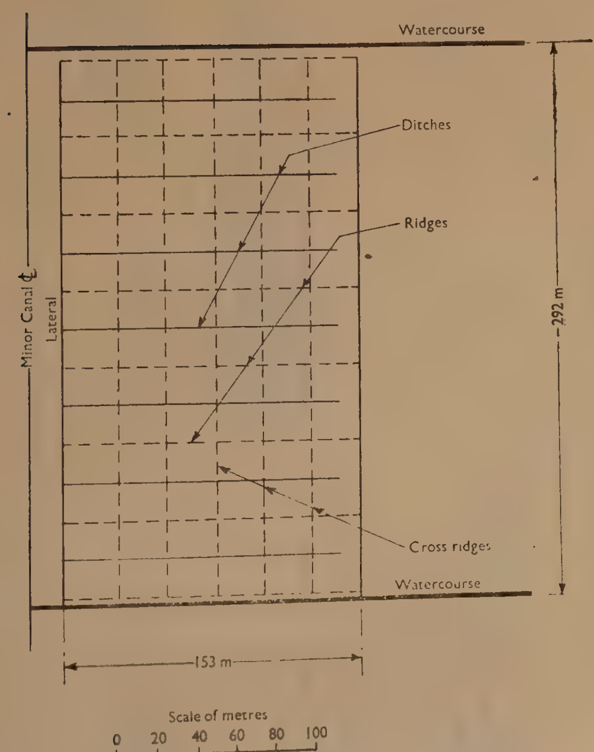


FIG. 4.—TYPICAL LAYOUT OF FIELD PLOT

TABLE 1.—DESIGN DATA FOR WATERWAY AND BANK SECTIONS OF CANALS

	Main and branch canals	Major canals	Minor canals
WATERWAY			
Command—maximum, i.e., height of water level above ground level	2·0 m	0·80 m	0·60 m
Manning's $\frac{1}{n}$	45	45	45
Side slope of cut	2:1	2:1	1:1 or 2:1
Bed width and water depth	To provide a water section to pass the required discharge as calculated by Manning's formula. The proportions of the channel to conform to the Lacey theory for a silt factor of 0·63, so far as is practicable	As for main and branch canals	Bed width 4·2 m minimum for canals dug by elevating grader. Otherwise, the channel dimensions to conform as far as possible to the Lacey theory for a silt factor of 0·63
Water slopes	7 to 15 cm per km	As high as the ground slope and limiting head on the upstream regulator will allow	For original design of banking, assumed to be flat. For the required section after silting the water slope to be as great as possible, limited only by the permissible head on the regulators controlling the reach

Note:—Commands may be increased above these values for short reaches of the canal.

BANK SECTION			
Bank cover, i.e., height of top of bank above water level	1·0 m	0·75 m	0·50 m
Hydraulic gradient through bank	8:1 from 0·50 m above design water level	8:1 from 0·30 m above design water level	7:1 from 0·20 m above design water level
Bank height—minimum	2·0 m	0·75 m	0·70 m
Bank top width	Minimum 5·0 m	Minimum 2·0 m	1·0 m
Side slope—inner	2:1	2:1	2:1
outer	To suit position of outer toe to provide the correct hydraulic gradient and bank top width, with the proviso that the slope shall not be steeper than 2:1		
Berm width	The depth of excavation with a minimum of 2·0 m	Minimum 0·50 m	Minimum 0·50 m
Bank top cross-slope from inner crest	40:1	Flat	Flat
Bulking of excavation	25%	25%	25%
Settlement of banks	15%	15%	15%

although in the first instance these are excavated by mechanical plant. The tenant prepares the necessary channels for watering within the 10-acre plot (see Fig. 4).

Canal sections

During the construction of the Gezira scheme, sections for the various types of canals and field channels have been standardized and the factors affecting these bank shapes and water sections are summarized in Tables 1 and 2. Typical cross-sections of various channels are shown in Figs 5, 6, 7, 8, 9, and 10. The cube of excavation must satisfy the requirements of both the waterway and the borrow for the bank section. The criterion for excavation of main and branch canals is usually the section required for waterway. On the other hand, for major and minor canals, the criterion, with very occasional exceptions, will be the earthwork volume required for

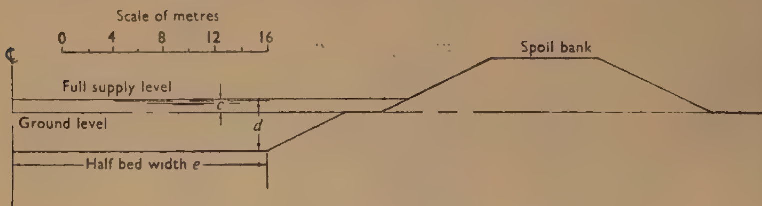


Fig. 5.—Main or branch canal

Notes: 1.—The cut is fixed by command C, water depth d, and bed width c. 2.—The command may vary from negative values of about 0·5 m to a command of 2·0 m. 3.—The bed width may vary from 50 m to 10 m. 4.—The water depth may vary from 4·5 m to 2·5 m. 5.—The length of reach between regulators is about 20 km.

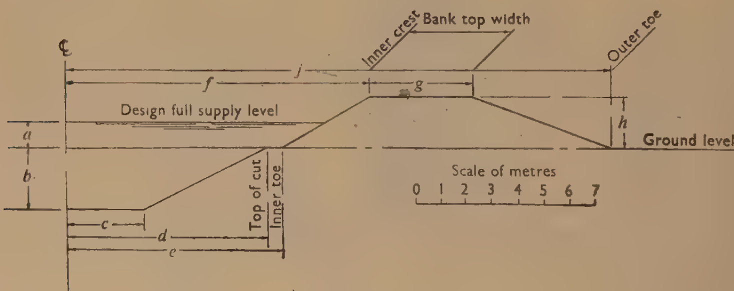


Fig. 6.—Major canal

Note: Dimension h is gross bank height, 15% greater than net bank height

TYPICAL DIMENSIONS: METRES

a	b	c	d	e	f	g	h	j
0·97	2·43	3·0	7·86	8·36	11·80	4·0	1·98	21·06
0·60	1·91	2·0	5·82	7·73	10·43	3·0	1·55	16·73
0·40	1·40	2·0	4·80	6·20	8·50	2·0	1·32	13·20
0·20	1·10	2·0	4·20	5·30	7·20	2·0	1·09	11·10

Fig. 11.—Circular night-storage weir

Fig. 12.—Pipe regulator

Fig. 13.—Movable weir—series I

Fig. 14.—Movable weir—series II

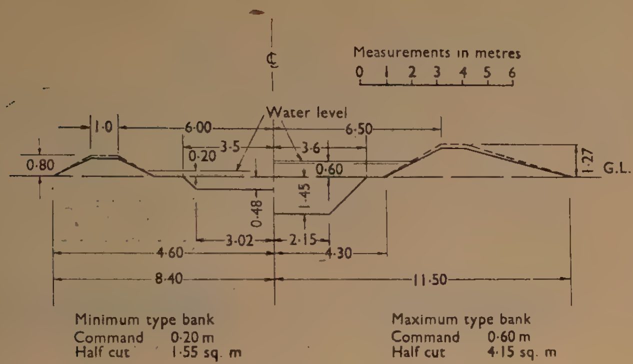


Fig. 7.—Minor canal

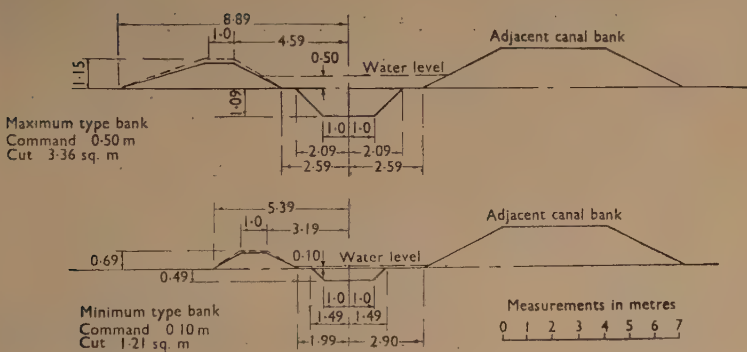


Fig. 8.—Minor canal carrier—single-bank "gannabia"

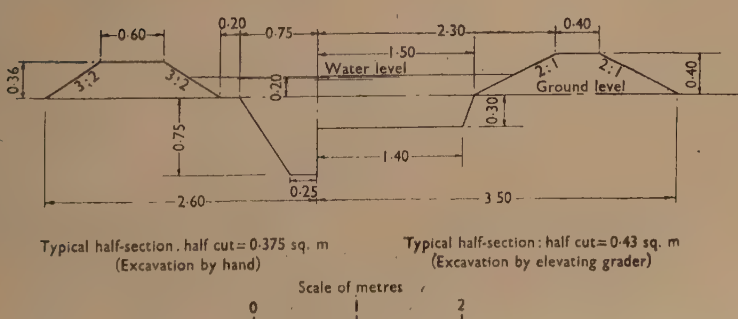
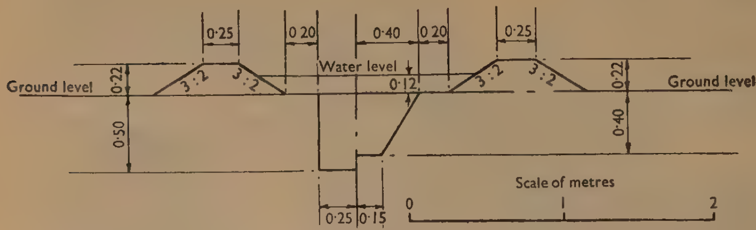


Fig. 9.—Watercourse



Typical half-section : half cut = 0·125 sq. m (Excavation by hand)

Typical half-section : half cut = 0·11 sq. m (Excavation by ditcher. Additional borrow to make up type bank by hand work)

Fig. 10.—Lateral

banking. Dragline excavators are used for the construction of all canals with the exception of minor canals; for these, elevating graders are usually employed, and the wide bed section of 4·2 m has been adopted to accommodate the plant in the bed of the canal. For excavating single-bank carrier minor canals, dragline excavators are used.

The excavation for field channels does not vary from section to section. The

Table 2.—Design data for field channels

	Watercourse	Lateral
Waterway		
Command minimum	10 cm	7 cm
,, maximum	20 cm	12 cm
Water slope minimum	5 cm per km	5 cm per km
Manning's $\frac{1}{n}$	50	50
Discharge per 12 hours	5,000 cu. m	2,000 cu. m
Bed width	1·0 m	0·60 m
Water depth	0·40 m	0·30 m
Bank section		
Bank cover	0·16 m (hand) 0·20 m (machine)	0·10 m
Bank height	0·36 m (hand) 0·40 m (machine)	0·22 m
Bank top width	0·60 m (hand) 0·40 m (machine)	0·25 m
Inner and outer side slope	3:2 (hand) 2:1 (machine)	3:2
Bulking of excavation	25%	25%
Settlement of banks	15%	15%

The dimensions for machine excavation are for work by a short-jib elevating grader.

layout is arranged so that commands do not exceed those which the standard bank can carry. In the case of watercourses, there is considerable variation between the type for hand work and machine excavation. The machine type detailed in Table 2 is the result of excavation by a short-jib elevating grader. Provided the cut is not less than 0·75 sq. m, and the bank height at least 0·36 m, the section can be varied to suit the plant. The water surface width should be between 3·80 m and 2·50 m which can be considered as maximum and minimum dimensions. Various types of plough excavators have been tried recently in the Sudan and it may be possible to excavate a watercourse in one pass of a plough instead of the ten or twelve runs required by the elevating grader. The plough will, of course, require a more powerful tractor.

Laterals have for many years been excavated by one pass of a Killifer ditcher, leaving a certain amount of tidying up to be done by the cultivator.

Water levels

Minor canals

The minimum head from water level in the minor canal to the highest ground level in the cultivation is, for continuous watering canals, 0·30 m, to allow for loss of head through the pipe to the watercourse and in the laterals. The maximum permissible head on the canal banks which will not cause excessive seepage or entail a bank width which would encroach on the road is 0·60 m. Spoil to provide adequate cross-section for these banks to withstand the above heads has to be borrowed from the channel section. The excavated area of the channel is therefore far in excess of the waterway required for the designed discharge. Consequently, water slopes will be very small until silting occurs and raises the water slopes to design, and on low discharges the water slopes will always be less than the designed slopes. For these reasons the original designed water surface is assumed flat from the point of minimum command (30 cm) to the point of maximum command (60 cm), where under normal conditions a regulator is placed. The minor canals are thus divided into reaches, their length varying between 1 and up to about 4 km according to the slope of the land. The intermediate regulators in continuous watering minor canals are pipe regulators (Fig. 12) and a minimum head of 10 cm is required for pipes of 0·91 m diameter and above; for smaller pipes a head of 5 cm is allowed.

Major canals

The level in the pool upstream of the major canal regulator must satisfy two conditions. It should be high enough to provide adequate head through the major regulator, and this may be 30 cm for a large standing-wave movable weir (Fig. 14) or 20 cm for an undershot gate (Fig. 15). The water level downstream of the major regulator should allow for adequate slope on the canal reach. The practice in the Gezira is so far as possible to accommodate the slope required by the Lacey theory using a silt factor of 0·63. On the other hand, with the present high price for the cash crop, cotton, lower water slopes are admitted, the cost of the resulting excavation required for clearance of the silt deposited being justified by the additional land brought under cultivation.

At the same time the pool level must be high enough to provide adequate head for the minor canal off-takes. For the Goneid pump scheme, where there is no "night storage" and the downstream water level in the minor canal will be reasonably constant, well-head pipe regulators (Fig. 16) are used, and for these a minimum head of 10 cm is usually provided. With a night-storage system, where the water level in

the minor canal will vary throughout the day (when the minor canal storage is emptying) and during the night (when the minor canal storage is filling up), a small type of standing-wave movable weir (Fig. 13) is used to control the discharge to the minor canal, and for this type of regulator a minimum head of 20 cm is provided. The level in the minor canal immediately downstream of the head regulator should allow for a minimum slope of 5 cm per km to the first intermediate regulator on the watering reach of the canal and, if possible, at least 10 cm per km on the carrier or "gannabia" length, if the minor canal is fed through such a channel.

Layout

The layout of the field channels, minor canals, and major canals is based on the field plot dimensions, and the width required for canals, watercourses, drains, and roads. The width of the plots has been standardized in the Gezira scheme to 280 m. This is suitable for 10-acre plots, and is in fact used for the Goneid scheme where the plots are only 5 acres. On uneven terrain it may be an advantage to reduce the width of the plot, and for smaller fields, it is possible that a narrower plot would be an advantage. As the distance between watercourses is reduced, so the number of field outlet pipes from the canal to the field channels is increased, and because the cost of these is appreciable, the agricultural plots should be made as wide as possible.

In addition to the net width of cultivation, distances are required for the inspection road and width of the actual watercourse, and the distance between adjacent field channels is (say):—

Width of cultivation	280
,, ,, field channel	6
,, ,, roadway	6
Total:	292 metres

The distance between minor canals is settled by the length of the field. There are certain considerations which must be satisfied:

(*a*) The field must be small enough to be watered by the standard watercourse. As has been previously stated the watercourses in the Sudan Gezira carry sufficient water to irrigate the 90-acre fields in about a week, if only watering by day, and twice that area, 180 acres, if continuous watering is allowed.

(*b*) The watercourse must be small enough to be maintained by the cultivator.

The Goneid layout was based on 90-acre fields, the standard Gezira area for watering by day only. (The Author has used "acre" when describing the Gezira scheme, a measure of area which is more generally known than the *feddân*, the local unit of area which is for all practical purposes the same as one acre, but is in fact 4,200 sq. m, or 1·038 acres.)

Thus for a 90-*feddân* field made up of 5-*feddân* plots measuring 280 m by 75 m the distance between the centre-lines of minor canals on the Goneid scheme is:

Half-width of minor canal	9·5
Roadway	7·0
Half-width of lateral	1·5
Eighteen plots each 75 m long	1,350·0
Eighteen laterals each 3 m long	54·0

Half-width of lateral	1·5
Minor drain	10·0
Roadway	7·0
Half-width of minor canal	9·5
Total:	1,450·0 metres

Goneid pump scheme design

Contour plans and soil survey

The first layout shows the main, major, and minor canals and is prepared on a contour plan which is to such a scale that the layout as a whole can be seen as one picture. The preliminary layout for an 800,000-acre extension to the Gezira scheme was made to scale 1:100,000. For the Goneid scheme of about 30,000 acres, a 1:50,000 contour plan was used. On this plan the results of the soil survey were shown to define the boundaries of the land which is suitable for irrigation. In the Gezira soils, the yields of cotton and the sodium values which occur in the soils have been correlated, and the following limiting sodium values are used:

Good soil	0–25
Medium soil	26–35
Bad soil	over 35

The sodium value is the measurement of exchangeable plus soluble sodium in composite 3-ft soil samples expressed in milligram equivalent per 100 g of clay. It is the practice in the Sudan to irrigate only areas of medium and good land, preferably with the larger part good.

Design criteria

The layout of canalization is defined by certain criteria:

(*a*) Canals and field channels may not cross a natural drainage line.

(*b*) Minor canals are so far as possible spaced at 1·450 km to accommodate 90-acre fields, and their lines should follow a natural ground slope of from 10 to 20 cm per km.

(*c*) The lines of watercourses, i.e., parallels between the minor canals, should so far as possible have slopes of at least 5 cm per km.

(*d*) The major canals, or pump channels, will have cross regulators at alternate minor canal off-takes, i.e., at intervals of 2·90 km. The water level upstream of these regulators should not be more than 0·80 m above the ground level, and at the same time this level should be high enough to satisfy the two conditions of design water levels described above. It should be 0·50 to 0·60 m above the upstream pool level of the next cross regulator, to provide adequate water slope on 2·8 km of canal (say 10 cm per km) and sufficient head on the regulator, 20 to 30 cm.

At the same time, the minor canal will require a pool level at least 55 cm above the ground level, which is made up as follows:

Water level in minor canal above cultivation	0·30
Water slope in first reach of the minor canal, assumed to be 5 cm per km for 3 km	0·15
Head through well-head regulator from major pool to the minor canal	0·10
Total:	0·55 metres

To achieve a regulator layout, a certain latitude is permitted. The slope of minor-canal lines may be reduced below 10 cm per km and the command at major regulators may be increased to 1 m for short reaches of canal.

Goneid pump scheme layout

The preliminary layout of the Goneid pump scheme is shown in Fig. 18. The head of the pump channel was settled to give a point of reasonable command as near as possible to the pump station to reduce the length of the rising main. The drainage line ABCD defined the west boundary of the main block of canalization, and on the east the rising ground settled the line of the pump channel, which must have a reasonable slope. The regular layout of minor canals Nos 3 to 15 was made first, and the line of the pump channel settled to avoid excessive commands as described above. The other canals, No. 1 North and No. 1 Right, were then fitted in to complete the layout.

The preliminary layout of canalization was next transferred to a scale 1:10,000 contour plan, which showed spot ground levels at 100-m intervals on lines 200 m apart. The exact kilometrages of the angles and minor canal off-takes on the pump channel were calculated, and at the same time the layout of the whole scheme was checked for proper working of the watercourses and laterals. These should follow a natural ground slope, but to preserve a regular layout a watercourse line may rise 10 cm and a lateral may rise 5 cm.

Setting-out in the field

The permanent survey marks on the contour plan of Goneid are concrete poles 26 ft high, placed at intervals of approximately 4·5 km. The exact location of these poles was not known, and for setting-out the canal lines, this grid was not used. The pump channel was set out by using the 1:10,000 plan to measure off the location for the first leg and then continuing by theodolite traverse. From the pump channel, the minor canals were set out by theodolite.

A more satisfactory method was previously practised in the Gezira scheme and is recommended for future projects. The survey and spot level plans are sited on "beacons," which are 4 in.-dia. pipes, protruding about 8 ft above ground. These beacons are placed at minute intervals of latitude and longitude, and are about 1,840 m apart. The exact distances between the beacons are measured up and recorded, and these distances are used for the calculations. The main and major canals are set out from these beacons. From the layout plan, the distance is measured from two points on each leg to convenient beacons, and these ties are calculated at intervals of about 3 km along the line. In the field, the line is set out from these ties and by lining-in with a theodolite between the tie points. The minor canals are set off by theodolite from the main and major canals, sited at correct kilometrage on the parent channel. A beacon tie at the extremity of each minor canal is calculated, and a check measurement is made in the field, to confirm the correct alignment of the minor, and thus the major, canal (Fig. 19).

A correct survey grid is essential where expropriation or renting of the land which may be privately owned is to be carried out. The settlement of the land prior to canalization can be mapped against a survey grid, and the position of the canalization, boundaries of the irrigation scheme, etc., can then be sited against the same grid. With this plan it is possible to distribute to the previous landowners the compensation or rent for the area taken over for irrigation.

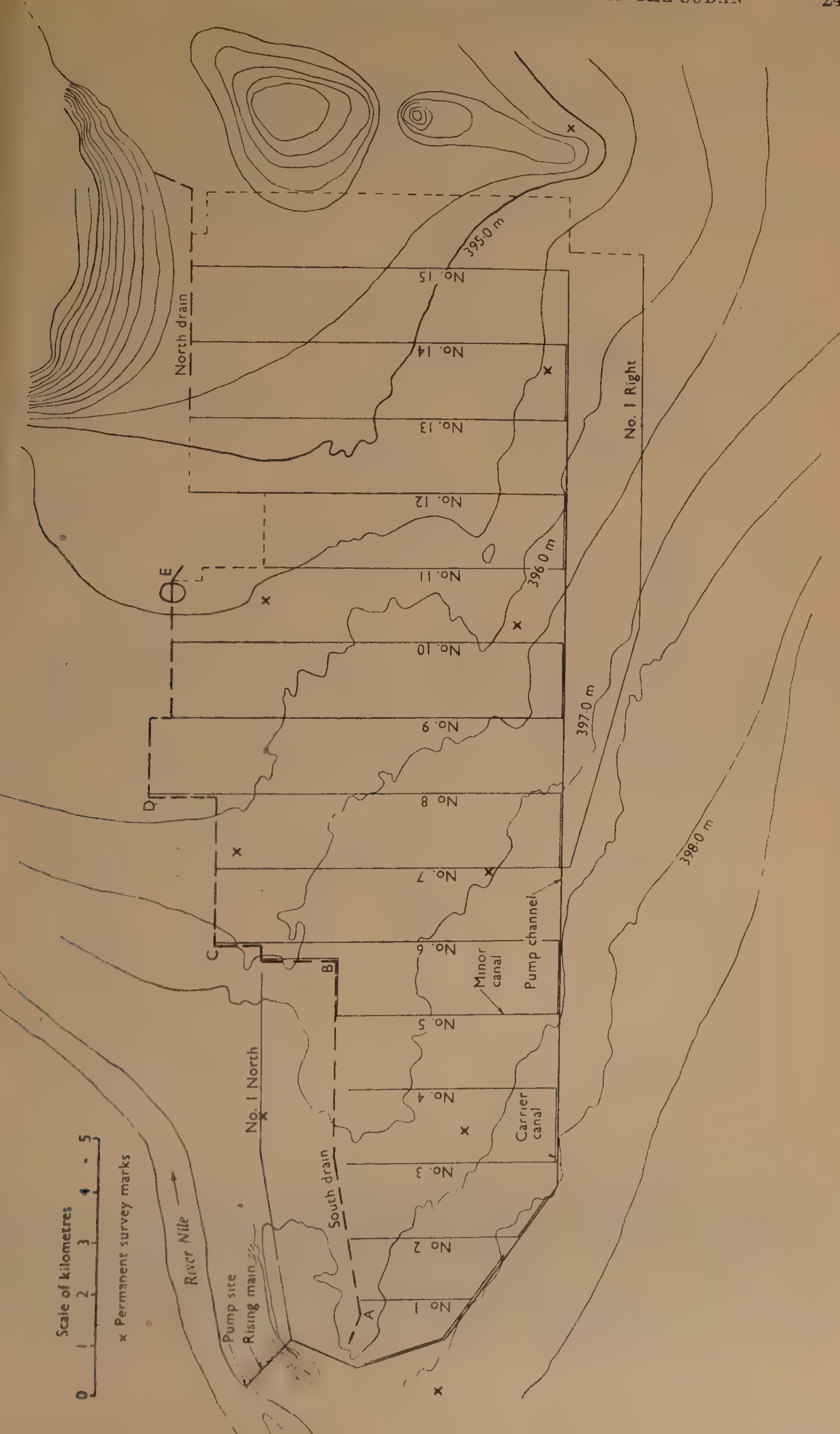


FIG. 18.—PRELIMINARY LAYOUT OF CANALIZATION: GONEID PUMP SCHEME

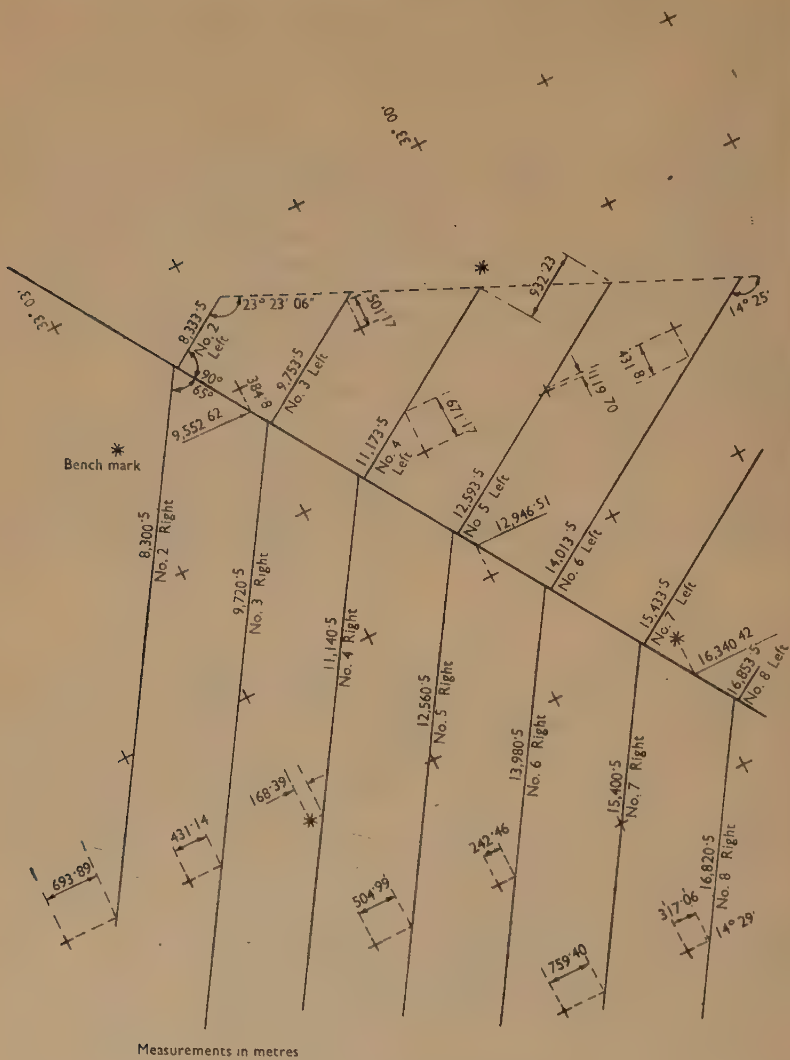


Fig. 19.—Beacon tie plan

Water requirements; canal and regulator discharges

As has been previously described, a single watering in the Gezira is taken to require a depth of 10 cm of water. This is equivalent to 420 cu. m per *feddân*. To permit two waterings each month, 900 cu. m per month, or 30 cu. m per day, are provided for each *feddân* under cultivation. The Goneid pump scheme is to crop 66% of the gross area at one time, and therefore 20 cu. m per day are required per *feddân* of

gross area. It is the practice to allow a certain latitude in the design of waterway of major and minor canals. The pumping capacity is sufficient to water 30,000 *feddâns* gross area in $18\frac{1}{2}$ hours' pumping each day; the distributary canals should pass the water in a rather shorter period each day and thus their capacity is fixed by the water required and the number of hours of watering each day.

If A denotes the gross area in *feddâns* to be watered by the canal, then the discharges for the capacity of waterways and regulators of the Goneid canal system are as given in Table 3.

TABLE 3

	Delivered per day: cu. m	Watering: hours per day	Discharge: cu. m per day
Waterway			
Pump channel	20·0 A	18	26·7 A
Minor canal	22·5 A	15	36·0 A
Regulators			
Pump channel	25·0 A	18	33·3 A
Minor canals	25·0 A	15	40·0 A

Canal design; minor canals

After setting-out in the field the lines of the pump channel and minor canals, and when these have been corrected to provide diversions round villages and other obstructions, longitudinal sections of the ground levels on the canal centre-lines are surveyed. The preliminary layout plan is then amended to show the new canal lines, and the final layout plan is completed (Fig. 20). The longitudinal section of a typical minor canal is shown in Fig. 21. This provides the following information:

Position and size of regulators
F.O. pipe offtakes to watercourses and the area served by each
Area served by the canal
Discharge
Velocity of water
Bed width
Water depth
Water slope
Bank level and slope
Full supply level and slope
Command with slope
Command flat water level

and land level on centre line of the canal.

The procedure for the canal design is to enter up the details of F.O. pipes and area served from the layout plan (Fig. 20). The water level at the first cross regulator of the watering length of the canal is then fixed to suit the levels required to command the highest ground level. As previously described, this is normally taken as 30 cm above the highest ground level, although for high ground at the head of the reach this may be reduced to 20 cm above this level to avoid additional cross regulators. The cross regulator is then sited to provide a command of not more than 60 cm on the

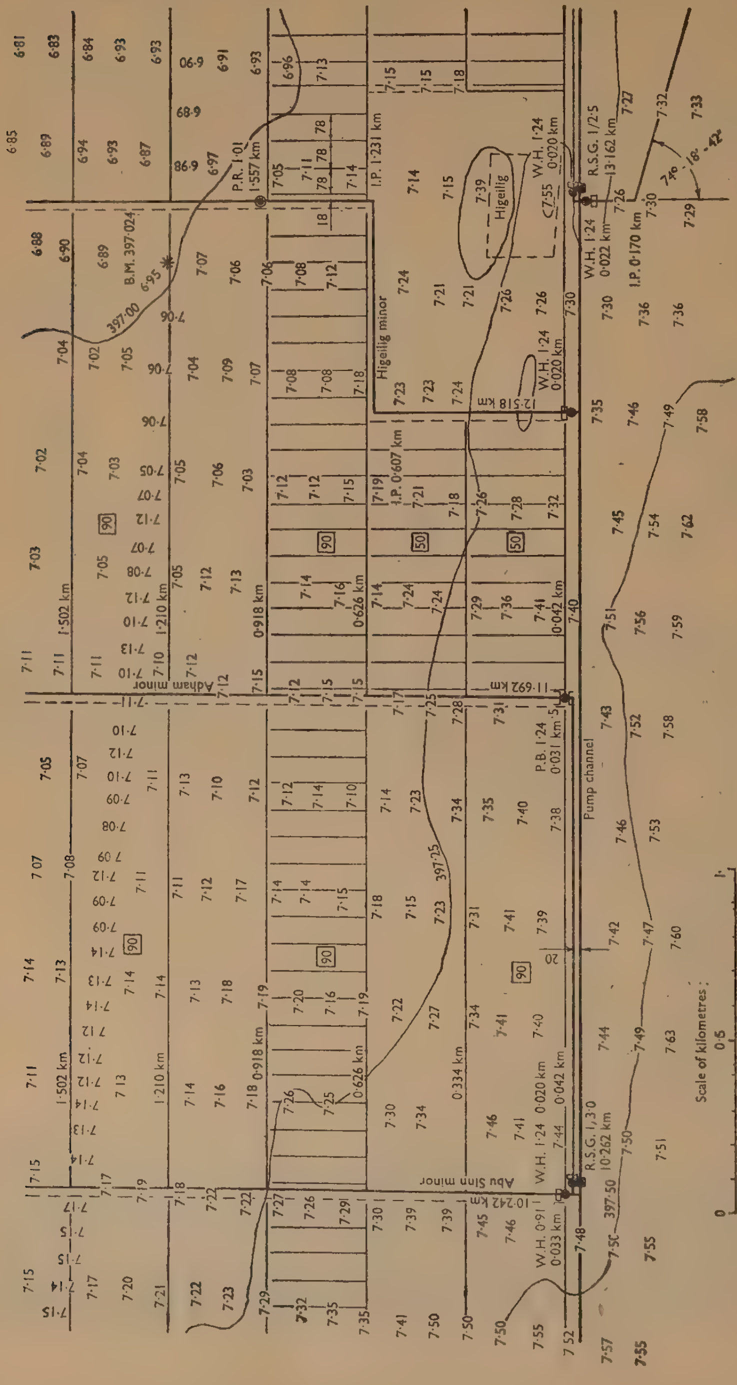


FIG. 20.—FINAL LAYOUT OF CANALIZATION : PART OF GONEID PUMP SCHEME

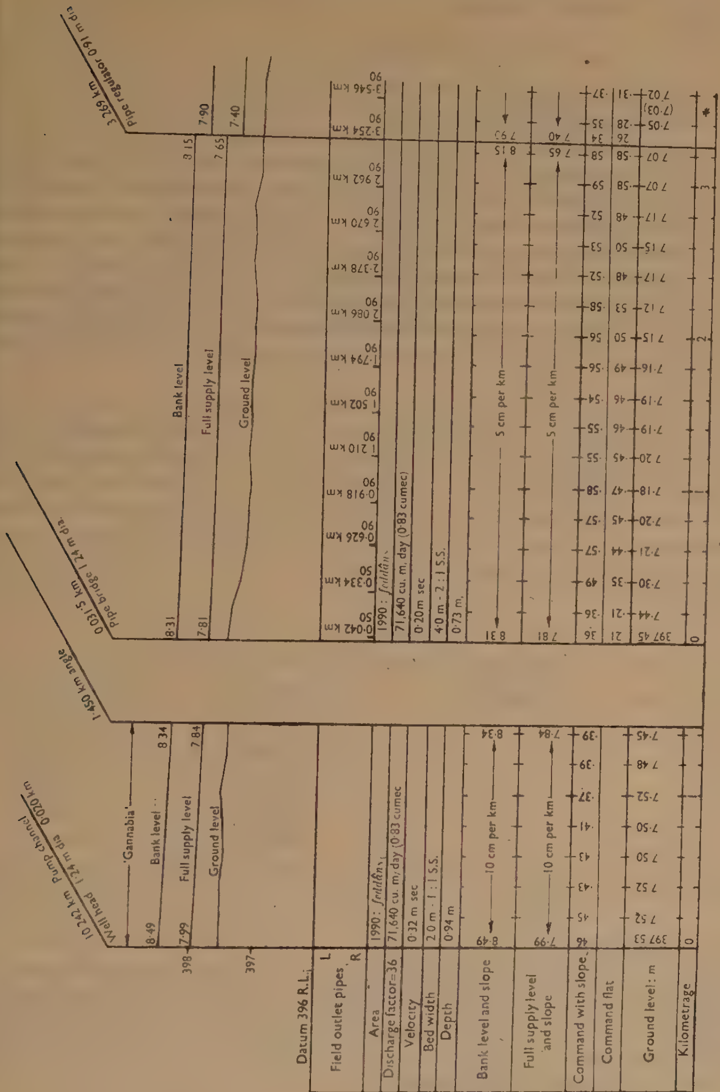


FIG. 21.—MINOR CANAL: PART LONGITUDINAL SECTION

ıch. As the excavation for first construction of minor canals is, with very occasional ceptions, to provide spoil for banks, there is no advantage in designing for a steep ter slope, and this is taken as 5 cm per km to settle the full supply level and command with slope for the reach.

The bank level is settled at 50 cm above the full supply level; the waterway section l width, depth, and velocity is calculated to pass the required discharge using nning's formula, $Q = \frac{1}{n}AR^{\frac{2}{3}}S^{\frac{1}{2}}$

where Q denotes discharge in cumecs

$\frac{1}{n} = 45$ (for Sudan Gezira conditions)

A denotes area in square metres

R ,, hydraulic radius in metres

S ,, water slope

This procedure is completed for each reach of the minor canal. The "gannabia," or single-bank carrier reach, has the water level settled by that required at the head of the first watering reach, and such water slope as can be accepted by the upstream full supply level in the pump channel. The head allowed through the pipe bridge (Fig. 17) between the carrier and watering reach is 3 cm. Although it is the usual practice to design the water section of watering canals with 2:1 side slopes, for the smaller single-bank carrier channels 1:1 side slopes are used; the same procedure is followed for obtaining bank levels, bed width, water depth, etc., as for the watering reaches of the minor canal.

The size of the cross regulators (Fig. 12) and the regulator off the pump channel (Fig. 16) is one of the five standard diameters used by the Sudan Irrigation Department and which range from 0·50 m to 1·24 m. It is the practice to allow 5 cm head through the regulator for the smaller sizes (0·50-m and 0·76-m dia.) and 10 cm head for the larger regulators. Diagrams have been established to give the calibration of these regulators, and the relation between diameter of pipe, discharge, and area served on the Goneid pump scheme is as given in Table 4.

TABLE 4

Regulator dia.: m	Design head: m	Discharge: cu. m per day	Max. area served: *feddâns*
0·50	0·05	10,800	270
0·76	0·05	32,000	800
0·91	0·10	62,000	1,550
1·01	0·10	78,000	1,950
1·24	0·10	120,000	3,000

Canal design; pump channel or major canal

As for the minor canals, the design is based on the longitudinal section from the survey of the ground levels on the centre-line of the canal. A part of the longitudinal section of the Goneid pump channel is shown in Fig. 22. This provides the following information:

Position of minor canal offtakes
Position and type of cross regulators
Area served
Discharge
Velocity
Bed width
Water depth
Water slope
Water level

Command with slope
land level
and kilometrage.

The position of the minor canal off-takes, the cross regulators and the area served s obtained from the layout plan. The discharge is then calculated from the area erved and the water requirements as described above (26·7 × area in cu. m per day). The water level at the downstream end of the reach is also settled by the requirements previously described, in that the level must command the minor canal offtakes and also provide adequate head and slope for the next reach downstream on the pump channel. The slope of the reach under design is then made as close as possible o the "Lacey" slope for a silt factor value of 0·63. The waterway is designed to pass the discharge calculated by Manning's formula, as for minor canals, and with proportions as near as possible to those derived from the Lacey formulae. If the reach 10·262 km to 13·162 km is considered in detail, then:

	"Lacey" section	*Design section*	
Discharge	6·55	6·55	cumecs
Slope	10·4	9·0	cm per km
Bed width	6·06	7·0	m
Water depth	1·42	1·47	m
Side slope of channel . .	2:1	2:1	

The Lacey dimensions are got from substituting a value of silt factor $f = 0{\cdot}63$ in the equations in metric units

$$P = 4{\cdot}836 \quad Q^{\frac{1}{2}}$$
$$A = 2{\cdot}36 \quad Q^{5/6}f^{-1/3} = 2{\cdot}64 \quad Q^{5/6}$$
$$S = 30{\cdot}87 \quad Q^{-1/6}f^{5/3} = 14{\cdot}3 \quad Q^{-1/6}$$

where P denotes perimeter of water section
A ,, area of water section
S ,, water slope in cm per km
Q ,, discharge in cumecs
f ,, Lacey silt factor

The area and perimeter of the "Lacey" section are established, and when the shape of the channel is settled, in this case a trapezoidal section with 2:1 side slopes, the water depth and bed width of the canal can be calculated.

It will be found that below a certain discharge, a trapezoidal channel with 2:1 side slopes cannot be found to suit the Lacey requirements for area and perimeter. For the smaller discharges either an elliptical section or a trapezoidal section with 1:1 side slopes can be calculated. The design section, to conform to Manning's formula for discharge, should enclose the "Lacey" section.

Having settled the water slope and dimensions of the section, the design details of the reach on the longitudinal section can be completed.

The capacity of the cross regulators should be as previously described, 33·3 A cu. m per day where A denotes the area served in *feddâns*. If an undershot gate (Fig. 15) is to be used, then the area of opening is calculated from the formula $Q = 3{\cdot}0\,A\,\sqrt{H}$

where Q denotes discharge in cumecs
A ,, area of gate opening in sq. metres
H ,, head, or difference between the upstream and downstream water levels, in metres

When A is known, a gate or battery of several gates can be selected from the standard range of gates used. It is the practice in the Sudan Irrigation Department to limit the gate opening to either 66% Groove height, or to a point where the underside of the gate is 0·60 m below the downstream water level. The range of standard sizes of undershot gates in the Sudan Irrigation Department is:

Width: metres		Height: metres
2·0	×	2·2
2·5	×	3·0
3·0	×	3·7
4·0	×	3·7

If for smaller major canal discharges a standing-wave movable weir (Series II) regulator (Fig. 14) is to be used, this should pass the discharge required of the regulator with an opening of 0·70 m out of the maximum travel of 0·84 m. The discharge is calculated from the formula $Q = 2{\cdot}3\ WD^{1\cdot 6}$ cumecs

where W denotes width of weir in metres
D „ depth over crest in metres.

With this information it is possible to select one of the standard range of movable weirs from 0·80 m to 3·0 m in width.

Surface-water drain design

In the Sudan Gezira, the cash crop, cotton, is planted in August, during the rainy season. Since a delay in sowing will reduce both the quality and the yield of the crop, and as the cotton plant is, in the early stages, likely to be killed by submersion, surface-water drains are constructed where there is flooding from rainwater. The layout of the main surface-water drains is shown in Fig. 18. As for the canals, the drain design is based on a longitudinal section which shows the following information (Fig. 22):

Ground levels
Water level and slope
Bed level
Bed width
Water depth
Catchment area
and discharge.

The catchment area is ascertained from the layout plan and contour of the area which is not under cultivation. It is the practice in the Sudan Gezira to work out the discharge from the formula $Q = 150A^{\frac{2}{3}}$

where Q denotes discharge in cu. metres per day
A „ area in *feddâns*

The design water level in the drain is a compromise that gives as good a water slope as possible and at the same time avoids excessive excavation at the outfall of the drain. If the drain is a "major" drain, carrying water from subsidiary drains, then the design water level of the major drain should be below the water level of the subsidiary drains. The design water level should so far as possible lie below the ground level, though to avoid excessive excavation, it will be necessary to allow a

"command," i.e., a design water level above ground level, of up to 0·20 m in certain low places.

Having settled the discharge required and the water slope, then a suitable section can be calculated using Manning's formula, as for canals. It has not been the practice to use the Lacey formulae for drain designs, but it is suggested by the Author that the Lacey section should be calculated for the design discharge, and then a

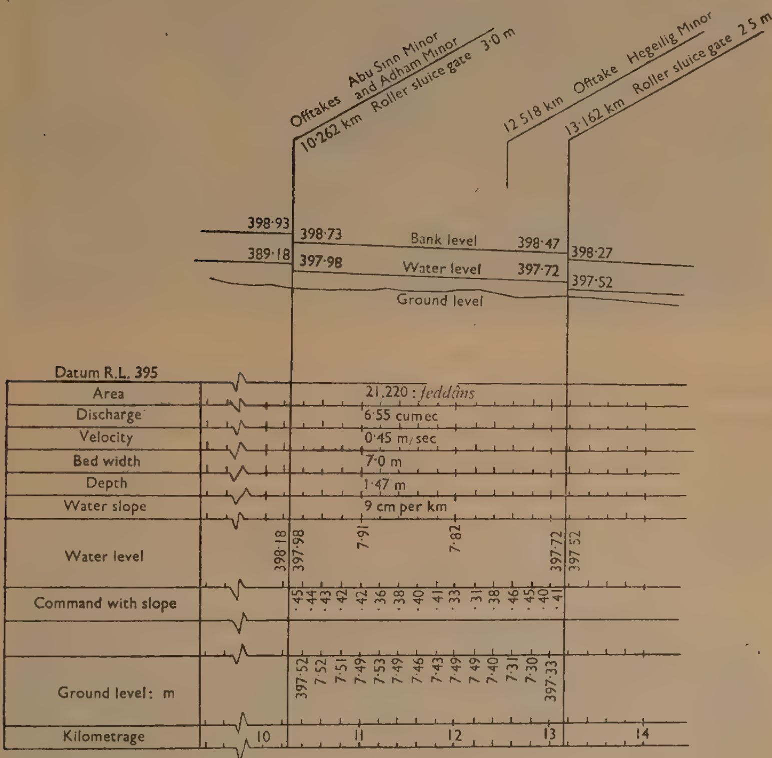


FIG. 22.—MAJOR CANAL: PART LONGITUDINAL SECTION

apezoidal section with 2:1 side slopes applied, to conform to Manning's formula d to envelop the Lacey section. It should be appreciated that the surface-water ains seldom work to the design levels and discharges. When flooding starts, the ain will have water levels well above the design levels, and as the land dries out e water slope will be steeper than design. For this reason the extremely small pacity of the drains is admissible; the design discharge allows for clearing 10% n-off from a 100-mm storm on 1,000 acres in 2·8 days and for larger catchment areas 10,000 acres 10% of a similar storm will be cleared in 6 days. Another feature ich allows for the small capacity of the drains is that the storms are usually local understorms, and there is seldom heavy rainfall over the whole catchment area.

Setting-out sheets for earthwork

These sheets are prepared from the longitudinal sections of the canals and drains to give the information necessary for setting-out the cut and banks in the field. The form of setting-out sheet for minor canals for the Goneid pump scheme was altered from previous practice to a basis of the number of cuts required by the elevating grader. This was not found satisfactory; the cube excavated by each run or cut of the elevating grader varies with the skill of the driver and the nature of the soil, and in this Paper therefore the usual type of setting-out sheet is described.

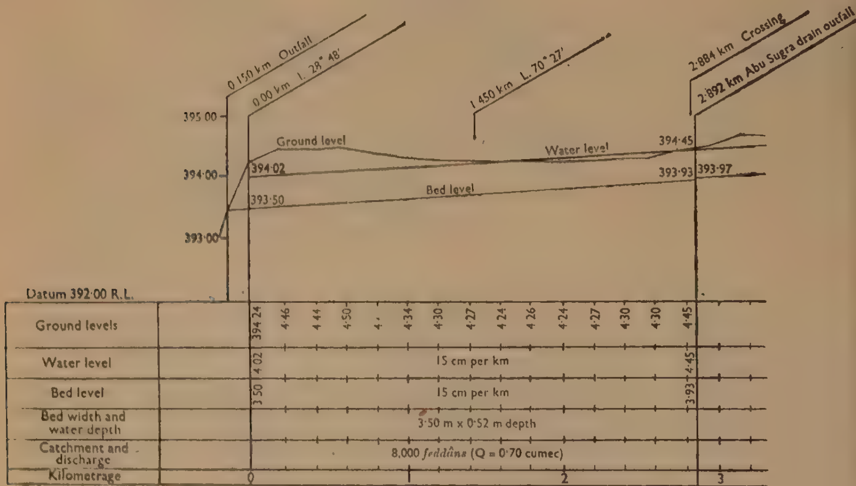


FIG. 23.—DRAIN: PART LONGITUDINAL SECTION

Typical setting-out sheets are given in Tables 5, 6, and 7. The dimensions and excavation required for making the various canal banks are calculated to conform to the types specified in Table 1. In practice, these calculations are not made for each canal, but design sheets are prepared giving the detail dimensions required at 1-cm intervals over the normal range of commands for each type of canal. For major canals (or pump channels) and for single-bank "gannabia" carrier canals it is possible that the excavation will be settled by the waterway required. It is therefore necessary to calculate the critical command for each reach, which is that command where the excavation for waterway is the same as the excavation required for making the banks. For commands below the critical command, the excavation is to the dimensions required for waterway and for commands above the critical command the excavation is that required to provide spoil for banks.

The net bank height is increased by 15% to give the gross bank height, which allows for settlement of the excavated spoil. The equivalent area of cut is obtained from the net bank area, increased by 15% for settlement, and reduced by 25% for bulking of the excavated spoil. When the excavation is for banks, the depth to dig is such that section with the design bed width will produce the "equivalent area of cut."

Fig. 15.—Undershot gate

Fig. 16.—Well-head pipe regulator

Fig. 17.—Pipe bridge

TABLE 5 (a).—SETTING-OUT SHEET (OFFICE USE)

MAJOR CANAL OR PUMP CHANNEL

Goneid pump channel—full cut for two banks

Reach	*Bed width*	*Water depth*	*Critical command*
0·170 km — 10·262 km . . .	9·0 m	1·62 m	0·50 m
10·262 km — 13·162 km . . .	7·0 m	1·47 m	0·41 m
13·162 km — 16·062 km . . .	5·5 m	1·27 m	0·27 m
etc.	*etc.*	*etc.*	*etc.*

km	Command: m	Net bank height: m	Gross bank height: m	Bank top width: m	Bank bottom width: m	Equivalent area of cut: sq. m	Depth to dig: m	Bed width: m	Type cube: cu. m
10·262			Regulator block 55 m						
10·297	0·45	1·20	1·38	2·00	7·50	10·49	1·14	7·0	1,070
10·400	0·44	1·19	1·37	2·00	7·40	10·28	1·12	7·0	2,037
10·600	0·43	1·18	1·36	2·00	7·30	10·09	1·10	7·0	1,999
10·800	0·42	1·17	1·35	2·00	7·20	9·90	1·08	7·0	1,980
11·000	0·42	1·17	1·35	2·00	7·20	9·90	1·08	7·0	2,013
11·200	0·36	1·19	1·37	2·00	7·40	10·23	1·11	7·0	2,023
11·400	0·38	1·18	1·36	2·00	7·30	10·00	1·09	7·0	1,978
11·600	0·40	1·16	1·33	2·00	7·10	9·78	1·07	7·0	1,945
11·800	0·41	1·16	1·33	2·00	7·10	9·67	1·06	7·0	2,025
12·000	0·33	1·20	1·38	2·00	7·50	10·58	1·14	7·0	2,139
12·200	0·31	1·22	1·40	2·00	7·70	10·81	1·16	7·0	2,081
12·400	0·38	1·18	1·36	2·00	7·30	10·00	1·09	7·0	2,069
12·600	0·46	1·21	1·39	2·00	7·60	10·69	1·15	7·0	2,118
12·800	0·45	1·20	1·38	2·00	7·50	10·49	1·14	7·0	2,027
13·000	0·40	1·16	1·33	2·00	7·10	9·78	1·07	7·0	1,381
13·142	0·41	1·16	1·33	2·00	7·10	9·67	1·06	7·0	
								Total reach	28,885

TABLE 5 (b).—SETTING-OUT SHEET (FIELD USE)

MAJOR CANAL OR PUMP CHANNEL

Goneid pump channel—full cut for two banks

km	Half bed width: m	Excavation					Bank dressing		
		Centre-line to							
		Top of cut: m	Inner toe: m	Outer toe: m	Depth to dig: m	Type cube cu. m	Centre-line to inner crest: m	Bank top width: m	Gross bank height: m
10·262	Regulator block 55 m								
10·297	3·50	5·78	6·92	14·42	1·14		9·32	2·00	1·38
						1,070			
10·400	3·50	5·74	6·86	14·26	1·12		9·24	2·00	1·37
						2,037			
10·600	3·50	5·70	6·80	14·10	1·10		9·16	2·00	1·36
						1,999			
10·800	3·50	5·66	6·74	13·94	1·08		9·08	2·00	1·35
						1,980			
11·000	3·50	5·66	6·74	13·94	1·08		9·08	2·00	1·35
						2,013			
11·200	3·50	5·72	6·83	14·23	1·11		9·21	2·00	1·37
						2,023			
11·400	3·50	5·68	6·77	14·07	1·09		9·13	2·00	1·36
						1,978			
11·600	3·50	5·64	6·71	13·81	1·07		9·03	2·00	1·33
						1,945			
11·800	3·50	5·62	6·68	13·78	1·06		9·00	2·00	1·33
						2,025			
12·000	3·50	5·78	6·92	14·42	1·14		9·32	2·00	1·38
						2,139			
12·200	3·50	5·82	6·98	14·68	1·16		9·42	2·00	1·40
						2,081			
12·400	3·50	5·68	6·77	14·07	1·09		9·13	2·00	1·36
						2,069			
12·600	3·50	5·80	6·95	14·55	1·15		9·37	2·00	1·39
						2,118			
12·800	3·50	5·78	6·92	14·42	1·14		9·32	2·00	1·38
						2,027			
13·000	3·50	5·64	6·71	13·81	1·07		9·03	2·00	1·33
						1,381			
13·142	3·50	5·62	6·68	13·78	1·06		9·00	2·00	1·33
					Total reach	28,885			
13·162			Regulator block—55 m						
13·197	2·75	4·87	5·93	12·17	1·06	1,670	8·05	2·00	1·22
13·400	2·75	4·93	6·02	12·42	1·09	1,645	8·20	2·00	1·25
13·600	2·75	4·87	5·93	12·17	1·06	1,597	8·05	2·00	1·22
13·800	2·75	4·83	5·87	12·07	1·04	1,559	7·97	2·00	1·21
14·000	2·75	4·79	5·81	11·93	1·02	1,550	7·87	2·00	1·18
14·200	2·75	4·81	5·84	12·00	1·03	1,583	7·92	2·00	1·20
14·400	2·75	4·87	5·93	12·17	1·06	1,645	8·05	2·00	1·22
14·600	2·75	4·95	6·05	12·45	1·10	1,775	8·23	2·00	1·25

TABLE 6 (a).—SETTING-OUT SHEET (FIELD USE) FOR MINOR CANAL, GANNABIA

Gannabia Adham (No. 6) single-bank excavation by dragline excavator. Bank top width 1·0 m

km	Command: m	Half bed width: m	Centre-line to Top of cut: m	Centre-line to Inner toe: m	Centre-line to Outer toe: m	Centre-line to Inner crest: m	Depth to dig: m	Bank height net: m	Bank height gross: m	Area of cut: sq. m	Type cube: cu. m
		Gannabia takes off pump channel 10·242 km—Well head 1·24 m—0·020 km									
0·055	0·46	1·00	2·02	2·52	8·46	4·44	1·02	0·96	1·10	3·07	223
0·200	0·45	1·00	2·00	2·50	8·35	4·40	1·00	0·95	1·09	3·00	518
0·400	0·43	1·00	1·97	2·47	8·14	4·33	0·97	0·93	1·07	2·86	572
0·600	0·43	1·00	1·97	2·47	8·14	4·33	0·97	0·93	1·07	2·86	572
0·800	0·41	1·00	1·93	2·43	7·92	4·25	0·93	0·91	1·05	2·72	544
1·000	0·37	1·00	1·86	2·36	7·49	4·10	0·86	0·87	1·00	2·46	492
1·200	0·39	1·00	1·90	2·40	7·71	4·18	0·90	0·89	1·03	2·59	563
1·435	0·39	1·00	1·90	2·40	7·71	4·18	0·90	0·89	1·03	2·59	304
1·450	Angle and pipe bridge (1·24 m)										
										Total cube:	3,788

Waterway section: Bed width 2·0 m. Water depth 0·91 m. Side slopes 1:1. Critical command: 0·25 m.

TABLE 6 (b).—SETTING-OUT SHEET, MINOR CANAL, WATERING REACH

Adham Minor (No. 6) double-bank excavation by elevating grader. Bank top width 1·0 m

km	Command; m	Half bed width: m	Centre-line to Top of cut: m	Centre-line to Inner toe: m	Centre-line to Outer toe: m	Centre-line to Inner crest: m	Depth to dig: m	Bank height net: m	Bank height gross: m	Area of cut: sq. m	Cube: cu. m
			Pipe bridge (1·24 m)—0·0315 km								
0·060	0·21	3·00	3·50	4·58	8·42	6·00	0·50	0·71	0·82	3·25	228
0·200	0·21	3·00	3·50	4·58	8·42	6·00	0·50	0·71	0·82	3·25	552
0·400	0·35	2·75	3·50	4·30	9·25	6·00	0·75	0·85	0·98	4·69	938
0·600	0·44	2·50	3·50	4·12	9·88	6·00	1·00	0·94	1·08	6·00	1,200
0·800	0·45	2·50	3·50	4·10	9·95	6·00	1·00	0·95	1·09	6·00	1,200
1·000	0·47	2·40	3·50	4·06	10·09	6·00	1·10	0·97	1·12	6·49	1,298
1·200	0·45	2·50	3·50	4·10	9·95	6·00	1·00	0·95	1·09	6·00	1,200
											and so on

Note.—For excavation by elevating grader, the depth of cut is in steps of 0·10 or 0·25 m. The field engineer arranges the run of the elevating grader and excavates to the maximum cut required on that run.

TABLE 7.—RAINWATER DRAIN

Setting-out sheet

km	Half bed-width: m	Centre-line to		Depth to dig: m	Area of cut: sq. m	Cube: cu. m
		Top of cut: m	Inner toe: m			
— 0·150	1·75	—	—	—	—	
— 0·100	1·75	2·15	4·15	0·20	0·78	98
0·000	1·75	3·23	5·23	0·74	3·68	736
0·200	1·75	3·61	5·61	0·93	4·98	996
0·400	1·75	3·51	5·51	0·88	4·63	926
0·600	1·75	3·57	5·57	0·91	4·84	968
0·800	1·75	3·37	5·37	0·81	4·15	830
1·000	1·75	3·13	5·13	0·69	3·37	674
1·200	1·75	2·99	4·99	0·62	2·94	588
						and so on

CONCLUSION

The Sudan Irrigation Department, in addition to the construction of the structures and canals necessary for the scheme, also builds the necessary houses and road bridges, and, in fact, completes the project. The style and method of construction of these ancillary works will vary according to the locality, as also will the design of the irrigation structures. The Author in this Paper has concentrated on the canalization, for which methods of design and construction have been developed over many years in the Sudan, and it is thought that this information may be of general use for the design and construction of irrigation schemes.

ACKNOWLEDGEMENT

The Author wishes to record his thanks to the staff of Projects Division of the Sudan Irrigation Department, who undertook the detail design work of the canalization of the Goneid pump scheme, and especially to Mohed. Eff. Kubawi and Mohed. Eff. Rashid Sid Ahmed.

The Author also wishes to record his thanks to the Director, Ministry of Irrigation, Sudan Government, for permission to publish this Paper, and to Mr P. Hall, B.Sc.(Eng.), M.I.C.E., lately Assistant Director, Development, whose design sheets are the basis of design of canalization in the Ministry of Irrigation.

TABLE OF APPROXIMATE CONVERSION FACTORS

1 centimetre	=	0·39 inch
1 metre	=	39·4 inches
,, ,,	=	3·28 feet
1 kilometre	=	0·6214 mile
,, ,,	=	1,094 yards
,, ,,	=	3,280 feet
1 sq. m	=	10·76 sq. ft
1 cu. m	=	35·31 cu. ft
,, ,, ,,	=	1·31 cu. yd
1 inch	=	2·54 centimetres
1 foot	=	0·305 metre
1 yard	=	0·914 ,,
1 mile	=	1·61 kilometre
1 sq. ft	=	0·09 sq. m
1 cu. yd	=	0·76 cu. m

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The Paper, which was received on 10 October, 1955, is accompanied by sixteen diagrams and seven photographs from which the half-tone page plates and the Figures in the text have been prepared.

CORRESPONDENCE on this Paper should be forwarded to reach the Institution before 15 August, 1956. Contributions should not exceed 1,200 words.—SEC.

CORRESPONDENCE

on a Paper published in Proceedings, Part III, August 1955

Paper No. 6039

" Sagging twin 0·4-sq.-in. copper equivalent S.C.A. conductor " †

by

Graham Philip Sturton, B.Sc.(Eng.), Grad.I.C.E.

Correspondence

Mr N. G. Simpson (Transmission Consultant, British Insulated Callenders Construction Co. Ltd) observed that the conductor stringing for the majority of overhead lines was now a well-established technique. The advent of the British Super Grid, however, by the adoption of large twin S.C.A. conductors on long spans had warranted the consideration of hitherto unimportant factors and the general application of a greater degree of precision to erection methods. The Paper dealt effectively with those aspects of the problem.

On the question of overtensioning, some doubt might well be expressed as to the usefulness of reducing the sag of the conductors by as little as 25% for so short a time as 20 min. The operation of overtensioning was intended to deal with two problems; to bed down the conductor strands; and to reach the stage when the conductor behaved according to elastic modulus which remained constant throughout the conductor's life. To do that the conductor had to be loaded up to a tension which would not be exceeded during its life and was, in fact, something like the maximum working tension. The bedding down of the strands was dealt with in the normal operation of running out and tensioning. To deal effectively and quickly with the second point, it would be necessary to impose a complicated and expensive construction operation, with attendant hazards to clamps, pulling-up gear, and possibly to structures. In any case Mr Simpson believed that it was unnecessary for it could be dealt with in time by the conductor itself under service conditions. Some time before the programme of actual stringing of those large conductors he had been concerned with the installation of an experimental line at his firm's own works, when several spans had been erected with three sets of 0·4-sq. in.-S.C.A. conductors complete with insulators and spacers. The object had been to perform handling and running-out trials, to string the conductors to 12½% more than their correct tension, and finally over a period to take observations of sag variations, conductor twist, and vibration aspects. Furthermore the conductors for the trial had been manufactured at two different factories from machines using different methods of stranding so that a comparison could be made as to their subsequent behaviour under similar stringing conditions. Observations over a period of 14 months had shown no significant difference in the rate of settlement or twist, despite the two different stranding operations. With regard to the rate of settlement of the conductors arising from the application of the 12½% overtensioning, rather more than 30% of the settlement had occurred over the first 48 hours and practically

† Proc. Instn Civ. Engrs, Part III, vol. 4, p. 549 (Aug. 1955).

the whole of the remaining settlement during the following 6 months, with little or no further settlement during the following 8 months.

The experimental line consisted of three spans and the consistency of behaviour appeared different from that indicated in Fig. 3, where there was as much as 4 in. difference in sag between conductors Nos 1 and 2 after about 30 days. However, in actual practice sections were longer, and inconsistencies were likely to be more apparent.

The efficiency with which sags could be adjusted and conductors lined up with twin conductors depended largely on the use of carefully designed and high-quality large-diameter freely moving sheave blocks. He thought that aspect of construction might with advantage have been brought out with more emphasis in the Paper. Turning then to the section devoted to sag correction, and in particular to Fig. 1, an obviously new departure in the technique of sag adjustment could not be criticized in principle, but he wondered if such precision was warranted in practice in view of other uncertainties. If the sheave blocks operated efficiently the need in any case for that rather complicated operation would be lessened. Incidentally if the spans of a section were reasonably equal the adjustments which the Author made should presumably balance out. What was the net adjustment after allowances have been made for each span?

Perhaps the most interesting, and for that matter controversial, parts of the Paper were those devoted to tower deflexions and the extent to which the Author made allowance for them in his sagging operations. Once again the principle could not be criticized, but when several other practical aspects and uncertainties were considered it appeared a little doubtful whether the end justified the means. For example, had account been taken of the variations in tower height which were considerable in some cases? It was possible to have a difference between minimum and maximum height of about 60 ft. There was also some variation in conductor erection tension for sections with different equivalent spans and erection temperatures; for example, for sections with an equivalent span of 600 ft that tension at 40°F was 7,700 lb. The comparable figure for 1,500 ft at 80°F was 5,700 lb. They were admittedly extreme cases, but the effects on the deflexion would also vary in a similar manner.

In addition, when initially loading a tower there was liable to be some settlement of the foundations, which in turn caused tower deflexion. Since tower deflexions were dependent upon the reactions provided by the backstays and since, in practice, the length, slope, and direction of the stays varied owing to local site conditions, the value of the reactions and corresponding tower deflexion would also change. In the Paper it was inferred that the deflexion of any one type of tower was always the same. Would the Author confirm whether that inference was correct, and if not would he say how those factors had been taken into account?

In similar work with which Mr Simpson had been concerned the use of backstays for sagging operations had been reduced to almost negligible proportions. He agreed wholeheartedly with the principle of stringing conductors at one level at one time to avoid torsional effects. Apart from that he adopted a "leap-frog" method of stringing to minimize the effects of tower deflexion. For example, he strung the earthwire and top of four conductors on one section, then moved to the adjacent section and repeated the operation. A return was then made to the first section to string the lower conductors. Backstaying was largely dispensed with, so avoiding expense and ground disturbance. On the other hand a certain amount of delay was incurred in switching gangs and operations, but overall he regarded that procedure with the minimum use of backstays to be more economical and technically satisfactory.

Finally there was one important aspect which would appear to negative to a large extent the method whereby the Author took into account that aspect of tower deflexions. The effect which a certain linear tower deflexion had on the sags of adjoining conductor sections would vary considerably with the length of those sections. Taking an extreme case for the purposes of an example, that effect was liable to be very great in the case of a short single-span section and negligible in a long multi-span section. That was apparent when it was realized that the difference between the conductor and section lengths, in the

case of a 500-ft single span, compared with a section length of 12,000 ft at 40°F, was 1·2 in. and 242 in. respectively.

Having all those aspects in mind Mr Simpson suggested that with a selective plan of stringing as between sections, tower deflexions might be ignored for long section lengths, but should definitely be considered, perhaps with even greater accuracy, for abnormally short or single-span sections.

The Author, in reply, referred to over tensioning and explained that the figure of 25% given in the Paper referred to erection tensions which were 15% higher than design tensions on the line concerned; that resulted in overtensioning to approximately 40% of design tension. The real purpose of that overtensioning was to impose on the conductors equal loads of such magnitude that no subsequent treatment during the sagging operations would be likely to cause unequal non-elastic stretch.

He was interested to learn of the experimental line erected by Mr Simpson's firm. If full details and results could be published they would be very useful. Observations of non-elastic stretch of the conductor referred to by the Author had been continued. The results had been given by Mr McCullagh.[1] It was found that even after 12 months the sags were still increasing. At the end of that time the increase in sag for a single span of 712 ft was 28% and the increase in sag for a span in a section of ruling length of 1,177 ft was 15%.

The running-out blocks used on construction were of very good quality and consisted of light aluminium sheaves mounted on phosphor-bronze roller bearings. The required net adjustment calculated from the sag-correction chart would vary under differing constructional conditions. To quote an example, if one conductor of a pair was, say, 3 in. lower than its mate in five spans of 1,200 ft in a section being strung, then 5 × 0·4 in., i.e., 2 in., would be the net "cut-out" allowance.

All observations were made on standard-height towers, for those made up the vast majority of the route. It was agreed that towers with excessive extensions in very long or short spans would not obey the general rules but such extreme cases were rare. The variation in reaction of backstays arising from different site conditions would be nullified by tensioning the backstays until the required tower deflexions were obtained. The deflexions of towers of the same type were found to be reasonably consistent. The specified earth pressures were so low that settlement of foundations under erection conditions should be negligible.

It was noted with interest that Mr Simpson's firm still used the "leap-frog" method of stringing. The Author's firm had developed a system of running out the six conductors on both circuits simultaneously and to continue that system with "leap-frog" sagging would necessitate leaving conductors lying on the ground for longer periods with consequent increased risk of damage. It would be interesting to experiment with sagging adjacent sections simultaneously, thus entirely eliminating the need for backstays on half the section towers of a line. That would enable two "sagging" gangs to operate at the same time and might possibly be the most economical system of all.

The Author agreed that in extremely long sections the harmful effect of tower deflexions would be less noticeable on the sags concerned, but considered that it was worthwhile correcting for them in all cases.

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[1] G. R. McCullagh, "Overhead Line Erection." *Electrical Review*, Dec. 1955.

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